

Open Research Online

The Open University's repository of research publications and other research outputs

Interpretations of a constructivist philosophy in mathematics teaching

Thesis

How to cite:

Jaworski, Barbara (1991). Interpretations of a constructivist philosophy in mathematics teaching. PhD thesis The Open University.

For guidance on citations see [FAQs](#).

© 1991 The Author



<https://creativecommons.org/licenses/by-nc-nd/4.0/>

Version: Version of Record

Link(s) to article on publisher's website:

<http://dx.doi.org/doi:10.21954/ou.ro.0000dff0>

Copyright and Moral Rights for the articles on this site are retained by the individual authors and/or other copyright owners. For more information on Open Research Online's data [policy](#) on reuse of materials please consult the policies page.

oro.open.ac.uk

DX 94611

UNRESTRICTED

**Interpretations
of a
constructivist philosophy
in
mathematics teaching**

Barbara Jaworski BSc BA

**Thesis offered for the degree of
Doctor of Philosophy
of the
Open University
in the discipline of
Mathematics Education**

May 1991

Author's number: E0328516

Date of submission: 14th May 1991

Date of award 9th August 1991

To
John

Acknowledgements

This study owes a great deal to the teachers who were subjects of my classroom research. To Felicity, Jane, Clare, Mike, Ben and Simon, my very sincere thanks for their interest and cooperation, and for the time which they so generously gave.

Many of my colleagues have played an important part in terms of support and encouragement and extreme tolerance. I have particularly appreciated their willingness to listen, to talk over ideas and to offer their perceptions. In particular I should like to thank all members of the Centre for Mathematics Education at the Open University and my close colleagues in the School of Education at the University of Birmingham. It would be impossible to list everyone who has been of help, but I should like especially to thank Peter Gates, Sheila Hirst, David Pimm, Stephanie Prestage, Brian Tuck and Anne Watson.

Three people are owed especial gratitude:

John Mason, who has been an inspiration over many years and who has ever been willing to engage with ideas and offer his own particular gift of enabling me to reconstruct what I know for greater sense and coherence.

Christine Shiu, who has supervised this study giving generously of her time and friendship, experience and sensitivity, and her own particular brand of care and attention to detail.

John Jaworski, who has given not only his love and forbearance, humour and support, but also his time and expertise in the presentation of this manuscript.

I shall not be forgiven if I do not give credit to *Princess Boris-in-Ossory* and *Frankincense* for their contribution to this work. They were mainly responsible for my not getting cold feet during the many drafts of this thesis.

ABSTRACT

This thesis is a *research biography* which reports a study of mathematics teaching. It involves research into the classroom teaching of mathematics of six teachers, and into their associated beliefs and motivations. The teachers were selected because they gave evidence of employing *an investigative approach to mathematics teaching*, according to the researcher's perspective. A research aim was to characterise such an approach through the practice of these teachers.

An investigative approach was seen to be embedded in a *radical constructivist* philosophy of knowledge and learning. Observations and analysis were undertaken from a constructivist perspective and interpretations made were related to this perspective.

Research methodology was *ethnographic* in form, using techniques of *participant-observation* and *informal interviewing* for data collection, and *triangulation* and *respondent validation* for verification of analysis. Analysis was qualitative, leading to emergent theory requiring reconciliation with a constructivist theoretical base. Rigour was sought by research being undertaken from a *researcher-as-instrument* position, with the production of a *reflexive account* in which interpretations were accounted for in terms of their context and the perceptions of the various participants including those of the researcher.

Research showed that those teachers who could be seen to operate from a constructivist philosophy regularly made high level cognitive demands which resulted in the incidence of high level mathematical processes and thinking skills in their pupils.

Levels of interpretation within the study led to the identification of *investigative teaching* both as *a style of mathematics teaching* and as *a form of reflective practice in the teaching of mathematics*. These forms were synthesised as *a constructivist pedagogy* and as *an epistemology for practice* which may be seen to forge links between the theory of mathematics teaching and its practice.

The research is seen to have implications for the teaching of mathematics, and for the development of mathematics teaching itself through professional development of mathematics teachers.

In the halls of memory we bear the images of things once perceived, as memorials which we can contemplate mentally, and can speak of with a good conscience and without lying. But these memorials belong to us privately. If anyone hears me speak of them, provided he has seen them himself, he does not learn from my words, but recognises the truth of what I say by the images which he has in his own memory. But if he has not had these sensations, obviously he believes my words rather than learns from them. When we have to do with things which we behold in the mind ... we speak of things which we look upon directly in the inner light of truth ...

(St. Augustine, *De Magistro*, 4th century AD¹)

We can, and I think must, look upon human life as chiefly a vast interpretive process in which people, singly and collectively, guide themselves by defining the objects, events, and situations which they encounter. ... Any scheme designed to analyse human group life in its general character has to fit this process of interpretation.

(Blumer, 1956, p 686²)

¹The St. Augustine quotation is taken from H.S.Burleigh (ed.) *Augustine: Earlier writings*, Westminster Press p 96

² Quoted in Denzin, 1978

TABLE OF CONTENTS

PART ONE – THEORY

CHAPTER 1 – BACKGROUND AND RATIONALE	1
<i>An investigative approach</i>	2
The origins of investigations	2
The purposes of investigations	2
The status of investigational work in mathematics teaching	5
An investigative approach to mathematics teaching	7
<i>The research study</i>	9
A statement of purpose	9
The structure of this thesis	10
The fieldwork	10
My own position in the research	11
the contribution of the study	12
 CHAPTER 2 CONSTRUCTIVISM	 13
<i>What Constructivism is</i>	13
<i>Constructivism and knowledge</i>	16
<i>Constructivism, meaning and communication</i>	17
<i>Constructivism and the classroom</i>	23
<i>Challenges to constructivism</i>	25
 CHAPTER 3 THE TEACHING OF MATHEMATICS	 31
<i>The implications of learning for teaching</i>	31
The influence of Piaget	31
Construction of mathematical concepts	33
Hierarchies of mathematical concepts	35
Two kinds of learning	39
Pupil construal and its recognition	41
<i>The role of the teacher for mathematical learning</i>	43
The Zone of Proximal Development	43
Language and the social environment	46
<i>The trouble with mathematics teaching</i>	54
<i>My own study</i>	59

PART TWO – RESEARCH

CHAPTER 4 METHODOLOGY	61
<i>Introduction</i>	61
<i>An initial choice</i>	61
<i>Ethnography, or an ethnographic approach</i>	63
<i>The place of theory in an ethnographic approach</i>	66
<i>Validation and Rigour</i>	69
<i>My own study</i>	71
Interpretive enquiry	73
Data collection	74
Data analysis	79
Theoretical perspectives	82
Verification	84
Terminology	86
 CHAPTER 5 PHASE ONE RESEARCH	 87
<i>Introduction</i>	87
<i>Stages of Involvement in the Phase 1 work</i>	87
Stage1.–The introductory work	87
Stage2.–Teaching lessons myself	88
Stage3.–Pairs of lessons of the two teachers	88
Data collection and analysis across the three stages	89
<i>Analysis and reflection</i>	89
Stage 1 – Initial Observations	89
Stage 2 – Teaching lessons myself, observing others	91
Stage 3 – Pairs of lessons taught by Felicity and Jane	99
<i>Conclusions</i>	108
 INTERLUDE A	 111
<i>The ‘fit’ with radical constructivism</i>	112
Relating constructivism to teacher-development	114
<i>Implications for Phase 2</i>	115
 CHAPTER 6 PHASE TWO RESEARCH	 117
<i>Introduction</i>	117
<i>Methodology</i>	119
Data Collection	119
Data Analysis	121

<i>The study of Clare's teaching</i>	122
Introduction	122
Lessons from which data were collected	122
Analysis of the Autumn term lessons	123
Analysis of recorded lessons	137
Students' and Teacher's views	150
Concluding my characterisation of Clare's teaching	155
<i>The study of Mike's teaching</i>	160
Introduction	160
Lessons from which data were collected	160
Management of Learning	160
Sensitivity to Students	167
Mathematical Challenge	174
Mike's own thinking – and responses from students	179
<i>Conclusion to the chapter</i>	185
 <i>INTERLUDE B</i>	 189
Teacher and researcher awareness	189
Recognition of an issue - the teacher's dilemma	192
The researcher's dilemma	194
Significance	195
Implications for Phase 3	197
 <i>CHAPTER 7 PHASE THREE RESEARCH</i>	 201
Introduction	201
Methodology	202
Data Collection	202
Data Analysis	203
The Study of Ben's Teaching	204
Lessons from which data were collected	204
The teaching triad	205
Didactic versus Investigative teaching	207
The Moving Squares lesson	211
The Vectors lesson	229
Tensions and Issues	242
The teacher's dilemma	242
The didactic/constructivist tension	245
The didactic tension	247
Constructivism and the Teaching Triad	252
The Study of Simon's Teaching	254
Lessons from which data were collected	254
Consolidation of graphs	255
The teaching triad	268
Conclusion	269
Conclusion to the Phase 3 work	271

PART THREE – CONSEQUENCES AND CONCLUSIONS

CHAPTER 8 REFLECTIVE PRACTICE	275
<i>Thinking and reflection</i>	276
<i>The teacher-researcher relationship</i>	278
Stage 1 – Reflecting	280
Stage 2 – Accounting for	282
Stage 3 – Critical analysis	286
<i>Reflecting on the conceptual model</i>	291
<i>Teaching knowledge and Teaching wisdom</i>	293
<i>The reflective teacher</i>	294
<i>Developing investigative teaching</i>	297
<i>My own development as a reflective practitioner</i>	299
<i>Reflective practice is ‘critical’ and demands ‘action’</i>	302
<i>Conclusion</i>	304
 CHAPTER 9 CHARACTERISTICS OF AN INVESTIGATIVE APPROACH	 307
<i>The classrooms</i>	307
<i>The teaching role</i>	313
Establishing meaning	315
Engendering mutual trust and respect	316
Encouraging responsibility for own learning	317
Establishing an investigative approach	319
<i>The wider issues</i>	320
Building of mathematical concepts	320
Social issues	321
<i>Conclusion</i>	322
 CHAPTER 10 BRINGING THEORY CLOSER TO PRACTICE	 323
<i>Introduction</i>	323
<i>A constructivist pedagogy</i>	324
The teaching triad	326
Teacher-pupil interactions	328
Tensions and issues	329
<i>An epistemology for practice</i>	330
<i>A critical appreciation of this study</i>	332
<i>Methodological implications</i>	333
<i>Future directions</i>	334
<i>Conclusion</i>	335

REFERENCES 337

APPENDICES

1 – Chronology and conventions	1A
2 – Constructivism – a historical perspective	5A
3 – Phase One lessons	9A
4 – Phase Two lessons	25A
5 – Phase Three lessons	53A

TABLE OF FIGURES

3.1	The fruit metaphor	42
6.1	Research chronology	121
6.2	Lessons observed with Clare	123
6.3	Clare's diagram	134
6.4	The teaching triad	155
6.5	Lessons observed with Mike	160
B.1	Links between theory and practice	199
7.1	Lessons observed with Ben	205
7.2	Ben's view of the teaching triad	207
7.3	Moving Squares	212
7.4	Pat's diagram	231
7.5	Drawing a vector	232
7.6	Colin's drawing	233
7.7	Ben's diagram	235
7.8	Luke's explanation to Danny	239
7.9	Lessons observed with Simon	254
7.10	Linking the triad with constructivism	271
7.11	Constructivism and the triad	272
7.12	Elaborating the teaching triad	272
8.1	The teacher-researcher relationship	279
8.2	The reflective process for the teacher	296
9.1	Characteristics of an investigative approach	320

TABLE OF DATA ITEMS

4.1	Research methods – a summary	71
4.2	Significance	80
4.3	Thoughts in preparation	82
5.1	Use of my own terminology	89
5.2	An early significant event	90
5.3	Current thinking	91
5.4	Observations in an early lesson	92
5.5	Felicity's comments	93
5.6	My questions to Felicity	94
5.7	Felicity's response	94
5.8	Aims for lesson 1	95
5.9	Reflecting on my first lesson	96
5.10	Tessellations	100
5.11	Conversations with teachers	103
5.12	Use of transcript for verification (1)	105
5.13	Use of transcript for verification (2)	107
6.1	Some of Clare's words from lesson 1	124
6.2	More of Clare's words from lesson 1	124
6.3	Statements made by Clare during Fractions	126
6.4	Excerpt from field notes in Fractions 2	129
6.5	Talking with Nigel in lesson 2	131
6.6	Remarks on students after Fractions 1	132
6.7	It's a cuboid	138
6.8	Lines crossing	144
6.9	Jaime	148
6.10	Introduction to Billiards	161
6.11	Continuing with Billiards	165
6.12	But what do you do?	168
6.13	Is it accurate?	171
6.14	Phil 1	173
6.15	Phil 2	177
6.16	Trust	181
6.17	Is there an answer?	182
6.18	Does the teacher know the answer?	182
B.1	Sense-making	189
B.2	Hypotheses	191
B.3	Clare – prodding and guiding	193
B.4	Essence of teaching	196
7.1	Management of Learning	206
7.2	Very didactic	208
7.3	A conjecture I agree with	209
7.4	Planning for the Moving Squares lesson	212
7.5	Can we move diagonally?	214

7.6	You're the teacher, aren't you?	214
7.7	Freedom v Control	217
7.8	The highest authority in the classroom	218
7.9	Two by two equals five	221
7.10	Teacher knows the answer	223
7.11	Pupils' views of investigating	225
7.12	Drawing a vector	231
7.13	Making questions more interesting	237
7.14	Making incorrect exam answers explicit	250
7.15	Introduction to consolidation of graphs	257
7.16	Reflections on transcribing audiotape	258
7.17	Explaining	261
8.1	Luke's $3AB = AB + 2AB$	281
8.2	A threatening question triggers reflection	284
8.3	Difficult questions	285
8.4	Anticipating a question	285
8.5	Threatening questions	286
8.6	Just a cut-and-stick lesson	287

PART 1

THEORY

CHAPTER 1

BACKGROUND AND RATIONALE

The research which is reported in this thesis is a study of mathematics teaching. It involved participant observation of the classroom practice of six secondary mathematics teachers and extensive exploration of their motivations and beliefs. It began as an enquiry into an investigative approach to the teaching of mathematics – the teachers studied employed a classroom approach which could be described as investigative according to popular connotations in the mathematics education community in the U.K. which have developed over several decades. It consists of interpretations, made from a constructivist perspective, of the events which took place in a number of mathematics lessons and the beliefs which motivated these events; also of issues arising from the interpretations made. The relationship between the researcher and the teachers, and their respective development of knowledge and practice, played an important role in the study which led to considerations of the relationship between investigative teaching and reflective practice.

Throughout this study, the constructivist philosophy on which interpretations are based is my own. In particular, in speaking of teachers as operating from within a constructivist philosophy, it must be clear that this is my judgement. However, a major thrust of my research has been the pursuit of perspectives of the teachers, which has involved *their* interpretations of events in which we participated. Associated with this are interpretations by pupils of the events in which they too have participated. Eisenhart (1988) states that ‘the researcher must be involved in the activity as an insider, and able to reflect on it as an outsider’. So, it is my task, as researcher, to ‘make that world understandable to outsiders, especially the research community’ (Eisenhart, 1988). This thesis is a reflexive account (e.g. Ball, 1990) of my study in which I juxtapose interpretations with details of the methodology and thinking which has led to these interpretations. It is in this that the rigour of the research lies. However, the reader is no less an interpreter, and what is construed, finally, will be the reader’s interpretation.

An investigative approach

THE ORIGINS OF INVESTIGATIONS

In contrast to the tasks set by the teacher – doing exercises, learning definitions, following worked examples – in mathematical activity the thinking, decisions, projects undertaken were under the control of the learner. It was the learner's activity. (Love, 1988, p 249)

Mathematical activity is Eric Love's term for a type of activity which was propagated in the United Kingdom during the 1960s and has come to be known subsequently as *mathematical investigation*. Children worked on loosely-defined problems, asking their own questions, following their own interests and inclinations, setting their own goals and doing their own mathematics. According to Love, the teachers involved 'viewed mathematics as a field for enquiry, rather than a pre-existing subject to be learned.' He makes the point that in this activity the children's work was seen as paralleling that of professional mathematicians, with the teacher's role involving provision of starting points or situations 'intended to initiate constructive activity'.

Such activity became more widespread through teacher-education courses in colleges and universities, and through workshops organised by the Association of Teachers of Mathematics (ATM). Particular activities or starting points became popular, and potential outcomes began to be recognised. For example a certain formula could be expected to emerge, or a particular area of mathematics might be addressed. Sometimes the outcomes were seen to be valuable in terms of the processes or strategies which they encouraged. In the beginning, people working *on* some initial problem or starting point could be seen to be *investigating* it, but over time, the object on which they worked came to be known as '*an investigation*'. Sometimes the term 'investigation' included also the strategies employed and the outcome achieved.

THE PURPOSES OF INVESTIGATIONS

There were many rationales for undertaking investigations in the classroom. Investigations could be seen to be *more fun* than 'normal' mathematical activity. Thus they might be undertaken as a treat, or on a Friday afternoon. They might be seen to promote more truly mathematical behaviour in pupils than a diet of traditional topics and exercises. They might be seen to promote the development of valuable mathematical processes which could then be applied in other mathematical work. They

could be seen as an alternative, even a more effective, means of bringing pupils up against traditional mathematical topics.

There were differing emphases, depending on which of these rationales motivated the choice of activity. For example, where investigations were employed as a Friday afternoon activity they were often done for their own sake. What mattered was the outcome of the particular investigation, and the activity and enjoyment of the pupils in working on it. It was taken less seriously than usual mathematical work. However, where the promotion of mathematical behaviour, or of versatile mathematical strategies was concerned, the investigation was just a *vehicle* for other learning.

This *other learning* might be seen as learning to *be mathematical*. Wheeler (1982) speaks of 'the process by which mathematics is brought into being', calling it *mathematization*:

Although mathematization must be presumed present in all cases of 'doing' mathematics or 'thinking' mathematically, it can be detected most easily in situations where something not obviously mathematical is being converted into something that most obviously is. We may think of a young child playing with blocks, and using them to express awareness of symmetry, of an older child experimenting with a geoboard and becoming interested in the relationship between the areas of the triangles he can make, an adult noticing a building under construction and asking himself questions about the design etc.

... we notice that mathematization has taken place by the signs of organisation, of form, of additional structure, given to a situation.

Wheeler elaborates by offering clues to the presence of mathematization under the headings of *structuration*, *dependence*, *infinity*, *making distinctions*, *extrapolating and iterating*, *generating equivalence through transformation*. For example, he suggests that 'searching for pattern' and 'modelling a situation' are phrases which 'grope' towards structuration; that, as Poincaré pointed out, all mathematical notions are concerned with infinity – the search for generalisability being part of this thrust. Others have tried to be more precise about elements of mathematization, offering the student sets of processes, strategies or heuristics through which to guide mathematical thinking. Most notable was George Polya whose famous film 'Let us teach guessing' promoted *guess and test* routines and encouraged students first to get involved with a problem then to refine their initial thinking. He offered, for example, stages in tackling problems: *understanding a problem*, *devising a plan*, *carrying out the plan*, *looking back* (1945 p xvi); or ways of seeing or looking at a problem: *mobilization*; *prevision*; *more parts suggest the*

whole stronger; recognising; regrouping; working from the inside, working from the outside (1962, Vol II p 73). He advised students that 'The aim of this book is to improve your working habits. In fact, however, only you yourself can improve your own habits' (ibid). In similar spirit were processes or stages of operation offered by Davis and Hersh (1981) and by Schoenfeld (1985). In the U.K., much work in this area has been done by John Mason who has suggested that *specialising, generalising, conjecturing* and *convincing* might be seen as fundamental mathematical processes describing most mathematical activity, and has offered other frameworks through which to view mathematical thinking and problem solving (see for example, Mason et al 1984; Mason, 1988a).

The problem with such lists of processes, or stages of activity, is that they can start as one person's attempt to synthesise mathematical operation, and become objects in their own right. It is possible to envisage lessons *on* specialising and generalising. Love points to two disadvantages, first that the particularity of the lists fails to help us decide whether some aspect that is not included in the list is *mathematizing* or not; and second that the aspects start out as being descriptions, but become prescriptive – things that *must* happen in each activity. (1988, p 254)

One result of this emphasis on process was that a polarisation arose in mathematical activity between *content* and *process*. Traditionally, in what was taught as mathematics, the mathematical topics were overt and any processes mainly covert. Little emphasis had been put on process, and indeed little evidence of use of process seen in pupils' mathematical work. Alan Bell (1982) made the distinction,

Content represents particular ideas and skills like rectangles, highest common factor, solution of equations. On the other side there is the mathematical process or mathematical activity, that deserves its own syllabus to go alongside a syllabus of mathematical ideas; I would express it as consisting of abstraction, representation, generalisation and proof.

Although common sense indicates that content and process would most valuably go hand in hand, moves to make process more explicit were in danger of turning process into yet more content to be learned rather than a dynamic means of enabling learners to construct mathematical ideas for themselves (Love, 1988). However, in schools, the mathematics curriculum was moving steadily towards a differentiation between mathematical content and process

THE STATUS OF INVESTIGATIONAL WORK IN MATHEMATICS TEACHING

Investigating became more widely seen as a valuable activity for the mathematics classroom, supported by the Cockcroft report (DES, 1982), which included *investigational work* as one of six elements which should be included in mathematics teaching at all levels (para. 243). In paragraph 250, the authors wrote:

The idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in very many fields.

They recognised that investigations might be seen as extensive pieces of work, or 'projects' taking considerable time to complete, but that this need not be so. And they went on:

Investigations need be neither lengthy nor difficult. At the most fundamental level, and perhaps most frequently, they should start in response to pupils' questions, ...

The essential condition for work of this kind is that the teacher must be willing to pursue the matter when a pupil asks "could we have done the same thing with three other numbers?" or "what would happen if ...?"

Despite this advice, investigations in many classrooms have become separate pieces of work, almost separate topics on the syllabus. This has been supported, legitimised, and to some extent required by the advent of the General Certificate of Secondary Education (GCSE) in which an assessed element of *coursework* is now a requirement. *Coursework* consists of extended pieces of work from pupils which are assessed by teachers and moderated by an examination board. Boards have responded to National Criteria for this assessment by producing assessment schedules for such coursework, often expressed in process terms. It has meant that many teachers, often under some duress, have undertaken investigational work for the first time in order to provide coursework opportunities for their pupils, and see it as being quite separate from their normal mathematics teaching. Thus the particular processes required by the examination board are nurtured or taught without reference to mathematical content which is taught separately and assessed by written examination.

Quite separately from the GCSE requirement, authors of some published mathematics schemes introduced investigational work as a semi-integral part of the scheme. These were, in the main, individualised schemes, for

example, SMP, KMP and SMILE¹, in which children worked 'at their own pace' and followed an individual route set by their teacher. The investigations were built into these routes, but were separate from other parts of a route. In some cases, as part of the final examination at 16+, pupils were required to undertake an investigation under examination conditions. A consequence of this was that investigations set as examination tasks were rather stereotyped, and could be undertaken by applying a practice-able set of procedures – for example by working through a number of special cases of some given scenario, looking for a pattern in what emerged and expressing this pattern in some general form, possibly as a mathematical formula. Often such sets of procedures were learned as a device for tackling the investigations rather seen as part of being more generally mathematical.

Thus, two forms of investigation have become 'the state of the art'. In the first, pupils undertake some extended piece of work, in which they investigate some situation and write this up as coursework to be assessed. In the second, pupils work on stereotyped tasks or problems according to a routine which the teacher expects will lead them to a resolution of the problem. It is often the case that the traditional mathematics syllabus is taught alongside this investigational work, that the two types of work do not interrelate, and that the processes inculcated for the latter are not seen to be valuable in the former.

Of course, there are many classrooms in which this is not the case and in which teachers *do* link investigational work with traditional mathematical topics in differing degrees. Indeed there have been attempts to teach the mathematics syllabus through investigations, and courses have been devised to link investigational work integrally with the teaching of topics. One such course *Journey into Maths* was devised for lower secondary pupils, and typically provided lists of content and process objectives for each topic (Bell, Rooke, & Wigley 1978/9). Other such courses have been devised by groups of teachers, some working under the aegis of ATM, and recognised by an examination board for assessment purposes. Where this was the case, the merging of investigational work and syllabus topics allowed for a more overt linking of process and content.

¹ SMP is the School Mathematics Project. KMP is the Kent Mathematics Project, SMILE is an individualised scheme in School Mathematics, pioneered by the Inner London Education Authority.

AN INVESTIGATIVE APPROACH TO MATHEMATICS TEACHING

An investigative approach to teaching mathematics might be seen as a way of approaching the traditional mathematics syllabus which emphasises process as well as content. I would see it taking the advice quoted from Cockcroft above, but going beyond this to the active *encouragement* of questions from pupils and the inquiry or investigation which would naturally follow. It is akin to '*inquiry teaching*', Collins (1988):

Inquiry teaching forces students to actively engage in articulating theories and principles that are critical to deep understanding of a domain. The knowledge acquired is not simply content, it is content that can be employed in solving problems and making predictions. That is, inquiry teaching engages the student in using knowledge, so that it does not become 'inert' knowledge like much of the wisdom received from books and lectures.

However, Collins goes on to say:

The most common goal of inquiry teachers is to force students to construct a particular principle or theory that the teacher has in mind.

I have philosophical difficulties with this statement which might be to do with the language in which it is expressed, rather than what the author means by it. Speaking from a constructivist philosophy, and as a teacher, I do not believe that I can *force* a pupil to construct, and in particular I cannot force a *given* construction. However, there are many principles or theories in the required mathematics syllabus which pupils are required to know, and which the teacher has responsibility to teach. Thus an important question, which this study addresses, concerns *how* pupils will come to know, and what teaching processes will promote this knowing.

Another word much used in connection with learners coming to an understanding of given concepts is *discovery*. Elliot and Adelman (1975) contrast *inquiry* with *discovery*:

The term *inquiry* suggests that the teacher is exclusively oriented towards 'enabling independent reasoning', and therefore implies the teacher has unstructured aims in mind. On the other hand *discovery* has been frequently used to describe teaching aimed at getting pupils to reason out inductively certain preconceived truths in the teacher's mind.

It is therefore used to pick out a structured approach. Although the guidance used in both inquiry and discovery approaches will involve *not-telling* or explicitly indicating pre-structured learning outcomes there is a difference. Within the inquiry approach there are no strong preconceived learning outcomes to be made explicit, whereas within the discovery approach there are. In discovery teaching, the teacher is constantly refraining from making his pre-structured outcomes explicit. In inquiry teaching this temptation is relatively weak.

It appears, from these quotations, that Elliot and Adelman's perception of 'inquiry' differs somewhat from that of Collins; and that there are similarities between Collin's 'inquiry' and Elliot and Adelman's 'discovery'. So called *discovery learning*, promoted in the 1960s (e.g. Bruner, 1961) was criticised because it seemed either to be directed at pupils discovering (in the space of a few years) theories which had taken centuries to develop; or it was not discovery at all, when pupils were somehow guided to the results which teachers required. It was also suggested that many research studies into the value of discovery methods in teaching mathematics were not convincing of its value over didactic methods (Bittering, 1968). One of Polya's books is called *Mathematical Discovery*. It is not, however, directed at the discovery of mathematical theories or concepts, but rather at the personal development of a set of heuristics which will enable successful problem solving.

A danger is that *investigative* will be seen as just another word, like *inquiry*, or *discovery*, used to describe teaching or learning, whose meaning will be debated as above. As a teacher I had a sense of what I understood by an investigative approach to teaching, and I tried to articulate this in Jaworski (1985b). I presume that other teachers who undertake investigational work in the classroom, beyond the doing of isolated investigations, also have a sense of what an investigative approach means, not necessarily the same as mine, or of others. The value in speaking of *an investigative approach* is not in some narrow definition, but in its dynamic sense of what is possible in the classroom in order to encourage children's mathematical construal. Love talks of 'attempting to foster mathematics as a way of knowing', in which children are encouraged to take a critical attitude to their own learning, similar perhaps to the attitude which Polya was trying to encourage in his readers. To do this, Love suggests that children need to be allowed to engage in such activities as:

Identifying and expressing their own problems for investigation.

Expressing their own ideas and developing them in solving problems.

Testing their ideas and hypotheses against relevant experience.

Rationally defending their own ideas and conclusions and submitting the ideas of others to a reasoned criticism. (1988, p 260)

Such statements are indicative of an underlying philosophy for the classroom which will have implications for the mathematics teacher. I believe that they support overtly the constructivist² stance that knowledge is a construction of the individual. Children *will* build their own mathematical concepts whether they are told facts or asked to investigate situations. *Telling* facts seems to close down possibilities, whereas *investigating* opens them up. Telling or explaining on the part of the teacher seems a very limited way of encouraging construction. However, not-telling (ever!) seems particularly perverse. An investigative approach to teaching mathematics, as well as employing investigational work in the classroom, literally *investigates* the most appropriate ways in which a teacher can enable concept development in pupils. I see it encouraging exploration, inquiry, and discovery on the part of the pupil, but not prohibiting telling or explaining on the part of the teacher.

The research study

A STATEMENT OF PURPOSE

The purpose of my research has been to explore of what such an investigative approach consists. I have started from a constructivist theory of learning and asked what are its implications for classroom teaching. I see an investigative approach being a bridge between the theory of constructivism and the practice of teaching. I have chosen to observe teachers who, according to my judgement, have been employing an investigative approach to some degree. Through observations of their practice and scrutiny of their philosophies of learning and teaching, I have sought to characterise their teaching, and in so doing, to come to a greater understanding of the design of teaching for concept construction in pupils and the issues which this raises. Not least of the issues are those which concern the teacher's own involvement in the design process.

² I explore *Constructivism* in more detail in Chapter 2.

THE STRUCTURE OF THIS THESIS

Part I of this thesis is concerned with theory. Chapter 2 focuses on constructivism, its history as a theory of knowledge and learning and its implications for education. Chapter 3 looks more particularly at the role of mathematics teaching in enabling concept development in pupils.

Part II is my account of the research study. This starts in Chapter 4 with methodology, and continues through Chapters 5, 6 and 7 with accounts from my three phases of research. Introductory details of the fieldwork are provided below.

Part III is devoted to consequences and conclusions of the research. It is in three chapters, Chapter 8 focusing on reflective practice, Chapter 9 on characteristics common to investigative teaching in the classrooms studied, and Chapter 10 drawing together the various strands of the research in making links between theory and practice. A brief rationale for this structure is provided after the section on fieldwork below.

THE FIELDWORK

The fieldwork for this research was conducted during the period from January 1986 to March 1989³. It occupied three phases, each taking just over six months to complete. Each phase involved one secondary school, two experienced mathematics teachers and two classes of pupils - one for each teacher. I studied lessons of the teacher with their chosen class, spending approximately one day per week in the school over the period of research. I talked extensively with each teacher about their teaching of this class, and occasionally saw lessons with other classes. I also sought the views of pupils in each of the schools. In writing of these experiences, I have changed the names of schools, teachers and pupils to preserve anonymity.

In January 1986 I formally began Phase 1 of classroom observations at Amberley, a large 11–18 comprehensive school in a small town in the East Midlands, although I had been working with teachers in this school during the previous year. This was a pilot phase in which questions and methodology would evolve and it continued until the summer of 1986. The teachers I observed were Felicity and Jane.

Phase 2 of the research began in September 1986 and continued until March 1987. It took place at Beacham, a large 12–18 comprehensive

³ A research chronology is provided in Appendix 1

school in a new city in the South Midlands. The teachers with whom I worked were Clare and Mike, who was head of the mathematics department. It was during this phase that methodology became established, and I regard this phase as the first half of my main study.

Phase 3 took place between September 1988 and March 1989. I observed classes of two teachers, Ben and Simon, at Compton, a small 11–16 secondary modern school in a rural area in the Midlands. Ben was head of the mathematics department, and Simon had responsibility for information technology in the school. This phase formed the second part of my main study. Patterns which had emerged from Phase 2 were tested in Phase 3.

The methodology of the study was ethnographic in style involving, chiefly, strategies of participant observation and informal interviewing, and was conducted from a researcher-as-instrument position. Data collected was in the form of field notes, audio and video recordings with transcripts of these, pieces of writing from the teachers themselves, and one set of questionnaire responses from pupils. Some of the video material collected was used for stimulus-recall with teachers and pupils. Chapter 4 addresses the methodological issues involved in the study. However, methodological considerations pervade my reporting of the three phases of research.

MY OWN POSITION IN THE RESEARCH

The unavoidable linearity and constraints on structure and organisation in a thesis place demands on the reporting of the study which are in some sense artificial. A three dimensional network would offer more flexibility. I have chosen to offer a structure of

theory → research → consequences and conclusions.

However, this study charts a development in my own thinking with respect to the teaching of mathematics, its relation to a constructivist philosophy of knowledge and learning, and the investigative approach bridging the *practice* of teaching and the *theory* of learning. This development has influenced both theoretical and methodological considerations throughout the research and has drawn heavily on my reading during this time. Although I present my accounts, in Chapters 5 to 7, in the first person, I have felt that more is necessary to try to make links, convey a sense of the personal nature of this study, and add to its rigour. I have therefore included two interludes, between chapters 5 and 6, and 6 and 7, in which I

refer specifically to my own focus and emphasis at these stages in the research and its potential influence on the research.

An important consequence of my particular methodology in this study has been the relationship between teacher and researcher, and its link to teacher development which I claim is a consequence of an investigative approach to mathematics teaching. Chapter 8 is devoted to these ideas, which are linked to the various strands of my own thinking throughout the research in a model for reflective practice.

THE CONTRIBUTION OF THE STUDY

The main contribution of the study will be to knowledge of mathematics teaching – in particular to characteristics of teaching, and issues which teachers face in enabling pupil construal of mathematics.

The study presents a device, the *teaching triad*⁴, which arose from data and which has been found valuable for viewing and describing mathematics teaching. Its contribution to the design of teaching might form the basis of further research.

The study, further, has implications for teacher development – particularly with regard to the reflective teacher – and makes a contribution to methodology in terms of interpretive analysis of qualitative data, and reflexive reporting of qualitative research.

These contributions are elaborated in Chapter 10.

⁴ This device is a significant theoretical construct arising from this research. As such it will be mentioned on numerous occasions before it is introduced formally in Chapter 6. It consists of three strands, which characterise aspects of teaching, and their inter-relationships. The strands are: Management of Learning (ML), Sensitivity to Students (SS) and Mathematical Challenge (MC).

CHAPTER 2

CONSTRUCTIVISM

As I have emphasised in Chapter 1, the constructivist philosophy from which interpretations are offered is my own. It is therefore the purpose of this chapter to present the view of constructivism on which my study is based.

It has been argued (see for example Richardson, 1985) that modern constructivism has its origins in the thinking and writing of Kant, owes much of its current conception to the works of Piaget and Bruner, is evident in the writing of influential educational psychologists such as Donaldson, and underpins an important influence for classroom practice in the United Kingdom – the Plowden report.

My chief source in presenting a view of constructivism and showing its relevance to my work and thinking is the writing of Ernst von Glasersfeld. Constructivism, although internationally recognised as a theory which has much to offer to education, and in particular to mathematics education, has had a ground swell in the United States during the 1980s. Von Glasersfeld has been one of its leading proponents and has written extensively about its historical base and its applicability to education.

What Constructivism is

I begin with a definition:

Constructivism is a theory of knowledge with roots in philosophy, psychology, and cybernetics. It asserts two main principles whose application has far reaching consequences for the study of cognitive development and learning, as well as for the practice of teaching, psychotherapy and interpersonal management in general. The two principles are:

- 1** knowledge is not passively received but actively built up by the cognising subject;
- 2** the function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality.

To accept only the first principle is considered *trivial constructivism* by those who accept both, because that principle has been known since Socrates and, without the help of the second, runs into all the perennial problems of Western epistemology. (von Glasersfeld, 1987a)

As my chief interest in constructivism is in its relation to the teaching and learning of mathematics, I shall pursue those aspects of the above definition which relate to my area of interest. *Cognising subjects*, in my terms, refers to pupils in the classroom, the teachers who teach them, the researchers who study them, and indeed to the readers of this thesis. The first principle says that we all *construct* our own knowledge. We do not passively receive it from our environment. It is von Glasersfeld's claim that this would be unprofound without the power of the second principle. In contrast to *trivial* constructivism, which indicates the acceptance of the first principle only, *radical* constructivism indicates espousal of both principles. The second principle implies that an individual learns by adapting. What we each *know* is the accumulation of all our experience so far. Every new encounter either adds to that experience or challenges it. The result is the organisation for each person of their own experiential world, not a discovery of some 'real' world outside. Piaget (1937) claimed that "L'intelligence organise le monde en s'organisant elle-même" ('Intelligence organises the world by organising itself'¹)

In classroom terms, if, for example, a pupil needs to know the *area of a triangle*, she might use a number of methods which have been part of her previous experience. This experience might suggest that there is only one value for the area of the triangle, but if her various methods when applied throw up more than one value her experience is challenged. She then has to re-examine her methods and her current concept of *area*. If as a result she discards a method because she thinks that it is now inappropriate, or changes her view of *area* to believe that there might be more than one value, her experience has been modified. She will have come to know more about finding area of triangles. Next time she comes to a question on areas of triangles, it will be this new experience which will condition her thoughts. However, what she now *knows*, or believes, says nothing about the reality of triangles, their area, or methods of finding area. If there exist any absolutes regarding triangles, areas or methods, her developing experience tells her nothing about what they are. (In Chapter 7, I refer to a pupil Phil, who was in the situation which I describe here. I consider his teacher's *coming to know* more about Phil's conceptions, and consequent effort to create dissonance to bring Phil up against the contradiction in his reasoning. See p 184.)

Thus, knowledge results from individual construction by modification of experience. Constructivism does not deny the existence of an objective

¹ Cited in von Glasersfeld (1984)

reality, but it does say that we can never know what that reality is. We each know only what we have individually constructed. Von Glasersfeld wrote:

If experience is the only contact a knower can have with the world, there is no way of comparing the products of experience with the reality from which whatever messages we receive are supposed to emanate. The question, how veridical the acquired knowledge might be, can therefore not be answered. To answer it, one would have to compare what one knows with what exists in the 'real' world – and to do that, one would have to know what "exists". The paradox then, is this: to assess the truth of your knowledge you would have to know what you come to know before you come to know it. (1983)

Radical constructivism, thus, is radical because it breaks with convention and develops a theory of knowledge in which knowledge does not reflect an "objective" ontological reality, but exclusively an ordering and organisation of a world constituted by our experience. (1984, p 24)

If von Glasersfeld's second principle, quoted earlier, implies there is no world outside the mind of the knower, it could according to Lerman (1989) imply that "we are certainly all doomed to solipsism". However, Lerman refutes this, pointing out that the hypothesis recognises *experience*, thus valuing the interactions with others in the world around us. He states:

Far from making one powerless, I suggest that research from a radical constructivist position is empowering. If there are no grounds for the claim that a particular theory is ultimately the right and true one, then one is constantly engaged in comparing criteria of progress, truth, refutability etc., whilst comparing theories and evidence. This enriches the process of research.

These views have major implications for the classroom. The teacher who wants pupils to know, for example, about Pythagoras' theorem, possibly because the syllabus requires it, has her own construal of what Pythagoras' theorem is or says. It is very easy for a teacher to dwell in an ontological state of mind regarding Pythagoras' theorem, acting as if there *is* an object known as Pythagoras' theorem, that she *knows* it, and that she wants pupils to *know* it too. The last two *its* refer to the same object. *It* is well defined. *It* exists. *It* can be conveyed to pupils so that they too will know *it*. If the pupil's *it* seems in any substantial way to differ from the teacher's *it*, then the teaching is regarded as less than successful.

Constructivism and knowledge

Von Glasersfeld's definition of constructivism continues with:

The revolutionary aspect of Constructivism lies in the assertion that knowledge cannot and need not be 'true' in the sense that it *matches* ontological reality, it only has to be 'viable' in the sense that it *fits* within the 'real' world's constraints that limit the cognising organism's possibilities of acting and thinking. (von Glasersfeld, 1987a)

The words *match* and *fit* are used very particularly here. It is von Glasersfeld's claim that we can never hope to construct a *match* with reality, because we can never know that reality. The best we can do is come up with a *fit*. He uses the analogy of opening a door by putting a key in a lock. Many keys will *fit* the lock. The key does not need to *match* the lock perfectly to open the door. In his words,

From the radical constructivist point of view, all of us, scientists, philosophers, laymen, school children, animals, and indeed, any kind of living organism – face our environment as the burglar faces a lock that he has to unlock in order to get at the loot. (von Glasersfeld, 1984, p 21)

In construing the world around us we need to construct explanations which fit the situations we encounter. Any fit will do, until it comes up against a constraint. A master key may open all the doors in my corridor. If, however, someone changes their lock, the master key may no longer fit. I then need either a new master or an extra key. The changed lock is a constraint which I must take into account.

Biologists use the word 'viable' to describe the continued existence of species, or individuals within species, in a world of constraints. The species adapts to its environment because all individuals which are not viable are eliminated and so do not reproduce. The Darwinian notion of the survival of the fittest might imply that some are fitter than others, but in fact the crucial requirement is to fit, somehow, or die. So, to say that the fittest survive is meaningless. The fit survive; the others do not. In cognitive terms, a lack of fitness is rarely fatal. In von Glasersfeld's words, "Philosophers, however, rarely die of their inadequate ideas" (1984). Ideas, theories, rules and laws are constantly exposed to the world from which they were derived, and either they hold up, or they do not. If they do not, then they have to be modified to take the constraints into account. Where the unviable biological organism would fail to survive and therefore die, a person's knowledge would evolve through

modification, as in the example of Phil mentioned above. In the history of science some theories have been discarded when new experience has shown them to be inadequate – Aristotle’s crystal spheres for example, and the flat earth theory. In other cases, for example in many of Newton’s theories, limitations have been recognised, but the theory itself has prevailed with its limitations taken into account. In recent research in mathematics education the notion of ‘conflict discussion’ has been used as a deliberate exposure of knowledge to conditions in which it is unviable (e.g. Bell and Bassford, 1989)

Where mathematical knowledge is concerned there has been much debate about whether mathematics exists in the world around us or whether it is a construct of the human mind. Descartes, and the Cartesian school in the seventeenth century, following in the tradition of Robert Grossetest and Roger Bacon, believed that “the mathematical was the only objective aspect of nature” (Crombie, 1952, Vol 2, p 160). Mathematics formed the basis of the inductive theory of scientific discovery in the Aristotelian tradition, in which observed objects were broken up into “the principles or elements which produced them or caused their behaviour”. (Crombie, *ibid*) Giambattista Vico, in 1710, in his treatise *De antiquissima Italorum sapientia*. (On the most ancient knowledge of the Italians) said that, “mathematical systems are systems which men themselves have constructed”. Richard Skemp spoke of *inner reality*, which corresponds closely with notions of the adaptation of experience, by the individual, in developing a consistent view of the world. He wrote recently (Skemp, 1989) “Pure mathematics is another example of a widely-shared reality based on internal consistency and agreement by discussion within a particular group.” I return to the idea of communicating such *inner reality* in the next section. For a more detailed historical perspective on constructivism see Appendix 2.

Constructivism, meaning and communication

Fundamental to teaching and learning is a consideration of how communication takes place, of how meanings are shared. In the teaching of mathematics it is also fundamental to ask *what* meaning and *whose* meaning? Von Glasersfeld wrote:

As teachers ... we are intent on generating knowledge in students. That after all is what we are being paid for, and since the guided acquisition of knowledge, no matter how we look at it seems predicated on a process of communication, we should take some interest in how this process might work.

Although it does not take a good teacher very long to discover that saying things is not enough to "get them across", there is little if any theoretical insight into why linguistic communication does not do all that it is supposed to do. (1983)

As I grow in experience, as an individual, I continually develop and modify conceptions as a result of everything which happens to me. For example I do not touch things which I know to be hot, because I have learned from experience that burning is unpleasant and destructive; when I heard about and saw pictures of people landing on the moon, I revised my conceptions of interplanetary travel; I recently bought an oyster knife and have developed a fairly successful method of opening a shell without covering the oyster in grit.

In terms of these three experiences I could be said to have certain knowledge. It is knowledge which is very personal to me. My visions of interplanetary travel might differ greatly from those of other people. Someone else may have a much better method of opening oysters than the one which I have developed. However, I believe that many other people share my reluctance to touch hot objects, and I believe that they would have reasons very similar to mine. This belief is well founded because I interact with others and have means of sharing concepts of hotness. Two people might agree that their tolerance of hotness differed, that their concepts of what was too hot to hold were different. I might find an expert in opening oysters who could share other methods with me. People's alternative conceptions in these examples are not surprising. I do not expect that everyone else will have the same beliefs or the same experience which I have. Our knowledge in these areas differs, but there are ways in which it can come closer through communication.

However, when it comes to mathematics, subtle shifts in perception of knowledge take place. There are many mathematical operations or objects which I *know*. I know how to *subtract* one number from another. I know *Pythagoras' theorem* and how to use it to find lengths in triangles. I know what is meant by the *empty set*. Implicit in these examples of my knowledge is that I know *numbers*, *triangles* and *sets*. I could start to identify what this knowledge consists of. For example, what do I know about numbers, about triangles? How do I know these things? If I have to

teach some pupils about Pythagoras' theorem, what is it that I should want *them* to know?

As a constructivist I recognise that all this knowledge is of my own construction, resulting from repeated modification of my experience. What I know about a triangle is my own personal construct of triangle, my own inner reality. I have confidence in it because it fits with my experiences of triangles as I encounter them. These experiences include interactions with other people who have their own constructs of triangle, and the accord between what *I* understand of triangle and what I perceive of other's constructs of triangle reinforces my own knowledge. I am very confident of it. However, I can remember encountering the idea of a triangle on the surface of a sphere, and being seriously challenged. Could my concept of triangle take this new object into account? It was very tempting to exclude the new object, and restrict my notions to ones of triangles in the plane, whose angle sum was 180° . When I teach pupils about Pythagoras' theorem, and find myself referring to triangles, I have to be aware that their constructions of triangle are likely to be different from mine and different from those of each other². Indeed in teaching, the very words I use are my own words with my meanings and the pupils in hearing my words will interpret them according to their meanings. Alan Bishop (1984) writes:

Given that each individual constructs his own mathematical meaning how can we share each other's meanings? It is a problem for children working in groups, and for teachers trying to share their meanings with children individually ...

If meanings are to be shared and negotiated then all parties must communicate ...

Also communication is more than just talking! It is also about relationship.

Von Glasersfeld said:

If you grant this inherent subjectivity of concepts and, therefore, of meaning, you are immediately up against a serious problem. If the meanings of words are, indeed, our own subjective construction, how can we possibly communicate? How could anyone be confident that the representations called up in the mind of the listener are at all *like* the representations the speaker had in mind when he or she uttered the particular words? (1987b, p 7)

² This became very obvious in Ben's 'Kathy-Shapes' lesson, when a group of girls was tackling areas of triangles in which their image of 'vertical height' differed from mine and from that of the teacher. See Appendix 5

It is here that notions considered earlier, regarding the construction of knowledge, become useful. The biological notions of viability and fit are as applicable to sharing of meaning as they are to construction of knowledge. Communication is a process of *fitting* what is encountered into existing experience and coping with constraints such as clashes in perception. When I attempt to communicate with another person, various sensory exchanges take place. I am likely to listen to the other, and to look at them and observe their gestures. I can interpret voice tones, pausing and emphasis, facial expressions, hand movements, body postures and so on. When I speak myself, I hear responses which I can try to make sense of in terms of my own meanings and intentions. There is an extensive literature in the areas of language, semiotics and philosophy regarding how meaning is constructed and communication achieved. For example, Sperber and Wilson (1986) write of the importance of *relevance* to communication between individuals - that any person, being addressed by another, makes sense of what is said by making assumptions about its relevance to their common experience. Thus the interpretation made would be conditioned by the mutual experience of the people concerned. Stone (1989) referred to a term 'prolepsis', dating from ancient rhetorical scholarship, and introduced by the modern linguist Rommetveit (1974) to speak of the way in which a person in speaking might presuppose some unprovided information.

Rommetveit argues that the use of such presuppositions creates a challenge for the hearer ... which forces the hearer to construct a set of assumptions in order to make sense of the utterance. ... This set of assumptions essentially re-creates the speaker's presuppositions. Thus the hearer is led to create for himself the speaker's perspective on the topic at issue. (Stone, 1989)

In constructivist terms what the hearer creates is her *own* perspective. However, successful communication might depend on this being close to the perspective of the speaker. The implications of *non-verbal* communicative devices are less well known. Wood (1988), while admitting that "research into non-verbal dimensions of communication and their effects on teaching and learning is sparse", nevertheless states that "there is some evidence which suggests that problems of understanding which one might expect to occur when people are 'out of tune' ... do arise". For example, "Some of the problems of mutual understanding that one experiences when talking to people from other linguistic communities may arise not only from difference in the sounds that they make but also from the timing of their movements."

Paul Cobb, an American mathematics educator who is currently investigating teacher education programmes based on a constructivist philosophy, has written about the implications of a constructivist philosophy for perceptions of classroom communication in mathematics:

Constructivism challenges the assumption that meanings reside in words, actions and objects independently of an interpreter. Teachers and students are viewed as active meaning makers who continually give contextually based meanings to each others' words and actions as they interact. The mathematical structures that the teacher 'sees out there', are considered to be the product of his or her own conceptual activity. From this perspective mathematical structures are not perceived, intuited, or taken in but are constructed by reflectively abstracting from and reorganising sensorimotor and conceptual activity. They are inventions of the mind. Consequently the teacher who points to mathematical structures is consciously reflecting on mathematical objects that he or she had previously constructed. Because teachers and students each construct their own meanings for words and events in the context of the on-going interaction, it is readily apparent why communication often breaks down, why teachers and students frequently talk past each other. The constructivist's problem is to account for successful communication. (Cobb, 1988)

Cobb seeks to justify a constructivist view of teaching and learning rather than the more common view which a metaphor of *transmission* might describe. This more common view is characterised by common phrases or expressions, such as,

I *got* the idea *across*

I didn't *get* what you said

I *did* adding of fractions with the class.

I feel pressured to *get across* (*cover*) a large volume of information

I'm trying to *give* students the skills and techniques they need.

The teacher is a medium for *delivering* curriculum to students.

(Davis and Mason, 1989)

Davis and Mason suggest that, "A constructivist perspective challenges the usual transport metaphor which underpins a good deal of educational discussion, in which knowledge is seen as a package to be conveyed from teacher to student."

Cobb makes the distinction that holders of a transmission view need to justify the *breakdown* of communication, perhaps in terms of limitations of memory, or failing to take account of all that was said, because the transmission view is predicated on *handing over*. Providing that the

teacher *hands over* the required knowledge, perhaps by giving a ‘good’ exposition of it, all the pupil has to do is accept it. Thus the anomaly lies in cases where learning appears *not* to have taken place. Whereas, in the case of a constructivist view, the reverse is true – successful communication needs to be *accounted for*. Since in constructivist terms a *match* in meaning, between teacher and pupil, can never be known, even if it were achieved, how is it then possible for meanings to be shared at all? Yet we know of cases where people did appear to understand each other.

Within a constructivist framework, the assumption that successful communication is not a norm, can be a positive rather than a negative influence. Much of the mis-communication which takes place in teaching and learning is exacerbated by the assumption, from a *transmission* viewpoint, that it should not have occurred. As a result participants, customarily, do not look out continually for evidence of common or alternative conceptions, with a view to modifying what they have said or done where necessary. Constructivists *have* to behave in this way, being constantly aware that the other person’s interpretation might be very different to that which they themselves wished to share. This level of awareness promotes a healthier possibility of people moving consciously closer in understanding. It is the teacher’s task to promote this attitude in students. Cobb says:

The teacher’s role is not merely to convey to students information about mathematics. One of the teacher’s primary responsibilities is to facilitate profound cognitive restructuring and conceptual reorganisations. (Cobb, *ibid*)

Davis and Mason elaborate a methodology for communication which is based on the sharing of fragments:

Even the most radical constructivist will agree that there are aspects or fragments of experience which different observers can agree on.

The basis of the methodology to be elaborated is that effective construal begins with fragments that can be agreed between people ... and weaves these into stories which can be discussed, negotiated and acknowledged as appropriate to a particular perspective. (Davis and Mason, *ibid*)

I shall argue that *effective construal*, which is related to *successful communication* is the root of *successful learning*.

Constructivism and the classroom

Von Glasersfeld claimed that there were certain consequences of a constructivist philosophy for a teacher in the classroom.

In education and educational research, adopting a constructivist perspective has noteworthy consequences:

1. There will be a radical separation between educational procedures that aim at generating *understanding* ('teaching') and those that merely aim at the repetition of behaviours ('training')
2. The researcher's and to some extent also the educator's interest will be focused on what can be inferred to be going on inside the student's head, rather than on overt 'responses'.
3. The teacher will realise that knowledge cannot be transferred to the student by linguistic communication but that language can be used as a tool in the process of guiding the student's construction.
4. The teacher will try to maintain the view that students are attempting to *make sense* in their experiential world. Hence he or she will be interested in student's 'errors' and indeed, in every instance where students deviate from the teacher's expected path because it is these deviations that throw light on *how* the students, at that point in their development, are organising their experiential world.
5. This last point is crucial also for educational research and has led to the development of the teaching experiment, an extension of Piaget's clinical method, that aims not only at inferring the student's conceptual structures and operations but also at finding ways and means of modifying them. (von Glasersfeld, 1987a)

Thus, in order to help a pupil, the teacher has to understand something of a pupil's conceptual structures, not just affect the pupil's responsive behaviour. Von Glasersfeld's third point supports my remarks on communication and meaning, in the last section, and goes further to suggest that teachers can powerfully employ language to help pupil construal. Pupil construal may be seen in terms of pupils actively *making sense* of what they encounter.

Implicit in this is that the teacher is construing pupils' construal. 'Getting inside the pupil's head' involves the teacher in constructing a story about the pupil's conceptual level – Chapter 3 offers a descriptive metaphor for this involvement (see p 42) – and 'using language to guide pupils' construction' involves devising appropriate responses as a result of the story constructed. The teacher's construction, no less than pupils' constructions, needs supportive or constraining feedback. This can be provided potently by pupils' errors or apparent misconceptions, which can be the basis for diagnosis by the teacher and subsequent modification of

the teacher's vision of the pupil's conception. The teaching experiment to which von Glasersfeld refers is a research device developed by Steffe (e.g. 1977) and explored by Cobb and Steffe (e.g. 1983). It involves an interviewer in interacting with a child by talking with her, setting tasks and analysing the outcome of the tasks in a cyclical fashion, which allows the interviewer to build their own construction of the child's construal. It is thus a device designed as a consequence of the four earlier observations.

I shall go further here and talk about the researcher. In studying the investigative teaching of mathematics it has been my task to observe teachers and students and make my own constructions regarding both the teacher's construal of pupil learning, and the pupils' construal of mathematics. Von Glasersfeld's five observations above are as relevant to *my* activity in this study as they are to a teacher's in promoting mathematical learning.

Cobb (1988) characterises teaching as a continuum on which *negotiation* and *imposition* are end points. Imposition involves the teacher in attempting to constrain pupils' activities by insisting that they use prescribed methods. Negotiation, on the other hand, arises from a belief in the value of communication through sharing meanings. As Bishop (1984) wrote:

The teacher has certain goals and intentions for pupils and these will be different from the pupils' goals and intentions in the classroom. Negotiation is a goal directed interaction, in which the participants seek to modify and attain their respective goals.

According to Cobb, although constructivism does provide a rationale for teaching by negotiation, this form of teaching requires far more of the teacher.

Ideally the teacher should have a deep relational understanding of the subject matter and be knowledgeable about possible courses of conceptual development in specific areas of mathematics. In addition, the teacher should continually look for indications that students might have constructed unanticipated, alternative meanings. But this requires that the teacher transcend the common sense transmission view of communication derived from everyday experience. (Cobb, 1988)

Challenges to constructivism

Objections to constructivism in the field of cognitive science arise from the as yet unresolved paradox that:

there is no adequate cognitive theory of learning - that is there is no adequate theory to explain how new organizations of concepts and how new cognitive procedures are acquired.

To put it more simply, the paradox is that if one tries to account for learning by means of the mental actions carried out by the learner, then it is necessary to attribute to the learner a prior cognitive structure that is as advanced or complex as the one to be acquired. (Bereiter, 1985)

According to Bereiter, no one has succeeded in accounting for how leaps in conceptualisation are made – and they *are made*, for example the leap from rational to irrational numbers – where “learners must grasp concepts or procedures more complex than those which are available for application” (Bereiter, *ibid*) The cognitive structures which allow the conceptual leap to be made, must be in place first. He refers to a theory of *innateness*, which Chomsky and Fodor claim is the necessary alternative to constructivism, that cognitive structures are innate and are merely fixed or instantiated through experience (Chomsky, 1975; Fodor, 1975). Bereiter cites Fodor (1980):

There literally isn't such a thing as the notion of learning a conceptual system richer than the one that one already has; we simply have no idea of what it would be like to get from a conceptually impoverished to a conceptually richer system by anything like a process of learning. (p 149)

Yet the notion appears to have manifestations in practice; hence the paradox to which Bereiter refers. Bereiter's response, rather than just to accept these objections, and the alternative theory proposed, is to look for means of 'boot-strapping', that is “means of progress towards higher levels of complexity and organisation, without there already being some ladder or rope to climb on.” He proposes a number of ways in which this might begin, but recognises that until some progress has been made the paradox will challenge a theory of individual construction of knowledge. I shall return to questions of 'boot-strapping' in my next chapter (see p 34).

Another major challenge to constructivism came from Kilpatrick (1987) when examining what constructivism *is* from the point of view of mathematics education'. He acknowledged that:

as one who stands outside both constructivism as a belief system and philosophy as a profession, I have decided that it would be unfair of me to claim that I know, let alone could tell you what it is. (1987, p 4)

Under a heading of 'What Constructivism Seems Not to Be', Kilpatrick starts with the statement:

As a theory of knowledge acquisition, constructivism is not a theory of teaching or instruction. (p 11)

I do not believe that this is in contention, and have read nothing in the literature to suggest that constructivism *is* a *theory of teaching*. Kilpatrick adds:

Nonetheless, constructivists have sought to derive implications for practice from their theory, and in some writings the implication seems to be drawn that certain teaching practices and views about instruction presuppose a constructivist view of knowledge. (ibid)

I should not wish to claim that any particular consequences of constructivism derive *only* from constructivism. Kilpatrick quotes the five consequences from von Glasersfeld, quoted in the previous section, as an example of his claim above. I do not interpret von Glasersfeld as claiming that these are *only* consequent on a constructivist theory. However, I do believe, with von Glasersfeld and with Cobb, that it is important that theories of teaching be consistent with theories of knowledge and learning. Thus the implication flows from the theory, constructivism, to the consequences, for example the five propositions of von Glasersfeld, and if a practitioner follows a constructivist belief then there is likely to be evidence of such consequences in the practice.

Are these propositions indeed consequences of constructivist theory? Kilpatrick does not show that any of them are not. He argues that there is a too narrow insistence on the meaning of popular terms. For example *training* in the stricter sense might be interpreted as “forming habits and engendering repetitive behaviour”, but it might be used more loosely as a term which allows for practice which involves “explanations, reasons, argument, and judgement”

Making the distinction into a dichotomy ignores the contexts in which the two terms [*teaching* and *training*] are used interchangeably but may be useful if it can be defined. (p 12)

This seems to be splitting hairs. Training in the behaviourist sense implies the former and *not* the latter, and this seems to be the distinction which von Glasersfeld very particularly draws. It is possible to see a continuum of which teaching and training are at opposing ends. For many teachers in the classroom, finding appropriate places to be in this continuum constitutes a major issue. I am particularly interested in how teachers make their decisions. What I will claim is that if one starts from a constructivist perspective, this must influence those decisions.

Kilpatrick also takes up the language of *knowledge transfer*, for example, 'I *got* the ideas *across*', ridiculing the insistence on a literal treatment of such statements, arguing that they are elements of common usage which are not meant in such literal sense. In response to an admission from Cobb that constructivists "often manage to tie ourselves in linguistic knots", Kilpatrick offers:

A plausible alternative hypothesis is that it stems from an aversion to common language forms that other people find viable but that signal dangerous thoughts to constructivists. (p 14)

Kilpatrick claims that:

The teachers quoted (by Davis and Mason, p 21 above³) evidently have constructed a model of the world in which the transport metaphor provides a viable way of talking about instruction. That model is apparently wrong. (I am not sure how constructivists have come to know it is wrong, but I assume they have), so the task facing the constructivists is to change the teachers' model. (ibid)

I do not believe that any constructivist would say that any model is *wrong*. This is an ontological stance which constructivists take care to avoid. The language of *viability* and *fit* seems valuable here. The transport metaphor does not *fit* with the two principles of constructivism. Thus statements quoted by Davis and Mason, which imply a belief in the transport metaphor, are inappropriate for describing teaching and learning for anyone who is a declared constructivist. However, unless constructivists were also to take an evangelical stance, there would be no requirement on their part to change the model of any teacher. I personally take a constructivist stance and so for me a transport metaphor would be inconsistent with my belief. Thus in making observations in other teachers' classrooms, I have to take into account the way in which being consistent affects interpretations which I make. This raised hard questions for me in observing the lessons of Simon in Phase 3 (see Chapter 7).

There is a subtle point here concerning use of language, which Cobb may have been alluding to in the comments which Kilpatrick quotes. There are many forms of language in common use which people employ without thinking through their literal meaning and implication. It may be that people, using these forms, do not mean them in their literal sense. However, employing them without considering their underlying meaning, could imply that not much thought has been given to what they represent.

³ The discrepancy of dates (i.e. with Kilpatrick 1987) arises from the existence of Davis and Mason (1989) as an occasional paper distributed by the authors in 1986

When I catch myself using a familiar phrase in a way which seems inconsistent with my belief, I need to do more than just change the phrase. I need to examine the constructions which I am making which evoke the phrase, because it might be that there are deeper inconsistencies than just the use of language, and I could usefully learn from inspecting these more closely. Where other people are concerned, and teachers in particular, I feel that there is no harm in bringing to their attention some implications which might follow from literal interpretation of what they say. Their response might be that they had intended such sense to be made of their words, but experience suggests that they are often encouraged for the first time seriously to consider what lies behind the words. Such action on the part of the constructivist is not directed at changing the teacher, but rather at drawing attention to levels of awareness.

Kilpatrick emphasises that, just as models of learners treat the learner as someone who is attempting to make sense of the teaching encounter, so too, for consistency, should a teacher be treated as someone who is attempting to make sense of that same encounter. This notion is fundamental to my research in studying teaching. I go further and claim that the *researcher* is also attempting to make sense of that encounter. In setting up opportunities for discussion and negotiation with teachers and pupils, I am able to bring a wider perspective to the account which I am able to give of the encounters which I observe. This is methodologically important to my study.

I drew attention to Cobb's view that the transmission model takes successful communication as its norm, so that breakdowns in communication need to be explained, whereas the constructivist maintains that perfect communication is impossible, and that successful communication is something to reach towards. As Kilpatrick says:

One cannot deny that the world is full of classrooms in which much mis-communication about mathematics is taking place. (p 16)

He goes on however, to suggest that this 'negative' view of successful communication could be damaging to those who are striving hard in the classroom.

Few people respond well to claims that they are failing most of the time, especially when their own models of communication are signalling success. (ibid)

Giving benefit of doubt that this is not mischievous misrepresentation, it seems to be an apposite example of mis-communication between Cobb

and Kilpatrick. I see Cobb's view being that, in the transmission model, when communication is not successful, there must be a sense of failure, and that is particularly negative. If, after what a teacher regards as a particularly good explanation, a pupil gives evidence of having not understood, then the teacher must either blame the explanation or blame the child. Something has gone wrong somewhere. However, if the teacher works from a constructivist stand-point, then any evidence of successful communication is something to treasure. When communication is unsuccessful, this is no more than might be expected. It is not an indication of failure, or cause for blame. The teacher can look positively for other ways of making the communication successful. I must add that by *successful* I imply that evidence obtained from pupils responses indicates a *fit* with the teacher's own thinking, not that there has been a transfer of ideas from the teacher to the pupil.

One final point of Kilpatrick's which I wish to address is the eternal question about the relationship between constructivism and ontology. Von Glasersfeld (1985) has said, very overtly, that constructivism makes no ontological commitment:

(Constructivism) deliberately and consequentially avoids saying anything about ontology, let alone making any ontological commitments. It intends to be no more and no less than *one* viable model for thinking about the cognitive operations and results which, collectively, we call 'knowledge'.

However, is this statement itself indicative of an ontological stance? Kilpatrick (1987) suggests that many of the claims made by constructivists are ontological:

To reject "metaphysical realism" is to take an ontological stand. Cobb's (1983) eschewal of "realist language" expresses an ontological view. Contrasting radical constructivists with realists (Davis and Mason, 1989) by saying what constructivism is not, contributes to the construction of a constructivist ontology. (p 18)

The problem seems to lie in implications from the language used regarding what 'constructivism is' or what 'constructivism is not'. This brings us back to Cobb's admission of "tying ourselves in linguistic knots". It must be accepted that Cobb's constructivism and Davis and Mason's constructivisms are their own constructions, as is von Glasersfeld's, as is mine. When I hear von Glasersfeld say "constructivism is", I make my own construction of what this means according to my interpretation of it in the light of my experience. Yet if I subsequently claim that 'I am a constructivist', this might imply that there is *something* identifiable as

constructivist. The importance of this paradox is its recognition, particularly where consistency is involved. Ultimately for me it is my own construction which counts, and as what matters for me is to have a basis for acting in the classroom in a way which will best enable pupils to learn mathematics, the fine epistemological details are less important than a consistent approach to practice. Kilpatrick ends with a reference to George Polya and to his student, in the film “Let us teach guessing”, who claimed that she “sort of” believed the hypothesis which they had been exploring. Kilpatrick leaves the challenge:

Mathematics educators who are not ready to become born-again constructivists may well find that they can live viable lives as sort of constructivists. (ibid, p 23)

Leaving aside the heavy irony in this challenge, I feel rather happy to be a *sort of* constructivist – my own sort. Indeed, can I be anything else?

CHAPTER 3

THE TEACHING OF MATHEMATICS

As this study is of secondary mathematics teaching in the United Kingdom, Chapter 3 will address influences on, and research relating to, the classroom teaching of mathematics from a U.K. perspective, with particular reference to secondary teaching where this is possible, although much research and development has related more directly to primary education.

The implications of learning for teaching

THE INFLUENCE OF PIAGET

Perhaps the greatest single influence on education generally, and mathematics education in particular, has been the work of Piaget. Piaget is held to have been both a constructivist epistemologist (e.g. von Glasersfeld, 1982; Richardson, 1985) and a developmental psychologist whose work has influenced the 'child-centred pedagogy' (Walkerdine, 1984). His influence as an epistemologist will be illustrated in the next section – *Construction of Mathematical Concepts*. His influence on 'child-centred pedagogy' may be seen particularly in the Plowden Report (1967), which advocated a child-centred approach to primary teaching based on Piagetian theory. The report stated, "Piaget's explanations appear to most educationalists in this country to fit the observed facts of children's learning more satisfactorily than any other." (para 522). Piaget's work on children's stages of development of logical thinking, particularly with regard to mathematical tasks (e.g. Piaget, 1952), influenced the teaching of mathematics in forming the basis of the Nuffield Mathematics Project, "the first and most influential curriculum intervention into primary school mathematics in the 1960s" (Walkerdine, *ibid*).

Piaget's 'clinical method', in which he studied children's development in learning situations has formed the basis of much subsequent research into children's learning, for example the teaching experiment in early arithmetic to which von Glasersfeld (1987a) refers (see page 23 above). His documenting of children's responses to the given tasks provides, in my view, a benchmark for analysis of children's mathematical statements in current classroom situations. This is not in apportioning labels in terms of

Piaget's stages, since research has shown that secondary pupils in the UK mainly fit Piaget's *concrete* stage (Shayer, Kuchemann and Wylam, 1976). It is rather to gain insight into children's reasoning by comparison against Piaget's continuum – he claimed that “reasoning moves continually as a function of a ‘structured whole’” (Inhelder and Piaget, 1958).

However, not all of Piaget's influence is regarded favourably. For example, his theory expressed in terms such as, “Each time one prematurely teaches a child something he could have discovered himself, the child is kept from inventing it and consequently from understanding it completely” (Piaget, 1970, p 715), could be seen to exert a restricting influence on teachers' perceptions of the teaching act. Walkerdine quotes from an interaction between a teacher and a pupil in which the teacher perceived the pupil, who was working on *place value*, as having gone “too far, too fast”. Walkerdine claims, “His [the pupil's] failure was also understood as *her* [the teacher's] failure – that she had ‘pushed’ him – the worst sin of the child-centred pedagogy; she had not allowed him to go ‘at his own pace’”. (1984, p 193). The Plowden Report had placed emphasis on children's reaching states of readiness to learn, and on their firsthand experience – acting rather than listening – with the teacher leading from behind. Notions of letting a child develop at her own pace became closely linked with those of discovery learning and the undesirability of teacher intervention. Edwards and Mercer (1987) point out that the Plowden Report did not uncritically endorse ‘discovery learning’, indeed “the committee in fact commented upon its widespread misapplication, and advised caution in its use”; nevertheless “as is often the case, cautionary comments do little to dampen the resonance of a text's major themes.” (p 37)

One of the main limitations of Piaget's work with regard to its relevance to classroom learning is seen to be in its model of the learner as an individual rather than as a cultural participant. Bruner (1985) suggests that in this model “a lone child struggles single-handed to strike some equilibrium between assimilating the world to himself or himself to the world”. Smedslund (1977) has argued that in making statements about children's logicity, Piaget has ignored social and contextual implications of the tasks on the children's thinking. He claims that “children who failed on tasks were often simply described as non-logical, the problem of criteria of understanding has received relatively scant attention in the Piagetian literature.”

Two issues arise in relation to classroom teaching of mathematics. The importance of the mathematical development of the individual child is undisputed, but how should this be fostered within a group of thirty children? And, how far can the social structure of the classroom affect the learning of each child within it?

CONSTRUCTION OF MATHEMATICAL CONCEPTS

The view of Piaget as a constructivist epistemologist might be seen through his theories of the construction of knowledge in terms of *assimilation* and *accommodation* into *action schemes*. Piaget wrote, "All knowledge is tied to action, and knowing an object or an event is to use it by assimilating it to an action scheme" (Piaget, 1967, translated by von Glasersfeld, 1982). Von Glasersfeld refutes the idea that action schemes are stimulus-response mechanisms, and that Piaget's theory may be classified as 'interactionist'. Rather he argues that Piaget, speaking of 'cognitive adaptation', intended a process more akin to *viability*, as discussed in Chapter 2 (see p 20), than to comparison with some ontological reality (1982, p 615).

Skemp (1971) discusses a model, very close to that of Piaget, to describe *mathematical* concept building. Piaget's *action schemes* are paralleled by Skemp's *schemas* – mental structures with two main functions, to "integrate existing knowledge", and to act as "a mental tool for the acquisition of new knowledge" (Skemp, 1971, p 39). To *understand* something, according to Skemp, is to "assimilate it into an appropriate schema" (ibid, p 46). This fits well with the constructivist principle of 'coming to know' (see Chapter 2, p 14).

Skemp proposes a hierarchy of mathematical concepts in which 'higher order' concepts are those which are abstracted from other concepts. This hierarchy is likely to involve a number of levels each with a variety of possible classifications involving relationships and transformations between different concepts. Concepts are rarely formed in isolation, and any individual will form their own structures of interrelated concepts, or their own *schemas*. Skemp warns that the adaptability of a schema in accommodating to new situations may frequently be difficult. He writes, "if [the accommodation] fails, the new experience can no longer be successfully interpreted and adaptive behaviour breaks down – the individual cannot cope." (1971, p 44) Skemp emphasises the difference between *assimilation* of experience to an existing schema, which "gives a feeling of mastery and is usually enjoyed", and *accommodation*, which

requires modification of the schema: "A schema is of such value to an individual that the resistance to changing it can be great, and circumstances or individuals which impose pressure to change may be experienced as threats - and responded to accordingly." (ibid). This might be seen as a warning to teachers. He claims that an important feature of accommodation is that the original schema is not overthrown, but becomes part of the new one.

So our meaning of schema has now expanded to mean a structure of conceptualised knowledge. We have further noted that concepts and schemas cannot be communicated directly. Each individual has to construct them for himself, in his own mind. (Skemp, 1989, p 72)

The more abstract the schema becomes, the greater difficulty there will be for a pupil in constructing it, and the more need for help. Thus:

the right kind of teaching can greatly help the construction of mathematical schemas (ibid).

The learning paradox (Bereiter, 1985) – if each pupil can only construct for themselves, how is construction of higher level concepts achieved? – is to be resolved through effective teaching. Skemp addresses directly the role of the teacher. "Though the first principles of the learning of mathematics are straightforward, it is the communicator of mathematical ideas, and not the recipient, who most needs to know them.". He offers two 'simple' principles which require "much hard thinking" for their application:

- (1) Concepts of a higher order than those which a person already has cannot be communicated to him by definition, but only by arranging for him to encounter a suitable collection of examples.
- (2) Since in mathematics these examples are almost invariably other concepts, it must first be ensured that these are already formed in the mind of the learner. (1971, p 32)

The 'hard' thinking must be on the part of the teacher, who is to supply the examples. This involves, for the teacher, not only an interpretation of what examples are "suitable", but also what concepts are prerequisite to those which are to be taught. Skemp (1971; 1989) offers his own *concept map* which a teacher (at primary level) might use in helping pupils develop their schemas.

Skemp seems to suggest here that one way to achieve the 'bootstrapping' of which Bereiter (1985) speaks (see p 25 above) is by the offering of suitable examples from which the learner can abstract the concept. This

provision of examples may be seen as part of the *scaffolding* which the teacher provides for the learner (Bruner, 1985).

Although such a theory of concept construction is very plausible, it is nevertheless difficult to know what constructions children have made in their mathematical learning. Brown (1979) relating children's problems with mathematics to their difficulties in forming conceptual structures writes, "The major difficulty, at least for secondary mathematics teachers, is our present lack of knowledge of the nature of these structures, including both the order in which and the processes by which such structures are formed." (p 357).

HIERARCHIES OF MATHEMATICAL CONCEPTS

The Nuffield Mathematics Project, mentioned above, was designed to encourage children's understanding through the *doing* of activity, and it embraced some notion of concept hierarchy. It was based on a concept map designed to plot the order of children's conceptual development. This was in the form of a partially ordered network, and was drawn up in conjunction with Piaget and the Geneva school, embodying operations which could be defined directly with reference to Piagetian literature, and organised according to the Piagetian stages of pre-operations, concrete operations, and formal operations (Brown, 1979). Brown points out that the name, 'concept map', might be misleading here, firstly because, "if 'concepts' are thought of as 'junctions' in the cognitive structure, they can only be built up gradually by the formation of the operational schemes which connect the concepts together" (ibid, p 357). It is also very difficult to say when any concept is completely formed, as this is dependent on links with other concepts. She adds that the 'map' does not try to show the nature of a cognitive structure, but rather a likely order of chronological development of the schemes which form part of the structure.

Skemp says of such conceptual analysis:

Teachers must first analyse the concepts so that pupils can re-synthesise them in their own minds. This is a huge job, and it is too much to expect busy teachers in classrooms to find time for this. But they are entitled to expect that it has been done by textbook writers, who may be regarded as indirect teachers; and it is important to be able to recognise whether or not this essential first step has been adequately done. (1989, p 69)

Skemp implies that the 'job' *can* be done. However, it is not obvious what this job *is*. Reservations of the sort which Brown indicates regarding the

Nuffield map can be levelled at whatever map is produced, however conscientiously. Indeed, if each learner is to build on existing schemas in constructing new concepts, then concept maps are very personal and can only be constructed with knowledge of the individual learner. Particularly at secondary level, where learners' existing schemas are very well established, the design of concept maps must fit fundamentally with the teaching role as the teacher identifies it; and there is danger in relying exclusively on concept analysis done by external agencies such as text-book writers. However, it is undeniable that teachers are busy and under pressure, so what *can* be done to help that would be valuable?

The CSMS (Concepts in Secondary Mathematics and Science) mathematics study was undertaken to address the need for such concept analysis at secondary level. Geoffrey Matthews in his foreword to Hart (1981) referred to the fact that while primary mathematics education had been given a great deal of attention, secondary mathematics education had been relatively neglected. The Nuffield project had "selectively and with caution", developed a hierarchy of mathematical concepts for primary level. Matthews felt that it was time that something similar was done at secondary level. However:

Finding just what secondary children are capable of learning, and where they are in their development, is indeed harder than at primary level, not only because previous research has been so scanty but also because the number of variables, and indeed the entire complexity of the problem increases with age.

Matthews saw his task as to "harass the research workers and constantly demand my hierarchy or 'concept tree' from which authors and teachers could determine proper order of topics and levels appropriate to various children".

The CSMS research involved the production of written test papers which were administered to about ten thousand children mainly in comprehensive schools in both rural and urban areas throughout the UK. The tests were designed in a problem-solving format in order to "probe understanding, rather than to test whether certain methods had been taught by a teacher" (Hart, 1981, p 1). Ten topics were covered including measurement, number operations, fractions, ratio and proportion, algebra and graphs. Where Piaget had done significant work in a particular topic, items on the test paper were adapted from tasks which he had used. For each topic, about thirty children in the appropriate age range from different schools were interviewed and recorded. Their replies were used to revise the tests

where language had been found difficult or ambiguous, and to find methods used and errors made by children when confronted with a mathematical problem.

The data from the wide scale testing were used to form a hierarchy in each topic. Two important criteria were *first the grouping of items within a topic* and the distribution of these groups on some scale from 'easy' to 'hard'; and *second the comparability between the scales for different topics*. An aim was to help a teacher, when planning a period of work for pupils, to identify aspects of a topic which were of comparable degree of difficulty and also to indicate what areas in other topics might be at a corresponding level when the teacher wanted to move onto another topic.

Data was analysed to allow grouping of items within a topic into levels which could then be compared across topics. Findings on levels were checked against the scripts of pupils who had participated in two or more topics. *Algebra*, for example, seemed to have four levels, whereas *vectors* had seven. Level three in algebra was found to have 'a high degree of correspondence' with level five of vectors. The research team commented on methods which the children tested had commonly used, what errors were prevalent and what sort of thinking contributed to these errors. Attempts were made to compare performances between different age groups. For each topic, implications for teaching were offered which resulted from data and analysis.

The notion of hierarchies on which the CSMS research is based comes in for severe criticism in Dowling and Noss (1991), where the editors question one of "the most widespread and influential assumptions of mathematics education" (p 4). One of the contributors, O'Reilly, argues that the influence of the CSMS study in legitimising hierarchies is misplaced, questioning both the theoretical basis and the research methodology which leads to conclusions drawn. O'Reilly claims, of the CSMS study, that

its 'hierarchies of understanding' rather than being universal in application are *at best* the results of particular teaching methods and conditions in England in the 1970s. (p 77)

Moreover, he claims that replication of the CSMS tests in other countries and cultures supporting the robustness of the CSMS levels, does no more than testify to the uniformity of school mathematics curricula worldwide.

Gates (1991), in his review of Dowling and Noss, refers to the CSMS levels with the remark “I certainly felt a lot of resonance with my own experiences of teaching mathematics”. Whether this implies that Gates simply fits this uniformity, or whether the resonance of Gates and others is indicative of some more fundamental validity of the levels is impossible to judge. However, whether a teacher’s teaching fits the uniformity suggested or otherwise, teachers are responsible (now by law) for ‘*delivering*’ given mathematical content to their pupils. The question which must be asked is how far the CSMS levels can be of help or support in this.

A teacher could find the CSMS mathematics report a resource in at least two ways. The detailed analysis of any particular topic could make a contribution to the planning for teaching the topic, as well as promote a greater awareness in the teacher of likely sources of difficulty of understanding and assist diagnosis of error. The comparison of topics could assist the teacher in planning a wider schedule of work and offering to pupils a variety of topics at comparable levels.

Hart, writing of implications of the study for mathematics teaching acknowledges:

The research was not based on classroom observation nor did we investigate the comparative merits of different textbooks so that although we have some information we cannot make pronouncements on how the children we tested were taught. (p 208)

She claims that the results of the CSMS research have “far-reaching implications for the teaching of mathematics at secondary level”, one of which is that mathematics is a “*very difficult* subject for most children”. She makes the point that as pupils of the same age group might be at different levels, it must be recognised that offering them the same mathematics is likely not to be helpful. “The type of mathematics given to the children must be tailored to their capabilities. It is impossible to present abstract mathematics to all types of children and expect them to get something out of it.” She goes on to suggest that “mixed ability teaching as an entity is therefore unprofitable” (p 210) This is questionable, since it makes assumptions, about the way in which pupils are taught, that she makes clear have not been a consideration in the CSMS data and analysis. As with the Piagetian research, there has been little attempt to link what pupils have done and said to the learning environment to which the measurements relate.

TWO KINDS OF LEARNING

One of the implications for teaching, reported in the CSMS study, concerned the teaching of *rules* or *algorithms*. While on the one hand teaching algorithms may provide pupils with short cuts to solution of problems, on the other:

The teaching of algorithms when the child does not understand may be positively harmful in that what the child sees the teacher doing is “magic” and entirely divorced from problem solving. (Hart, 1981, p 212)

CSMS claimed that for the most part children did not use teacher-taught algorithms. They either adapted what they had been taught, or replaced them by their own methods. Only when these methods failed them, did they see the need for a rule at all. One problem suggested was that in teaching the rules, teachers use simple examples which children can work out in other ways. They then assume that the pupils can apply the algorithms and will remember them when required. However, in terms of the *schemas* discussed earlier, this means that the algorithms are never seen in conceptual terms and are never linked satisfactorily to the relevant schemas. Thus, children are unable to apply them when idiosyncratic methods are inadequate.

Skemp distinguished two kinds of learning, *rote learning*, or *rote memorizing*, and *learning involving understanding*, or *intelligent learning*. (1971) He later refers to these as *instrumental* and *relational* understanding (1976) These two kinds of learning are paralleled by Brown’s *algorithmic learning* and *conceptual learning* (1979).

Skemp recognises that:

If learning how to do a particular job, memorizing a set of rules may be the quickest way. If, however, one wishes to progress, then the number of rules to be learnt becomes steadily more burdensome until eventually the task becomes excessive. (1971, p 43)

Brown points out that, “Even if the child has a fair-sized repertoire of algorithms and ‘facts’, without some conceptual background he is unlikely to be able to apply them in practical situations.”(1979, p 355)

In a recent study into “ways in which knowledge ... is presented, received, shared, controlled, negotiated, understood and misunderstood by teachers and children in the classroom”, Edwards and Mercer (1987) distinguish between *ritual* and *principled* knowledge.

What we are calling ritual knowledge is a particular sort of procedural knowledge, knowing how to do something. In many contexts of course, procedural knowledge is entirely appropriate and exactly what is required.

Procedural knowledge becomes ritual where it substitutes for an understanding of underlying principles.

Principled knowledge is defined as explanatory, oriented towards an understanding of how procedures and processes work, of why certain conclusions are necessary or valid, rather than being arbitrary things to say because they seem to please the teacher. (p 97)

Edwards and Mercer were not speaking of mathematical knowledge particularly, but their distinction parallels closely the ones made above, where:

ritual = instrumental = algorithmic,
and *principled = relational = conceptual.*

They show as a result of their study that although a teacher had a declared intention of working towards a principled knowledge in her pupils (of the operation of pendulums), nevertheless as a result of her cues, amongst other reasons, the knowledge which resulted was closer to being ritual than to being principled.

In practice it may be very difficult to distinguish in any pupil where understanding is truly principled rather than ritualistic to some degree. Brown, discussing the problem of expressing $\frac{3}{8}$ as a decimal, showed that a child could reach a satisfactory resolution of this using simple recall, an algorithmic procedure, a conceptual structure, problem solving strategies, or combinations of these (p 354). In a recent study, into mathematics teaching at primary level, Desforges and Cockburn (1987) identified difficulties in drawing conclusions about particular pupils' levels of understanding and their basis. They referred to particular children's work and behaviour:

And in making these observations it is clear that the distinction between 'procedural' and 'conceptual' competence, whilst useful to make a point, is an uneasy one. The only way to know a child has a concept is to give them something to do that involves the use of that concept. But such tasks can never be 'pure', they must always involve more than the concept itself. Indeed there can be no useful distinction between having a concept and being able to use it on a real life – probably messy – problem. (p 94)

There are now two important questions. Conceptual understanding is to be valued above procedural understanding, so how is such conceptual

understanding to be attained? This question is of fundamental importance to the teacher in deciding on a scheme of work and a classroom approach. But, also importantly, how is such understanding to be recognised?

PUPIL CONSTRUAL AND ITS RECOGNITION

The psychological basis for conceptual understanding in terms of *schemas* may be helpful for the teacher as a mental model of how a person structures knowledge. The concept map, or hierarchy of mathematical concepts, could be of value to the teacher in making decisions about the order of presentation of mathematical ideas to pupils. The teacher can use both of these in starting to answer the first of the above questions (How is such conceptual understanding to be attained?) and, to some extent, the second (How is such understanding to be recognised?). However, in the practical situation, particularly with regard to the second question, it is crucial to consider the ability of teacher and pupil to communicate and to see into each other's perceptions and intentions. The teacher particularly needs to be aware of pupil construal in order to make decisions about further provision.

The teacher-learner interface is of such importance in my study that I shall introduce a metaphor which I find valuable in describing the teacher's task. It concerns a soft fruit such as a plum or peach which has a rich layer of pulp on the outside surrounding a hard kernel which contains the seed from the plant. The person eating the fruit tastes and ingests the pulp, but stops when teeth come up against the kernel which prevents access to the seeds beyond. The analogy which I draw is between the pulp of the fruit and the behaviour of a pupil. Just as the eater tastes of the pulp, the teacher experiences and interacts with the behaviour of the pupil (see the diagram below). The kernel which forms a layer around the seeds can be seen as a boundary of awareness and emotion of the pupil within which construal takes place. Construal compares to the innermost seeds which are not available to the eater. Pupil construal is not available to the teacher. The formation, composition and modification of the pupil's schemas, with which construal is concerned, are opaque to the teacher. What is transparent is the *behaviour* of the pupil. The teacher can also gain insight into the levels of awareness and emotion of the pupil, which provide clues to what is being construed.

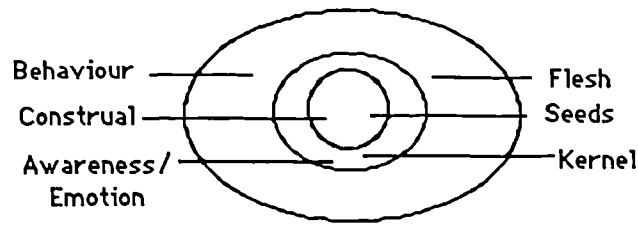


Figure 3.1: The Fruit Metaphor

The pupil, working on some mathematical task, talks with the teacher, and stimulated by the teacher's prompts and responses, reveals aspects of awareness which provide clues about construal, perhaps about schematic construction. The teacher, focusing on the pupil's activity and responses, adapts her own schematic representation of the pupil's level of understanding, and thus infers the pupil's needs.

One consequence of this metaphor for the teaching of mathematics is the importance of the relationship between teacher and learner. The barrier to the teacher, trying to gain access to pupil construal, is formidable. The teacher could be tempted to respond only to pupils' behaviour without making an attempt to go beyond. However, from a constructivist perspective, knowledge of pupils' constructions is vital in devising appropriate teaching.

Bauersfeld (1985) addresses this teacher-pupil interface when he writes:

Teacher and students act in relation to some matter meant, usually a mathematical structure as embodied or modelled by concrete action with physical means and signs. But neither the model, nor the teaching aids, nor the action, nor the signs are the matter meant by the teacher. What he/she tries to teach cannot be mapped, is not just visible, or readable, or otherwise easily decodable. There is access only via the subject's active internal construction mingled with these activities. This is the beginning of a delicate process of negotiation about acceptance and rejection. That is why the production of meaning is intimately and interactively related to the subjective interpretation of both the subject's own action as well as the teacher's and the peers perceived actions in specific situations.

He goes on to recognise further that, although a teacher and pupils may be working overtly on mathematical tasks, nevertheless,

Whenever we learn, all the channels of human perception are involved: i.e. we learn with all senses, ...

and he cites Dewey (1963) who wrote:

Perhaps the greatest of all pedagogical fallacies is the notion that a person learns only the particular thing he is studying at the time.
(p 48)

Thus a pupil's construal is of the total learning situation, which includes aspects of environmental and interpersonal relations within the mathematics classroom as well as the mathematics on which a lesson is focussed.

So far in this chapter, my focus has been on implications for teaching of the learner's construction of mathematical concepts. I shall now shift to the teaching/learning interface with consideration of the *social* context for mathematical learning and its implications for teaching.

The role of the teacher for mathematical learning

THE ZONE OF PROXIMAL DEVELOPMENT

If the teaching task is seen as enabling the development of conceptual understanding of mathematics in the pupils taught, then two strands may be identified: the *offering* of activities or tasks through which pupils will come up against mathematical ideas, and the *teaching approach* which will support the activities. It is likely that for any teacher these two strands are inextricably linked.

Many educational studies have sought to describe aspects of learning and the learning environment mainly from the learner's point of view without any reference to teaching. Desforges (1985) writes:

Prescriptions for the design of ideal learning environments often contain detailed analyses of the performance limitations of learners but take no account whatsoever of the performance characteristics of teachers. This is akin to designing aeroplanes on sound aerodynamic principles but in ignorance of the forces of gravity. (p 121)

Bruner (1985) claims that, "Too often human learning has been depicted in the paradigm of a lone organism pitted against nature" (p 25). While Piaget believed that learning resulted from the child's actions related to her external world, and that certain learning could not take place before the child was old enough to have developed the competence to learn, Bruner came to believe that appropriate instruction could hasten the learning process (cf Wood, 1988). Along with Vygotsky (1962; 1978), he believed that *language* was a fundamental ingredient of learning, that "language is a way of sorting out one's thoughts about things. Thought is a mode of

organising perception and action.” (Bruner, *ibid*) Vygotsky believed that the child’s social environment and the verbal interactions this included enabled the child’s concept formation. In Vygotsky’s words, “Human learning presupposes a special social nature and a process by which children grow into the intellectual life of those around them.” (1978, p 88) This has implications for instruction. According to Wood (1988, p 83), Piagetians would argue that “premature teaching serves only to inculcate empty procedures or learned tricks”, i.e. Skemp’s *instrumental understanding*. Vygotsky writes:

Our disagreement with Piaget centres on one point only, but an important point. He assumes that development and instruction are entirely separate, incommensurate processes, that the function of instruction is merely to introduce adult ways of thinking, which conflict with the child’s own and eventually supplant them. Studying child’s thought apart from the influence of instruction, as Piaget did, excludes a very important source of change and bars the researcher from posing the question of the interaction of development and instruction peculiar to each age level. Our own approach focuses on this interaction. (1962, p 116)

Whereas Piaget had not only *not studied* the influence of instruction on a child’s development, but had suggested that in some cases it could actually be harmful, Vygotsky was keen to research the effect of instruction on development. He grappled with the learning paradox of how the mind constructs the ‘toolkit of concepts, ideas and theories’ (Bruner, 1986, p 73) which allow it to reach the ‘higher ground’. Bruner puts great emphasis on Vygotsky’s concept, ‘the zone of proximal development’ (ZPD), which is “an account of how the more competent assist the young and the less competent to reach that higher ground from which to reflect more abstractly about the nature of things.” (Bruner, *ibid*) In Vygotsky’s words, the ZPD is:

the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers. (1978, p 86)

Whereas Piaget had suggested that instruction provided before the child was ready could be damaging, Vygotsky implied that, with appropriate instruction, there may be potential for the child to reach higher conceptual levels than she would be able to achieve naturally. Vygotsky went further:

Thus the notion of a zone of proximal development enables us to propound a new formula, namely that the only ‘good learning’ is that which is in advance of development. (*ibid*, p 89)

Bruner identifies a contradiction, which seems to be close to the learning paradox as expressed by Bereiter (1985):

On the one hand consciousness and control can come only after the child has already got a function well and spontaneously mastered. So how could this "good learning" be achieved in advance of spontaneous development since, as it were, the child's unmasterly reaction to a task would be bound initially to be unconscious and unreflective? How can the competent adult 'lend' consciousness to the child who does not have it on his own? (Bruner, 1986, p 74)

He refers to the implanting of a 'vicarious consciousness' in the child by his tutor, as if there is some 'scaffolding' erected for the learner by the tutor. Vygotsky had spoken of such scaffolding, but had been rather vague as to what it would entail, other than that it would be rooted in language. Bruner along with two others (Wood, Bruner and Ross, 1976) undertook some research to look at "what actually happens in a tutoring pair when one, in possession of knowledge, attempts to pass it on to another who does not possess it." (Bruner, 1986, p 75) Conclusions from this indicated that the tutor did indeed act as 'consciousness for two' for the children tutored. She demonstrated the task to be possible, kept segments of the task to a size and complexity appropriate to the child, and set up the task in a way that the child could recognise a solution and later perform it, even though she had been unable to perform it naturally before, or understand when it was told to her. The tutor did what the child could not do, and as the child became able to take on aspects of the task, these were handed over to the child, until eventually the child accomplished the task herself.

This raises a number of issues. The language used, particularly that of 'handover', suggests a transmission metaphor for teaching and learning. From a constructivist perspective, the child's construal of the tutor's words and actions must be the focus of consideration. The role of vicarious consciousness might in this respect be seen as creating space for pupil construal. The tutor is essentially close to the child, and is able to monitor the child's words and actions, coming up against her awareness of the concept. The notion of 'scaffolding' could result in dependency creation if the child became too reliant on the tutor's management. An extreme of the scaffolding principle is that the child never experiences the bewilderment of tackling a new task alone, and so is totally unprepared for any new task for which the tutor is not present. I discuss these ideas further in Jaworski (1990).

Wood, who worked closely with Bruner, went on to do further research in this area. He and colleagues (e.g. Wood, Wood and Middleton, 1978) set out to answer the questions:

How can we determine whether or not instruction is *sensitive* to a child's zone of development? When does it make demands beyond his potential level of comprehension? How can we be sure that instruction does not underestimate his ability? (Wood, 1988, p 78)

He talks of *contingent* instruction by the tutor – that is “pacing the amount of help children are given on the basis of their moment-to-moment understanding”. The scaffolding provided by contingent teaching could be suffocating of any initiative unless extremely sensitively applied. However, the metaphor of scaffolding seems to have potential for exploring teaching. It offers one means of *bootstrapping* (Bereiter, 1985). An important question seems to be what sort of scaffold would be appropriate in general problem-solving terms? I have suggested (Jaworski, 1990) that scaffolding might be interpreted in terms of a teacher's offering of strategies for thinking and learning, rather than for grasping a particular skill or concept.

LANGUAGE AND THE SOCIAL ENVIRONMENT

Vygotsky was concerned with language and its relation to the social environment enabling a child's concept formation. He wrote:

... children solve practical tasks with the help of their speech, as well as with their eyes and hands. This unity of perception, speech and action which ultimately produces internalisation of the visual field, constitutes the central subject matter for any analysis of the origin of uniquely human forms of behaviour. (1978, p 26)

and:

Human learning presupposes a special social nature and a process by which children grow into the intellectual life of those around them. (p 88)

This raises questions about how *mathematical* learning might be linked to language and social interaction. Jenner (1988) speaks of mathematics being seen traditionally as ‘dealing with universals’, and with an abstract nature which ‘reaches across cultural divides’. She further claims, “Mathematics is viewed as socially neutral and its content is held to be independent of the material world.” Bishop (1988) writes, “Up to five or so years ago, the conventional wisdom was that mathematics was ‘culture-free’ knowledge.”

My own experience supports these claims, and I believe that traditionally mathematics has been taught using the vehicle of natural language, as if that language bore little relation to the acquiring of mathematical concepts, and within a social structure, without regard to what influence that structure might have on the teaching and learning process.

There is a small but growing literature which concerns ways in which language and the social environment can influence the learning and teaching of mathematics. This has come, in parallel, through a concern with the language of mathematics and its relation to issues of language usage in learning more generally, and a concern for societal groups who might be seen as disadvantaged with respect to learning mathematics by the prevailing ethos of mathematical instruction. I shall treat these two strands separately below.

1: LANGUAGE AND MATHEMATICS TEACHING

The authors of the Cockcroft report wrote, “... mathematics provides a means of communication which is powerful, concise and unambiguous.” (DES, 1982, para. 3). Pimm (1987, p xvii) writes, “Mathematics is among other things a social activity, deeply concerned with communication.” Pimm explores the consequences for teaching of the perception of mathematics *as* a language, the metaphorical nature of the relationship being one of his main concerns, as well as the use of metaphor in mathematical expression. He quotes the mathematician René Thom as saying that the construction of meaning, rather than the question of rigour is the central problem facing mathematics education (ibid, p 7), and explores relationships between pupils making sense of mathematics and the language forms involved.

Austin and Howson (1979) review the literature in the field of language and mathematics. I shall here focus briefly on what seem to be the main issues for teachers of mathematics. These might be summarised under three headings, which I discuss briefly in the paragraphs which follow.

1. The use of mathematical language and problems of its similarity to and difference from natural language;
2. The importance of the articulation of mathematical concepts, their expression in normal language, and the associated imagery;
3. Mathematical symbolism and the reading and writing of mathematics.

The first of these concerns how the language of mathematics relates to natural language, and some of the difficulties this raises. The Cockcroft report states, "Children need also to learn that certain words are used in mathematics in ways that are not the same as those which they are used in ordinary speech" (para. 310). The mathematical usage of 'difference' was their particular example. However, there are particular mathematical terms, not in common usage, with which the learner of mathematics needs to be familiar. Pimm refers to a child's reference to a part of a pie-chart as a 'section', and the teacher's correction of this to the term 'sector', although *section* in this case seemed quite unambiguous (1987, p 60). The term *sector* labels a particular *type* of section of a circle which is important in considerations of the ratio of part to whole using angular measure. The teacher needs to be aware of words and forms of language used in mathematics which may cause difficulties for the learner, and pupils have to become aware of particular mathematical forms, reasons for their importance, and their conventional use.

Hughes (1988) goes further than this when he speaks of the difficulty faced by children when asked questions like, "how many is two and one more?". In a recognised context there seems no difficulty, e.g. in response to "how many is two elephants and one more?", the answer a child gave was "three". To the more abstract question, "how many is two and one more?", the same child said "six". There are questions about the mathematical, as opposed to the everyday use of language here, but there is also the question of abstraction. In how far is the language the problem, and in how far is it language complicated with abstraction with which pupils have difficulty in making sense?

The Cockcroft report (para. 246) put emphasis on the importance of *discussion* in the mathematics classroom, emphasising the value of pupils *own* expression of mathematical ideas and their negotiation with others. Such discussion requires the use of language. Pimm points to a common phenomenon – "when teachers ask pupils to try to articulate a difficulty they are experiencing, half-way through the resulting explanation pupils often say something like 'Oh, I see now, Thank you very much for helping me.'" (1987, p 23) The act of expressing has enabled them to clarify their thinking. Teachers can learn much of pupils' thinking and construal from listening to class discussion of ideas. For example, in Jaworski (1985a) I reported a class discussion about a poster in which a boy referred to seeing 'an octahedron with its mouth open'. Tahta (1970, p 27) offers a conversation with a seven-year old girl who refers to lines intersecting, or not intersecting a circle, as respectively *fighting* or

protecting. These pupils were expressing their own images in language which made sense to them and which was potentially revealing to the teacher. Pimm says of the latter example, "Who is to say without examining the idea further that perceiving geometric incidence and proximity in terms of threat and attack is not a mathematically useful way of viewing such situations." (1987, p 12). The notion of mathematical *usefulness* should perhaps be treated with caution if it could serve to repress the sharing of such images which are part of a pupil's total construal.

Traditionally, mathematics has been written largely symbolically. Pupils have been required to write their mathematical ideas using symbols and a requirement of the study of mathematical texts has been the ability to read and interpret symbolic forms. Skemp (1989) writes:

The power of mathematics is in the ideas. In the right partnership, symbols help us to make use of this power by helping us to make fuller use of these ideas. In the wrong relationship, a weak or barely existent conceptual structure is dominated by its symbol system, and mathematics becomes no more than the manipulation of symbols. Sadly this is the way it is for too many children." (p 99).

In summarising a chapter on the complexities of written mathematics, and use of symbols and notations particularly, Pimm claims:

Written mathematics is clearly not just spoken mathematics written down in words.

There is a widespread feeling that somehow only the fully symbolic representations are mathematical, and there is a strong tendency for teachers to move quickly to, for example, single letter variables. (p 136)

Pimm goes on to urge that attention be paid to pupils own spontaneous mathematical recording and its relation to context, suggesting that this might help pupils to write symbolic representations of their own mathematical ideas, and to appreciate conventional forms. However, recent research into the use of algebra in computer-based environments suggests that "the language of algebra cannot in some way be separated from the algebraic process and grafted on as a final step" (Sutherland, 1991, p 170). Sutherland suggests that, in the computer-based environment, symbolism is integral to the negotiation of generalisation for many pupils, rather than a final stage in expressing the generalisation.

However, what of pupils trying to read apparently disembodied forms of symbolisation in the text books which they are given? Shuard and Rothery (1984) offer, as an example, the following number sentence from a text, in

which the child is required to fill in the empty box: $24 \div 3 = \square \times 4$.
They comment:

It is easy to see why a child could fill in the answer '8' ... reading from left to right, the pupil completes what appears to him to be the first calculation, and then perhaps ignores the calculation of 8×4 , for which a box is not provided. Thus he needs to comprehend the sentence in this example as a whole, so that the missing number can be filled in to make the sentence true." (p 150)

How the child comes to comprehend the need to *see the sentence as a whole* must raise serious concern for teachers. Shuard and Rothery discuss methods of improving children's reading ability and conclude that their effectiveness needs to be investigated, and other methods need to be devised.

In all of the issues raised above, more research is required to make clearer the relationships between language use and mathematical teaching and learning.

2: THE SOCIAL ENVIRONMENT IN MATHEMATICS TEACHING

One aspect of language which was not addressed above was that of the bilingual child who is not only grappling with the difference between mathematical language and the English language, but whose natural language is not English and so is one stage further removed. Jenner (1988) states:

It is also possible that as teachers we assess bilingual children's mathematical ability without reference to their first language, again creating lower expectations and failing to recognise the pupil's full abilities by confusing fluency in English with mathematical competence. (p 75)

Jenner suggests that pupils should be encouraged to discuss mathematical ideas in their natural language, and should be encouraged to use this language when being assessed. There are complex issues for the teacher here, and again more research is needed.

Also currently much under scrutiny are issues of relating the learning of mathematics to pupils' ethnicity and gender. There is evidence of lower attainment in girls and among pupils from certain ethnic origins at some stages of education. (DES 1985a,b) Research is starting to consider how mathematical learning environments affect girls, or children from non-European origins, or from different social classes. In particular, what sort of environments support the learning of these children?

2.1: CULTURAL PERSPECTIVES

One approach is that of deliberately relating classroom activities to the cultures represented in the classroom. As well as providing familiarity for pupils concerned, and making evident respect and value for their culture, such activities can provide *new approaches to mathematical concepts*. For example, Emblen (1988), writing of her work with Asian children, talked of collecting different number names and symbols from children and parents of different cultures, e.g. Bengali, Urdu and Hindi, and comparing them. She said:

Nobinul noticed that in all the notations we had collected, two digits were used to express ten. This led to valuable work on the way numbers systems work and on 'tens and units'.

The Cockcroft Report (para. 224) suggests that the use of the Rangoli patterns used by Hindu and Sikh families to decorate their homes might form a basis for study of shape and space, and many other publications have offered cultural objects as a basis for mathematical study (see for example: Hemmings, 1980; Zaslavsky 1979). Woodrow (1989), valuing the possibilities offered, nevertheless gives a word of caution:

It is important not to import content merely in order to satisfy external requests and pressures. Tokenistic responses never meet such needs and usually result in increasingly heightened concern. Care must also be taken not to introduce such topics as marginal and trivial activities since this can imply a dismissive view of other societies and values.

To avoid tokenism, teachers need considerable knowledge of cultures represented in their classrooms and in society more generally in providing a multi-cultural approach to their mathematics teaching. The example from Emblen above seems of particular value to a teacher, indicating that more observations of this kind from teachers who are themselves learning about their pupils would be of great value in increasing the scant knowledge in such areas. Bishop (1988) expresses these ideas in more global terms:

The thesis is therefore developing that mathematics must now be understood as a kind of cultural knowledge, which all cultures generate but which need not necessarily 'look' the same from one cultural group to another. ... Mathematics is a pan-human phenomenon. Moreover, just as each cultural group generates its own language, religious belief etc., so it seems that each cultural group is capable of generating its own mathematics.

There are implications for the teacher of mathematics:

Inducting a young child into part of its culture is *necessarily* an interpersonal affair, and therefore teachers must be made fully aware of this aspect of their role. More than that they need to know about the values inherent in the subject they are responsible for, they need to know about the cultural history of their subject, they need to reflect on their relationship with those values, and they need to be aware of how their teaching contributes not just to the mathematical development of their pupils, but also to the development of their culture.

These are strong demands and, although they were directed at teacher-educators, nevertheless they present a serious challenge to the teacher already in the classroom.

2.2: GENDER AND CLASS

Isaacson (1988), writing about gender issues, quoted a mature woman speaking of her school experience of mathematics:

Especially in maths I couldn't stand the competitiveness of seeing the 'clever boy' (there's always one in every class!) do the exercises miles before everyone else and understanding things quicker than everybody which just reinforced the idea that I didn't have a 'mathematical brain'.

There are many theories concerning why boys at some levels are seen to do better at mathematics than girls, for example, the relative maturity of girls and boys at different ages; their differential early learning experiences; boys' superiority in spatial visualisation, and girls' preference for a collaborative rather than a competitive classroom environment. It has been suggested (Smith, 1986) that girls' performance improves when they are taught in single sex classes for some years within a mixed school. Scott-Hodgetts (1986), taking Pask's distinction of *serialist* and *holist* learning strategies (e.g. Pask, 1976) suggests that girls tend to serialist, and boys to holist strategies, whereas teaching at primary level is mainly serialist. Thus teaching reinforces the girls' strategies while encouraging the boys to become more versatile learners, providing boys with an advantage where secondary studies are concerned. Walden and Walkerdine (1985) suggest that girls' characteristics of femininity lead teachers to assume a lack of mathematical understanding when "indicators of 'real understanding' are to a large extent coterminous with those used to describe masculinity." In conclusion to the report of their study they recognise that their paper is able only to raise highly contentious and debatable issues. However, they say:

if we have succeeded in putting on the agenda central problems concerning the education of girls and women, our research will have been worthwhile. (p 108)

There is little evidence of the effects of children's social class on their mathematical learning. However, one study, undertaken by the Girls and Mathematics Unit of the Institute of Education in London, following the development of a group of girls, looked at how class difference cut across gender. At the age of four, 15 girls were identified as middle class and 15 as working class. At the age of ten, the gap between standards and lifestyles of the two groups had widened but, "more dramatic and depressing, however, was the huge gap in educational attainment between the groups". Comparing results from a standardised mathematics test the middle class girls were overwhelmingly more successful than their working class counterparts. (Walkerdine, 1989) The authors relate these test scores to the attitude and position of the girls within their classes and to their teachers' declared views of them, concluding that the working class girls particularly are disadvantaged (especially by the testing system in operation), but that all girls undergo pressures and face attitudes which negatively influence their mathematical achievement. Much of this disadvantage is put down to their teachers' attitudes.

3: THE SOCIAL CONTEXT

Although the many theories referred to above need much further research, their 'putting on the agenda' of problems and issues, pointed out by Walden and Walkerdine above, seems their most important consequence for teaching currently. Jenner (1988) states:

Western society places the highest value on the most abstract, thus creating an elitism which means many people feel alienated from mathematics, and apart from small groups, feel it has little to do with their lives. In mathematics education we need to re-examine our approaches to ensure pupils see mathematics as for and about everyone, thus promoting recognition of mathematics as an activity we can all engage in. (p 75)

Bishop (1988), contrasting mathematical education with mathematical training, states:

Surely a mathematical *education* [as opposed to mathematical training] should make the values explicit and overt in order to develop the learner's awareness and capacity for choosing.

Maxwell (1985), looking at values implicit in many of the situations which are offered as context for mathematical study, pointed out the powerful political messages that can be covertly conveyed. Making these overt

would at the same time emphasise that mathematics is not devoid of social implications, and that perhaps its teaching can be designed for support of more humane images.

Whereas much research in mathematics education has been directed at the individual learner, Bishop and Nickson (1983) suggest that such research, “should be directed away from the individual child as learner and towards an increased understanding of the effects of the social context of schools on the learning of mathematics.” Nickson (1989) suggests:

“Rather than simply looking for aspects of different cultures to exemplify mathematical ideas (thus supposedly providing familiar social contexts for learners), what we should be looking for is how their cultural perspective may affect their mathematical mode of thought.”

D'Ambrosio has written extensively of the cultural bases of mathematics education (e.g. 1986), and Bloor (1976) identified ‘social causes of mathematical thought’ in connection with accommodating cultural individuality within the mathematics curriculum.

Eales (1986), concluding a report on a study about gender conducted by his mathematics department, writes,

Good practice is girl friendly.

This is not the tautology it seems, or at least it has not been recognised yet! Emphasis on the learner, due regard for the individual, respect for each personality, the reduction of competition but encouragement of personal achievement will all improve the lot of girls.

These quotations point towards the need for a greater social awareness in the mathematics classroom, for awareness of the values related to the way mathematics is presented, for understanding of ways in which culture can influence mathematical thinking, and moreover for creation of environments based on mutual care and respect. These put very great demands on teachers, requiring both awareness of the complexity of the issues involved, and an open-ness of approach in exploring what might be possible. I observed teachers who were, in varying respects, tackling these issues.

The trouble with mathematics teaching

The teachers in my study, employing to some degree an investigative approach as outlined in Chapter 1, could be seen to attempt to address the

need which is expressed by Desforges and Cockburn (1987), recognising criticisms of children's mathematical competence:

It seems that children do not need further doses of basic skills training. Rather they need to acquire a 'feel' for number and other mathematical processes, together with a degree of intellectual autonomy which would enable them to go beyond routine calculations and to solve real-life problems involving mathematical thinking. They need to learn to use their skills with flexibility. They need to learn to think with mathematics rather than merely respond with routines. (p 3)

In a report on their study of the practice of mathematics teaching in first schools, Desforges and Cockburn claim that "The conservative nature of teaching practices has been most persistently criticised for the general rejection of methods of teaching considered to foster the attainment of *higher level learning goals* (such as learning to learn, problem solving, learning strategies)." (my italics). Their study supports the view that it is not that teachers do not share these goals, but it is the implementation of these goals which is prohibitively difficult. I shall consider their views in some detail as they raise issues and questions of direct relevance to my own study in which, to some extent, I have arrived at different conclusions to those of Desforges and Cockburn.

They devote an introductory chapter – 'The Trouble with Mathematics Teaching' – to explaining the situation in which mathematics teachers currently find themselves, particularly with respect to the views and expectations of the mathematics education community regarding innovations in classroom practice. They refute suggestions that teachers are not aware of the underlying pedagogic principles of the preferred practices:

It seems unlikely that the failure of these innovations rests on teachers' lack of appreciation of their underlying philosophy or rationale. At the heart of the call for more practical work, investigations and applications is a view of the learner as intrinsically motivated and inventive, a view of the syllabus as flexible and negotiable and a view of the teacher which leans towards power sharing rather than the autocratic. In general terms this model of learning, teaching and classroom relationships has been set before teachers for many decades. Since the popularisation of Piaget's work the virtues of the model have been extensively advertised. It seems to us inconceivable that teachers are not aware of the claims made for it. The resistance to the model may be seen to be broader and deeper than that. (p 11)

They go on to point out evidence that, although teachers are indeed aware of the pedagogic principles of suggested good practice, they nevertheless fail to convert them *into* practice and quote a poignant statement from a teacher:

I don't know why I keep going to meetings to learn more about becoming a better teacher. I already know how to teach ten times better than I ever can. (Brown, 1968)

The implementation of such principles typically involves the teacher in, according to Jackson (1968), a huge number of often conflicting demands, which Desforges and Cockburn paraphrase as:

She must attend to individual children while monitoring the rest of the class, supply corrective feedback whilst developing confidence and give children time to think while keeping her eye on the clock. The teacher distributes time to activities and attention and physical resources to children. She organises movement about the room, the composition of groups and the flow of events. (p 15)

They cite Doyle (1986) who has written about classroom organization and management, suggesting that there are many constraints on classroom activity in terms of the expectations of pupils and teachers, their individual needs and preferences, the necessary tasks and activities and a scarce supply of resources, which all contribute to difficult choices having to be made. Doyle claims that the complex classroom scene can only be organised fruitfully with the cooperation of the students, and that the work set is an important feature in sustaining this cooperation. If cognitive tasks of too high a level are demanded by the teacher, then students are likely to be less cooperative. Further, the rewards for tasks with higher level cognitive demands are elusive, with high levels of risk and ambiguity. Doyle paints a picture of a complex classroom setting, overloaded with information and events, requiring that the teacher select information and impose order on events. This has serious consequences for tasks with higher level cognitive demands. According to Desforges and Cockburn, paraphrasing Doyle:

Tasks with higher level cognitive demands increase the pupils' risks and the ambiguity involved in engagement and thus alter the commonly (and usually readily) established exchange rate in classrooms - that of an exchange of tangible rewards for tangible products. Pupils like to know where they stand. For this reason tasks demanding higher order thought processes are resisted or subverted by pupils. Resistance puts cooperation at risk. Teachers are lured into or connive at subversion and higher level task demands are frequently re-negotiated in the direction of routine procedures. (p 21)

Although Doyle's account is based on only a limited number of studies, and those related to the North American experience, Desforges and Cockburn find it "attractive" since it is unique and its issues transcend national boundaries¹. They therefore set out to answer a number of related questions:

Is it the case that the problems of classroom management produce and sustain the contemporary and limited classroom practice in teaching mathematics? If so, what do those problems look like to the teacher. What factors do teachers take into consideration in adopting management and teaching techniques and what factors force her to amend her goals or behaviours in the day-to-day world of the classroom? (p 23)

Their study focused on seven, experienced, and generally successful teachers, in depth. They observed and video-recorded lessons, talked with teachers and used the videotapes for stimulated recall. Their report presents views of teachers, pupils and researchers. Their methodology has many points of contact with my own.

Their conclusions celebrate the commitment of the teachers they observed to their pupils and to the teaching of mathematics. They are at odds with critics who:

imply that teachers lack commitment, will, effort, imagination, an appreciation of children's intellectual strengths and the industry to capitalise on them. (p 138)

and repudiate the interpretation that "routine work is the consequence of the routine and complacent minds of teachers and teacher educators". Instead, they provide evidence to "show clearly the intellectual quality, insight, imagination, and industry of the teachers as they struggled to deliver the curriculum under severe constraints" (p 139).

This struggle was in the face of:

a crowded curriculum and an attendant push for coverage;
the motives, skills and attendant capacities of children;
the low quality of conceptual support for the teachers;
the sheer quantity of information processing necessary on the part of the teachers to sustain motivation and learning among thirty extremely diverse children. (p 139)

¹ Subsequent research in North America has shown teachers who *were* able to modify their classroom practice, despite constraints placed on their operation. (Cobb, Yackel & Wood, 1988)

From this the authors conclude:

classrooms as presently conceived and resourced are simply not good places in which to expect the development of the sorts of higher order skills currently desired from a mathematics curriculum. (ibid)

Although they recognise that many higher order skills were in evidence, for example, children organising their own comings and goings, resourcing their own activities, and monitoring their own performances; there was no evidence of the application of these sorts of skills to mathematical problems or to the mathematical curriculum in general. They say:

The teachers showed that with considerable effort and skill the classroom can be made a civilised and thought demanding social environment. It did not follow - and does not follow - that it is a good environment for nurturing the sorts of academic skills that mathematics educators have in mind. (p 140)

In considering the way ahead, they demolish a number of current theories, ideologies and practices which relate to the nurturing of higher order skills in mathematical thinking and learning. I shall conclude references to this work with just two of these, which have particular implications for my own study.

The first is '*the notion of developing thinking skills through investigations work*'. Desforges and Cockburn make two observations:

- there is not a shred of evidence that investigations actually enhance the intellectual processes they are intending to nurture;
- investigations are high in risk, ambiguity and information processing demands. (p 151)

They conclude that:

We might therefore anticipate, in terms of our analysis, that classroom processes will operate to close down these risks and to convert open problems into predictable routines. (ibid)

The second refers to "*the 'inspired teacher' lobby*".

Set against formal, class teaching and all in favour of enquiry methods, these teachers recognise the problems of organisation and management that their approach faces. But they see no problem that cannot – indeed has not – been solved by a mixture of inspiration and devotion. They claim to have broken through the very barriers that we suggest limited even our very able teachers. Their classrooms draw on the spontaneous skills and interests of the children. They are in touch with the latest research on children's learning. The teachers have the capacity to monitor each individual child; they see when to

intervene and when to leave alone. They advance learning with a task here, a comment there and they learn from every exchange. (p 142)

The authors claim *never* to have seen such practice, despite searching for it for over ten years, except for a few moments or for special events. They urge any teachers who fit this mould to offer themselves for 'detailed, long term objective research'.

My own study

The teachers with whom I worked were constrained by the British educational system in that they were required to prepare pupils for nationally administered examinations at 16+. Thus, the conceptual hierarchies and modes of teaching which they developed had to fit the requirements of this system and the associated expectations of pupils, parents, school head-teachers and governors, local education authorities and politicians. This demanded their adherence to a mathematics syllabus designed by their chosen examination board and their preparation of pupils for success in the GCSE examinations. More recently, demands have increased with the advent of a National Curriculum, imposed by law, and including its own hierarchies of mathematical concepts with associated testing tasks, but this was not in place when my classroom studies were conducted.

One implication of a given syllabus, which has been emphasised by the introduction of the National Curriculum but not necessarily changed by it, is that teachers are not free to choose *what* mathematical topics they wish to teach. During my study, they were free to determine their own hierarchies within the given syllabus, and that might now be seen to have changed. In the first two phases of my research, pupils in the classes which I studied followed individualised commercial mathematics schemes during some of their lessons. These, to an extent, imposed their own hierarchies. However, in both cases, teachers used the scheme flexibly in that they interpreted it where they could to suit the children and class, and, in parallel with the scheme, they designed other lessons in which they themselves determined conceptual hierarchies. In the third phase of my research one teacher used no such scheme. He determined his own hierarchies and used published materials only to fit in with his own design.

In considering investigational work in the classrooms which I observed, I shall be concerned to address how these teachers approached the teaching of mathematical topics, how their approaches took into account the linking

of mathematical concepts, and what issues arose from these considerations.

I shall address the offering of activities and the way in which these have brought pupils up against particular mathematical ideas, stimulated involvement in mathematical thinking and encouraged development of higher order mathematical thinking skills.

The schools in which observations took place operated with mathematics classes of 28 or more pupils. These pupils, girls and boys, came from diverse social backgrounds although in none of the three schools was there a significant black or Asian community. The teachers worked under pressures which in my experience were typical of secondary education. Resources were scarce and had to be shared between teachers. All teachers had pastoral responsibilities. Assessment and administration took up a large proportion of their time.

I shall address how the teachers interacted with the pupils in their classes, how they created their particular classroom environment, what ethos prevailed and how this related to social issues.

I shall speak directly to the conclusions of Desforges and Cockburn which, although based on first schools, addressed issues which pertained no less to the secondary environments which I studied. I recognise the very limited nature of my study, and my particular selection of teachers, although neither I nor they would claim to be part of the 'inspired teacher lobby'. I, and I believe they, would readily recognise limitations of the classroom practice which I observed. However, high level cognitive demands *were* made and high level mathematical thinking skills *were* in evidence, despite the problems of classroom organisation, limited resources etc. which were as evident here as in Desforges and Cockburn's study.

Finally I reiterate that my over-riding concern in this study has been the way in which mathematics teaching can be designed for the enabling of effective construal of mathematical concepts, the issues arising from this, and its implications for mathematics teachers.

PART 2

RESEARCH

CHAPTER 4 METHODOLOGY

Introduction

... we should bear in mind that whilst the question of what we are to look at is by no means a trivial one, it is a little less important than the question of how we are to look at whatever we do look at. (Sharrock and Anderson, 1982)

In seeking to characterise an investigative approach to the teaching of mathematics¹, it was obvious that the subjects of my research would be teachers of mathematics and their pupils, and that the research would take place in mathematics classrooms and their relevant surroundings. What was less obvious in the beginning was the precise methodology which I should employ. Edwards and Furlong (1985) write,

Whatever their methodological persuasion, it is always tempting for researchers to present their final report as 'the best possible interpretation of events, inferior versions having been discarded along the way' (1985, p 21).

It is tempting in retrospect to imply that the methodology which I used was 'selected' for its suitability to my study, and in particular for its compatibility with my theoretical perspective. Although in general terms this is true, in practice the finer details of the methodology evolved as the study developed. This chapter explains my choice of a broadly ethnographic style of research and elaborates the development of its detail and the resolution of issues and tensions in its practical application.

An initial choice

Many studies of lessons, where focus has been on the interactions of teacher and pupils, have employed a process known as *systematic observation* or *interaction analysis*, based on the work of Flanders (1970). Broadly this has involved researchers in recording the incidence of certain predetermined features of such interaction. Hammersley (1986) suggests that this :

¹ The phrase 'an investigative approach to the teaching of mathematics' will be required frequently in what follows and for brevity it will be shortened to 'an investigative approach' or sometimes to 'investigative teaching'.

typically involves the observation of large samples of teachers and pupils by observers using a coding scheme in which the activities taking place at regular points in time or in particular time intervals (say every 3 seconds or every 25 seconds) are checked off.

Use of this approach would involve being able to identify critical features of an investigative approach for which to look out in selected classrooms. As a teacher myself, I had given much thought to what these critical features might be (Jaworski, 1985), and could have devised a coding schedule related to this thinking. However, observation of large samples would have been difficult both operationally – working alone on a part-time basis, and practically – a large sample of teachers employing investigative approaches would have been hard to find. More importantly, I wished to characterise investigative teaching from practical manifestations of it in classrooms, rather than from preconceived notions which I held. I followed a precedent set for this by Fensham et al (1986) who set out to characterise *alienation in schooling* in a similar way. This seemed to require a less prescribed study of classrooms than systematic observation allows. I therefore decided, in the very early stages of Phase 1 of my study, to look towards *ethnography* as a methodological approach.

An ethnographic approach seemed to involve a classroom observer in studying and trying to make sense of the whole activity of a classroom. Thus, rather than viewing from an overt given perspective, the observer would try to begin with a clean slate and write onto it some description of what was seen to occur, which could then form the basis of future analysis. I now realise that this simplistic view of ethnography was responsible for much of the methodological tension which I experienced during the early part of my research, particularly in its relation to my emerging theoretical base. For example, how far was 'what I saw' conditioned firstly by my own perspective and secondly by my own involvement in the activity which I observed?

However, the development of my research and of my methodology went hand in hand. Trying to take in the subtleties of ethnography as seen by others, in my early days when I had no basis to which to relate what I read, proved difficult. I had to start from the simplistic view which I expressed above, and learn from the questions which arose as I proceeded. This necessity was later supported by Ball (1990) who wrote:

The prime ethnographic skills cannot be communicated or learned in a seminar room or out of the textbook. Students can be prepared, forewarned, or educated in ethnography, but the only way to learn it is to do it. The only way to get better at it is to do more of it. My point is that ethnographic fieldwork relies primarily on the engagement of the self, and that engagement can only be learned enactively.

When I came to interpret an investigative approach in terms of a constructivist view of knowledge and learning, there seemed to be a consistency between the theoretical basis of my exploration and the methods used to explore. Eisenhart (1988) writes:

Numerous mathematics education researchers (I am thinking particularly of constructivists, of those interested in what teachers or students are thinking and actually doing in classrooms, and of those interested in the social context of mathematics education) are posing questions for which ethnographic research is appropriate.

Briefly, ethnography seemed to allow the construction of knowledge regarding investigative teaching, which would lead to the interrogation of my own previous knowledge and experience. This raised questions about the relation between theory and methodology on which I shall elaborate. However, the 'engagement of self' was from the beginning a central feature of my research.

Before elaborating the particularities of my methodology, I shall raise some issues about ethnography which are relevant to my study.

Ethnography, or an ethnographic approach

In a seminal paper, Hamilton and Delamont (1974) simultaneously attacked approaches to classroom observation which used a prespecified coding schedule, and made a plea for more attention to be paid to an alternative observational strategy, the *ethnographic*, also called *participant observation* or *anthropological observation*.

They updated their criticisms and reasons for preferring an ethnographic approach in a later paper, (Delamont and Hamilton, 1984), where they say:

Part of our attachment to the ethnographic is a desire to treat educational research as an 'open-ended' endeavour, where premature closure is a dangerous possibility.

They point out that prespecified coding systems, such as that of Flanders, depend totally on the observer's interpretation and fail to take into account any of the teacher's intentions. They say:

... by concentrating on surface features, interaction analysis runs the risk of neglecting underlying but possibly more meaningful features.

and go on to describe the alternative offered by ethnography:

The ethnographer uses a holistic framework. He accepts as given the complex scene he encounters and takes this totality as his data base. He makes no attempt to manipulate, control or eliminate variables. Of course the ethnographer does not claim to account for every aspect of this totality in his analysis. He reduces the breadth of enquiry systematically to give more concentrated attention to the emerging issues. Starting with a wide range of vision he 'zooms' in and progressively focuses on those classroom features he considers to be most salient.

An important issue for me has been that of 'totality' and the subsequent focusing of attention on perceived significant aspects of this totality, and I address issues related to 'significance' later in this chapter.

However, McIntyre and Macleod (1978), supporting the use of *systematic observation* techniques as an approach to classroom observation, made the point :

Any research undertaking reflects implicit values in the sense that the researcher focuses attention on some things to the neglect of others.

They went on to say that, while systematic observers make explicit their aspects of focus, it is very often not clear how far the *implicit values* of other researchers affect their conclusions. A consequence of this seems to be that the ethnographer, working to the ideals described above, must nevertheless be aware of personal interest, focus and emphasis and make overt recognition of this in analysing ethnographic data. My awareness of my own implicit values grew during my three phases of research and is one of the methodological developments which I shall chart.

Another criticism which proponents of systematic observation make of qualitative research methods is that it is very difficult to make and justify generalisations which might apply to other settings. Delamont and Hamilton (ibid) address this by recognising the difficulty, yet claiming that *some* degree of generalisation makes sense:

Despite their diversity, individual classrooms share many characteristics. Through the detailed study of one particular context it is still possible to clarify relationships, pinpoint critical processes and identify common phenomena. Later abstracted summaries and general concepts can be formulated, which may, upon further investigation be found to be germane to a wider variety of settings.

They claim that generalisations made from “good” ethnography are just as useful to both researchers and practitioners as those made from systematic observation, the great strength of ethnography being that it gets away from the simplistic behavioural emphasis of coding systems. In my own study, it will be important to consider how far the characteristics I draw from the classrooms I observe are indicative of investigative approaches more generally, or of relevance to other teachers wishing to interpret a constructivist philosophy in mathematics teaching.

In the paragraph above, emphasis of the word ‘good’ was mine. Delamont and Hamilton did not qualify their use of it. However, Lutz (1981) drew a distinction between ethnography and ethnographic methods:

*Ethnography ... is ... first and foremost a ‘thick description’. As such it involves many techniques and methods which can be described as ethnographic, including, but not limited to, participant observation, interview, mapping and charting, interaction analysis, study of historical records and current documents, use of demographic data, etc. But ethnography centers on the participant observation of a society or culture through a complete cycle of events that regularly occur as that society interacts with its environment. The principle data document is the researcher-participant’s diary. Ethnography is a holistic, thick description of the interactive processes involving the discovery of important and recurring variables in the society as they relate to one another, under specified conditions, and as they affect or produce certain results and outcomes in the society. It is not a case study which narrowly focuses on a single issue, or a field survey that seeks previously specified data, or a brief encounter (for a few hours each day for a year, or 12 hours a day for a few months) with some group. Those types of research are ethnographic but *not ethnography*. They may be good research, but when they are passed off as ethnography, they are poor ethnography and poor research.*

Lutz goes on to distinguish between macro- and micro-ethnography, the former involving the thick description of which he talks, but the latter occurring when the focus is on small groups or limited time-scales. This latter he says is ‘unfortunately’ consistent with the type of ethnographic work often encountered in education.

It applies to the study of small groups, often to a larger group, such as the whole class, and occasionally to single schools. This limitation tends to exclude studies of educational issues and questions in a broader and at least as important context – that of the school district-community, cultural perspective. I suggest that the narrow focus, while generating some important knowledge, fails to shed light on the more complex issues that account for much of what goes on (or doesn’t go on) in schooling.

I recognise that my own study must come in the realm of micro-ethnography, in Lutz's terms, and in what follows I shall talk about an ethnographic approach in this more limited sense, although I note that many writers use the term 'ethnography' more loosely than Lutz allows. I claim that my work fits Lutz's description of 'the interactive processes involving the discovery of important and recurring variables in the society as they relate to one another, under specified conditions, and as they affect or produce certain results and outcomes in the society'. The *society* in my case consists of the teacher and pupils within the mathematics classroom where my observations have taken place. I have focused on the teaching and learning of mathematics, and relationships in and outside the classroom where they have impinged on this teaching and learning. As I envisage this study being of interest to mathematics teachers and educators, rather than to administrators or policy makers, it is less important to have studied the whole educational scene. However, I challenge any attempt to describe the '*whole*', even in terms of the classrooms where my attention has been focused. Even Lutz's *thick description* must be a consequence of the researcher's knowledge and interpretation. Ball (1990) writes:

There is much that researchers do not know about the lives of those they study, but too often accounts fail to alert readers to the limits *within which the portrayal and analysis should be read*. ... Implicitly or explicitly, ethnographers claim too often to have produced definitive accounts of the settings they have studied.

The place of theory in an ethnographic approach

The question of the observer's unavoidable focus and emphasis, which may or may not be explicit, but which needs recognition, leads to questions relating theory and methodology. Furlong and Edwards (1977) claim that the separation of these is unrealistic since the researcher's theory "determines not only how the 'data' are explained, but also what are to count as data in the first place". Recognising that it may be the researcher's intention to present to the reader as full a description as possible so that the reader can "experience a sense of event, presence and action" (Kochman 1972 p xii), and be able to check the researcher's interpretation, they say nevertheless:

Although the ethnographer is committed to having as open a mind as possible during his period of observation, it is inevitable that he will begin his work with some preconceptions and some foreshadowed problems which will lead him to pay attention to certain incidents and ignore others. If he presents his observations as 'objective description', he is probably naively unaware of his own selectivity. On the other hand, if he follows a theory too closely, he will be accused of selecting observations to support his own point of view. (Furlong and Edwards, 1977)

There seems to be some skill in weaving a path between the two polarisations which are expressed here, and I am very much aware of the implications of this in my own work. An important theme of this study will be the rationalisation of patterns which emerge from data with my own theoretical base.

Glaser and Strauss (1967) refer to 'an opportunistic use of theory' which they call 'examplimg':

A researcher can easily find examples for dreamed-up, speculative, or logically-deduced theory after the idea has occurred. But since the idea has not been derived from the example, seldom can the example correct or even change it (even if the author is willing), since the example was selectively chosen for its confirming power. Therefore one receives the image of a proof where there is none, and the theory obtains a richness of detail that it did not earn.

They continue:

We have taken the position that the adequacy of a theory for sociology today cannot be divorced from the process by which it is generated. Thus one canon for judging the usefulness of a theory is how it was generated – and we suggest that it is likely to be a better theory to the degree that it has been inductively developed from social research. (p 5)

The inductive development of theory from social research, they call 'grounded theory'.

The question of theory exercises many writers in the area of ethnographic research. Hammersley (1990) argues that too little theory results from such research, perhaps as an over-reaction to 'positivism'. He cites *differentiation-polarisation theory* as seen in the work of Hargreaves (1967), Lacey (1966 and 1970), and Ball (1981), as a rare example of what is possible. He recognises the focus on theory of Glaser and Strauss, but points out that few examples of ethnographic work are based on their model. Hammersley suggests that ethnographic research on schools puts much emphasis on qualitative descriptions of behaviour, supported by

extracts from field notes or transcripts, but little on explanations for patterns discovered (p 102). Disputing criticisms of the 'narrowness' of his cited works he suggests that for the development and testing of theory to be pursued effectively, the research focus *has* to be narrow.

Woods (1985), also recognising the contribution of Glaser and Strauss to a general formula for theory generation, seems to concur with Hammersley in his view that "this general formula is not without difficulties ... the drift of ethnographic studies in education in Britain over the last decade has actually gone against its promise in the area of theory". In Woods' view, the very nature of ethnography as a descriptive approach concentrating on the construction of meaning of interactants has militated against theory building. However, he urges that the notion of *theoretical insight* should be given more credence, that is the creativity of the researcher in reflecting on the research and coming up with 'brilliant ideas' which stand up in further reflection. He claims that ethnography has suffered from Becker's (1958) argument that qualitative research should become more scientific, and urges that "theory can be aided by an artistic frame of mind" (p 71).

In my own research I have found notions of theory problematic. The *teaching triad*² might be regarded as an example of grounded theory, since it arose from close scrutiny of the data from one teacher and was checked against data subsequently collected. Indeed it might be thought to have *influenced* subsequent collection of data, so that this later data collection could be seen, in Glaser and Strauss's terms as 'theoretical sampling'. However, I believe that my overall pattern of data collection is more in the ethnographic tradition than theoretical sampling would allow. An example of this concerns the decision which I had to make regarding my level of explicitness in testing out the teaching triad in Phase 3. This is discussed further below (see p 82).

The basis of all observations was the desire to characterise an investigative approach to mathematics teaching, or subsequently an approach consistent with a constructivist philosophy of knowledge and learning. Inevitably, therefore, notions of constructivism underlie all attribution of salience in my observations. This is consistent with my quotation from Furlong and Edwards above. However, it might lay my work open to criticisms of 'examplifying' – that perhaps I have chosen what to look at in such a way that this adumbrates my conclusions. I have also not only been aware of

²The *teaching triad* arose from analysis of data from the teacher Clare in Phase 2, and was tested on data from the teachers Mike and Ben in Phases 2 and 3 respectively. Details will be found in Chapters 6 and 7.

the importance of theoretical outcomes (Hammersley, *ibid*), but also tempted by what Woods describes as 'brilliant ideas'. I have been wary of both these forces towards what might be spurious theory building. I have therefore needed to consider carefully the validity of my research in terms of decisions made and processes used to verify conclusions.

Validation and Rigour

Cicourel (1973) makes criticism of a work in which he claims that all descriptive statements are 'prematurely coded':

that is, interpreted by the observer, infused with substance that must be taken for granted, and subsumed under abstract categories without telling the reader how all of this was recognised and accomplished.
(p 24)

Hammersley (1990) criticises a statement by Pollard (1984) that "Mrs Rothwell felt a sincere caring duty towards the children in her class ...". He asks, "How do we know that the attribution to Mrs. Rothwell of a 'sincere caring duty' ... represents an accurate interpretation?" McNamara (1980) accuses researchers of 'an outsider's arrogance', in that "researchers impose upon straightforward examples of classroom discourse complex and elaborate analyses of their own devising" which are of "no intellectual, theoretical, or practical value to the teaching profession".

The crucial statement for me in Cicourel above is 'without telling the reader how all of this was recognised and accomplished'. I have tried to make clear in my reporting of data-collection and analysis during the three phases of this study what decisions I have had to make, why I have made them, and the limitations of the methods which I have used.

Ball (1990) speaks of the 'organic link between data-collection and data-analysis, and between theory and method. He emphasises that the rigour of ethnographic research lies in its *reflexivity*, that is 'the conscious and deliberate linking of the social process of engagement in the field with the technical process of data collection and the decisions that the linking involves'. This reflexivity demands a 'researcher-as-instrument position' which recognises the centrality of the researcher to the conduct and conclusions of the research and a presentation of a research report which draws the reader into a similarly reflexive position. Rigour resides in the ability of the researcher to convince the reader of the fit between data and

analysis, and this requires the researcher to justify subjective decisions made. Ball thus emphasises the importance of the 'I' in the writing of qualitative research:

The problems of conceptualizing qualitative research increase when data, and the analysis and interpretation of data, are separated from the social process which generated them. In one respect, the solution is a simple one. It is the requirement for methodological rigour that every ethnography be accompanied by a research biography, that is a reflexive account of the conduct of the research which, by drawing on fieldnotes and reflections, recounts the processes, problems, choices, and errors which describe the fieldwork upon which the substantive account is based.

I should have found it impossible to write my account in anything but the 'I' form. My account of data collection and analysis in the three phases of research is to some extent itself a research biography as it has often been difficult to separate my own thinking, both theoretically and methodologically, from my data analysis and reporting of this analysis. However, I have also included an overview of my thinking and its development throughout the research which has contributed to conclusions drawn. Thus the style of writing which I have adopted may be seen in Burgess's (1985a) terms as 'an autobiographical approach'.

A central feature of this research has been my delving into a teacher's deep beliefs and motivations which underlie the classroom interactions which I have observed. The close relationships, to which this has led, between myself and the teachers concerned has inevitably influenced what I report.

However, I have recognised the subjectivity of perception, striven hard for what Ball and others call *intersubjectivity* with the teachers whom I have studied, and have sought to triangulate data with the perceptions of pupils and other participants where this has been possible. My written accounts have been read by the teacher participants for respondent validation. As Ball (1982) points out, triangulation is no recipe for producing ultimately 'truthful' accounts, and respondent validation can throw up difficulties and contradictions. However, I believe that I have not claimed more of these techniques than they have usefully offered.

My own study

Early in 1988³ I wrote, of my study, that it '*involves in-depth exploration of the operation of certain mathematics teachers*' I continued:

Research Methods – a summary

This includes their planning for lessons; their activity in the classroom; their reflection on lessons; their evaluation and assessment of pupils; their perception of mathematics and pedagogy; their teaching strategies and philosophy.

The research [so far] involved participant observation of a teacher's lessons with some audio and video recording; discussions with the teacher before and after lessons; interviews with pupils and pupil questionnaires. Analysis of the resulting data involved significant aspects of the teacher's operation and attempts to relate these to the attitudes and possibly the learning of the pupils.

As a result of analysis it became possible (a) to speak about the characteristics of one teacher and their influence on the pupils observed, for example the way in which one teacher's strategies for encouraging management of learning affected pupils work and attitudes to lessons; and (b) to draw links *between* teachers, and identify how similar characteristics affected pupils. For example, two teachers in different schools had two types of lesson (i) where pupils followed individualised schemes chosen by the mathematics department of the school; and (ii) where pupils engaged in project work (or classwork) set by the teacher. Despite differences in age and ability ranges of the pupils concerned it was possible to notice common attitudes towards the two types of lessons and draw certain conclusions about their effects on pupils' learning.

Data item 4.1: Diary Extract (22.3.88)

In the remainder of this chapter, I shall elaborate the main methodological considerations of my study, although remarks on methodology will be an integral part of the accounts in Chapters 5, 6 and 7.

Much of this discussion will relate to my struggle with *objectivity* during the early stages of my research. Creating a tension for me as a researcher, was a strong notion that a researcher should aim for an *objective* view of a classroom. I was aware then that my observations were very subjective – I was too involved in what I was observing. I felt that it was the task of a

³ This was between my analysis of Phase 2 data and the start of Phase 3 field work – see Appendix 1 for a research chronology

researcher to be objective, and I did not know how I could reconcile the way things were with what I felt was required. Ball (1990) suggests that ethnographers should not be “closet positivists”, trying to impose positivistic values or attitudes on ethnographic research. I overtly eschewed a positivist paradigm, and feel that at this stage I was not so much insecure without the reassurance which the instruments of positivism can provide, as with my inability to become sufficiently *distant*.

Eisenhart (1988) states that “the researcher must be involved in the activity *as an insider* and able to reflect on it *as an outsider*.” (my italics). In Chapter 8, I speak of the importance of *distancing* in the teacher-researcher relationship in which the researcher’s probing has the effect of allowing the teacher to stand back to reflect on her own work, and to delve more deeply into her own motivations and beliefs than she would be likely to do alone. I recognise in retrospect that in the early stages of my research I found distancing for myself as researcher extremely difficult, and my concern with objectivity was a product of this. It was my fortune to have colleagues who provided this distancing function, so that over the period of the research I developed an ability to *reflect as an outsider*.

Blumer (1966), speaking of *symbolic interactionism* as a methodological approach, states:

One would have to take the role of the actor and see his world from his standpoint. This methodological approach stands in contrast to the so-called objective approach so dominant today, namely, that of viewing the actor and his action from the perspective of an outside detached observer. (p 542)

I have sought to know the teachers’ own beliefs, i.e. to see their teaching from their standpoint. To do this it has been necessary to distance myself from situations in which I have been participant. However, my construal of the teachers perspective cannot *be* the teacher’s perspective. My absorbing of the culture of the classrooms in which I have participated has never allowed me to stand in the teacher’s shoes and experience the teacher’s own thoughts, hopes and fears. Symbolic interactionism seems to present an unresolvable paradox to the researcher. At best the researcher can interpret what she experiences as honestly as possible with every attempt made to verify interpretations.

INTERPRETIVE ENQUIRY

I identify with Burgess (1985b), when he says:

For many qualitative researchers the main objective involves studying individuals in their natural settings to see the way in which they attribute meanings in social situations. In this context the main research instrument is the researcher who attempts to obtain a participant's account of the situation under study. (p 8)

I have been concerned with meaning-making at a number of levels. At one level it has been the meanings of the teacher in classrooms situations, in particular in interactions with pupils. The ways in which teacher and pupils make meaning together has been an important focus of my study. The interpretations of the teacher and the pupils of these interactions taken alongside my own interpretations have been the subject of my analysis, which again involves interpretation of the links involved.

Cohen and Manion (1989), speaking of Schutz's views on the study of social behaviour, write:

Of central concern to him was the problem of understanding the meaning structure of the world of everyday life. The origins of meaning he thus sought in a 'stream of consciousness' – basically an unbroken stream of lived experiences which have no meaning in themselves. One can only impute meaning to them retrospectively, by the process of turning back on oneself and looking at what has been going on. In other words, meaning can be accounted for in this way by the concept of *reflexivity*. For Schutz, the attribution of meaning reflexively is dependent on the person's identifying the purpose or goal he seeks.

Accounting for responses and actions in classroom situations has formed a major part of my work with the teachers and my subsequent analysis. Issues which I have come up against in this are extremely similar to what is described in the words of Cicourel, and Schutz below. Cicourel (1973), referring to Schutz (1964), writes:

When the observer seeks to describe the interaction of two participants the environment within his reach is congruent with that of the actors, and he is able to observe the face-to-face encounter, but he cannot presume that his experiences are identical to the actors. ... It is difficult for the observer 'to verify his interpretation of the others' experiences by checking them against the others' own subjective interpretations'... The observer is likely to draw on his own past experiences as a common-sense actor *and* scientific researcher to decide the character of the observed action scene. (Cicourel, 1973, p 36)

He cites Schutz (1964) in:

The observer's scheme of interpretation cannot be identical, of course, with the interpretive scheme of either partner in the social relation observed. The modifications of attention which characterise the attitude of the observer cannot coincide with those of a participant in an ongoing social relation. For one thing, what he finds relevant is not identical with what they find relevant in the situation. Furthermore, the observer stands in a privileged position in one respect: he has the ongoing experiences of both partners under observation. On the other hand the observer cannot legitimately interpret the 'in-order-to' motives of one participant as the 'because' motives of the other, as do the partners themselves, unless the interlocking motives become explicitly manifested in the observable situation. (Schutz, 1964, p 36)

Cicourel responds:

The observer cannot avoid the use of interpretive procedures in research, for he relies upon his member-acquired use of normal forms⁴ to recognise the relevance of behavioural displays for his theory. He can only objectify his observations by making explicit the properties of interpretive procedures and his reliance on them for carrying out his research activities. (Cicourel, 1973, p 36)

Such interpretations, and the issues involved in making them, are the substance of this research, and it is my task in presenting them to the reader to make their basis explicit. Cicourel's use of 'objectify' is interesting to me because this seems to mirror the sense in which I sought objectivity. Finally I feel that this 'objectivity' is the 'rigour' of which Ball (1990) speaks, and on which what I report will ultimately be judged.

DATA COLLECTION

I collected data chiefly through participant observation and interviewing. Burgess (1985b) refers to the former as "the most commonly-used qualitative method", pointing to Gold's (1958) typology of research roles which includes *participant-as-observer* and *observer-as-participant*. Eisenhart (1988) writes:

Participant observation is a kind of schizophrenic activity in which, on the one hand, the researcher tries to learn to be a member of the group by becoming part of it and, on the other hand, tries to look on the scene as an outsider in order to gain a perspective not ordinarily held by someone who is participant only.

⁴ Cicourel (1973, p 35) defines *normal forms* with reference to Schutz in terms of "a stock of preconstituted knowledge which includes a typification of human individuals in general, of typical human motivations, goals, and action patterns. It also includes knowledge of expressive and interpretive schemes, of objective sign-systems and, in particular, of the vernacular language." (Schutz, 1964, pp 29-30)

There are a number of decisions to make about one's role as a participant observer. Some people choose to be primarily an observer and less of a participant. Others choose to become very involved in the activities of the group. One's role may change during the course of the study, and decisions about role affect not only what one does during the study but also how one uses the results.

My very early work at the beginning of Phase 1 involved me as teacher and, separately, as participant observer in classrooms where my role was partly that of a subsidiary teacher. As I observed pupils, they asked me questions about their work and drew me into their thinking. I was a willing participant, virtually a second teacher, as it enabled me to get to know the children better and to address aspects of teaching and learning in which I was interested. The teachers also sought my views on particular practices. The lessons which I taught myself were designed to put into practice some of the theoretical ideas with which I was engaged. In all of this activity, I might be regarded as *participant-as-observer*. I interacted with the other participants and observed from this perspective.

I began Phase 2 with a determination to strive for a more 'objective' view of the classrooms in which I participated, and this included the following aim:

To remain professionally a researcher involved in participant observation of the classrooms I should visit, rather than being seen as another teacher in the room, or some expert from outside.
(Diary, September 1986)

I began to realise at this stage that use of words like 'participant observation' can be over-general in conveying a sense of what actually took place. In Phase 1, I had been very much part of the group, being regarded almost as a second teacher in the classroom, when I was not actually teaching myself. In Phase 2, I strove for almost the opposite, trying not to get involved in any aspects of teaching. In the beginning this involved trying to be as unobtrusive as possible in Clare's classroom, staying in one place, not initiating any conversations either with the teacher or with pupils. However, this had serious disadvantages in terms of what I could see and hear, so eventually, when my presence became familiar to pupils, I compromised by moving around the classroom, video-recording interactions, and addressing pupils more directly. My role in the phase was overall more of *observer-as-participant*.

In Phase 3, I had an overt aim to find out more about pupils' views in the classroom. I sat close to a group of pupils, or wandered around the room listening in to different groups, responding to their comments or questions

as appropriate and in some cases initiating dialogue with them. Occasionally such initial communications led to more extensive discussions. I felt most happy with this one of my various roles, having by this time rationalised some of my contradictory notions of objectivity. I came to realise that my very presence was a perturbation on the classroom, whatever my level of involvement, so it was incumbent on me to interpret what I experienced relative to this involvement.

Closely associated with my observations were the conversations which I held with other participants. In terms of research technique these might be regarded as informal interviews. Eisenhart (ibid) says of interviewing,

Interviews are the ethnographer's principle means of learning about participants' subjective views; thus, ethnographic interviews are usually open-ended, cover a wide range of topics, and take some time to complete.

She points out that interviews can take various forms from the very informal, 'much like having a conversation with someone', to the highly structured. In most cases my interviews were informal, starting with open-ended questions, although with pupils I often had to be more precise about what I meant than when talking with teachers. Many interviews with teachers were indeed more like conversations, although these varied in terms of my own involvement. With Clare I was very much a listener – often transcripts of our conversations consisted of lengthy portions of Clare's speech with only brief questions or interjections from me; with Mike I found myself engaging with him in discussing issues which arose⁵, and with Ben I felt able to be provocative and challenging.

My interviewing of teachers served different focuses in the three phases. In Phase 1 it was genuinely to question aspects of an investigative approach with the teachers, although I also recognised my role in stimulating the teachers' own thinking and influencing their developing awareness. For example I wrote, rather patronisingly I now feel, "My concern here is to help the teachers concerned identify aspects of their own practice which are of particular interest in the context of an investigational approach to learning mathematics" and "I want Felicity to talk about her own feeling/tensions – if she is unaware of them (i.e. they're subconscious) she can't communicate them. I have to somehow make her aware, help with the language of communication, start off *her* analysis of her *own* practice". I believe that this language, with its implicit message

⁵ Appendix 4 contains transcripts from 'conversations' with Clare and with Mike to illustrate this claim.

of trying to change the teachers became modified subtly with time to one involving a genuine wish to explore a teacher's own beliefs and motivations with her permission.

A difficulty in trying to explore beliefs, in Phase 1, was in getting Felicity and Jane to articulate, and thus make explicit, their ways of working with pupils which were either intuitive or based on experience, but which they had never consciously examined. One problem was the lack of language in which to speak of ways of working; another was what to focus on. When I raised particular instances myself, the teachers were quick to respond, and a language started to develop. In my seeking for objectivity at the time, I felt that it was almost impossible to be both a prompt for the teachers as I have described, *and* present an unbiased view of what occurred. I was overtly aware of operating at a number of levels simultaneously (see Appendix 3 for an account of this). I suggested, half in jest, that perhaps a second researcher was required, to present this unbiased view.

I realise now that what I was trying to achieve with Felicity and Jane, albeit expressed in the patronising way identified above, was actually realised in Phase 2 when I worked with Clare and Mike. I believe that this was in part due to my own developing awareness of what I was trying to achieve – for example I wrote as an objective for Phase 2, 'To try not to push my own agenda; rather to seek to find out what the teacher herself thinks, to try to enter into her thinking' – but in part too due to Clare and Mike operating much more confidently and overtly in an investigative approach to teaching, so that they were much less threatened by my probing and had language available to express their thinking. The discipline of listening, but trying not to promote particular views or lines of enquiry, was useful in enabling me to develop my ability to operate simultaneously and consciously at a number of levels. By holding back from offering my own views on the issues which Clare articulated, I was able to pay attention to the raising of issues and encouraging Clare to examine her own beliefs and motivations. For a time I saw this as being more objective about the issues as these appeared to Clare, yet I came to recognise my own interpretive levels even here. My own questioning of interpretations, and subsequent respondent validation by Clare herself of what I wrote, contributed to the ultimate rigour of the account which I presented.

I sought pupils' views or concerns about the classroom work or the way they were taught, in a number of ways. In Phase 1, I worked with pupils

talking with them as a teacher, and interviewed some of them, semi-formally, about their perceptions of lessons. I had no precise interview schedule, rather asking what they had thought of lessons which they had experienced, and following up with questions related to their responses. I obtained some very illuminating responses regarding their views of the types of lesson they experienced, and these mostly accorded with the teachers' views of how pupils perceived lessons. However, there were times when, in reflecting on my account of a lesson, I wondered how certain pupils had perceived particular events. Why had I not asked them? At the time my answer was that I had simply not thought to ask them. I resolved, as a result of this phase of work, to make an effort to ask pupils about significant episodes in my future work. An objective for Phase 2 was, 'to be alert to incidents in the classroom where the pupil's perception would be important to a comprehensive view of the event, and wherever possible to seek that perception'. However, I felt that I had not succeeded in this objective in Phase 2, as I shall explore further below. Towards the end of the Phase 2 work, I interviewed some pupils semi-formally as in Phase 1. In Phase 3, I sought to pursue my Phase 2 objective more overtly. I sat close to a group of pupils, or wandered around the room listening in to different groups, responding to their comments or questions as appropriate and in some cases initiating dialogue with them. Occasionally such initial communications led to more extensive discussions. I did little formal interviewing of the pupils, although I sometimes sought out individuals to ask particular questions. Overall I feel that I only scratched the surface of pupils' perspectives on their experiences, whereas I was able to delve deeply into the teachers' perspectives. This was to a large extent due to the time spent with each, little with pupils, a great deal with teachers. Associated with the time factor is the relationship which I was able to build. Measor (1985) speaks of the importance of the relationship in successful interviewing, and I believe that this is true. I worked hard at my personal relationship with the teachers, feeling that we developed mutual respect and in some cases friendship. With pupils I must have seemed a very distant figure in most cases.

The data which emerged from observation and interview was in a number of forms. It consisted of field notes throughout, although these were rather sketchy and unhelpful in the early stages of Phase 1; audio recordings of most of the interviews and of lessons in Phases 2 and 3, from which transcriptions were obtained; and video-recordings from Phase 2 lessons.

In addition to this data, which formed the basis of my analysis, I conducted a questionnaire with all of Clare's pupils in Phase 2. I discuss this in Chapter 6. I also used video-tapes for stimulated recall with the teachers Clare and Mike separately and together, and with Mike and one group of his pupils. This data contributed to my analysis alongside the interview data described above.

In addition to the above forms of data, I wrote my own reflective notes throughout the study. These consisted of day-to-day jottings regarding incidents which I had experienced and my own ideas and perceptions. Sometimes they were elaborations of anecdotes which had significance. Sometimes they involved incipient theorising – expressing patterns which I observed, or attempting explanations. Eisenhart (1988) refers to this type of data collection as *researcher introspection* in which “the ethnographer tries to account for sources of emergent interpretations, insights, feeling, and the reactive effects that occur as the work proceeds”.

DATA ANALYSIS

Analysis of data took various forms throughout the research, which might be broadly regarded as formal or informal. Informal analysis took place simultaneously with data collection. It consisted of reflection on data collected, recording of impressions, elaboration of perceived significance in terms of anecdotes or ideas, and of questions which emerged. These all fed subsequent data collection in terms of my conversations with teachers in which I validated my own impressions while seeking theirs. In Phase 1, this informal analysis was all that was done, although I recorded it more assiduously here for the benefit of the teachers concerned than was necessary in later phases.

More formal analysis of my data came close to what Glaser and Strauss (1967) call the *Constant Comparative Method* (p 105). This was engaged particularly in the case of data resulting from work with Clare⁶, and subsequently from that with Ben. It happened after fieldwork had been completed, and consisted of analysis of field notes, transcripts, and informal recordings. It involved close scrutiny of the data, “coding each incident into as many categories as possible, as categories emerge or as data emerge that fit an existing category” (ibid).

Initially, categories were indicated in the margin of the field notes or transcript. When a category was seen to repeat a number of times, the

⁶ I include details of my categorisation of the Clare data at Appendix 4

incidents were compared and possibly listed separately to begin a category profile. Sometimes it became obvious that the original category was inappropriate to describe the various incidents, and this allowed for finer tuning. Where an incident could fit a number of categories, application of Glaser and Strauss's injunction to use it only once in the most important category indicated was a hard discipline to follow. It was not always clear what were the criteria of importance. However, the integration of categories and their properties occurred far more naturally than I should have expected. It was partly a need arising from the unwieldy nature of a multiplicity of categories, and partly the rationalisation of some incidents belonging to more than one category. Appendix 4 shows examples of these stages in analysis of the Clare data. The ultimate delimiting of theory was not as clear cut as Glaser and Strauss suggest. In the emergence of the *Teaching Triad* from the Clare analysis it was the case of relationships presenting themselves as possibilities, rather than of relationships being evident.

An important issue for me throughout my research was that of *significance*. In the diary extract (22nd March, 1988) which I quoted earlier as Data Item 4.1, I talked of 'significant aspects of the teacher's operation'. The nature of this significance exercised me greatly, and I quote from a lengthy diary entry which I made at this time – after the Phase 2 field work and during the Phase 2 formal analysis – as it expresses my thinking at what was a crucial stage in the development of my study.

Significance

In the production of field notes or video [or audio] tape of classroom lessons automatic editing takes place. Some of this is explicit, for example deciding to focus on a particular group when the class are working in groups, or deciding to follow the teacher around from group to group. Some of it is implicit, for example, attention being caught by one event which leads to failure to notice others.

However, some editing is much more subtle and arises from the human characteristic of unconsciously attributing significance to some events while not to others. It is possible to believe that one is keeping an accurate record of what occurs in the lesson, in that reference is made to all events which are observed. However, any field notes show up evidence of stressing and ignoring. Whereas one event may be described only in brief general terms, another may have more vivid description including quoted speech or particular detail.

The 'significant' events in a lesson, as far as the observer is concerned, might be said to be those which are awarded the more detailed attention. The fact that I noticed something particularly meant that it had significance for me. As I look back over the two phases of this research I am aware that often my significances have been related to my own current thinking. At Amberley, I was interested in investigative work in the classroom and so found myself particularly noticing events and strategies relating to working investigatively. At Beacham, constructivism was very much part of my thinking, and my noticing here was undeniably conditioned by this.

Questions arise to do with how my current interest colouring my observation contributed to my perceptions of the teachers concerned. As I look back over transcripts of my conversations with Clare I notice that my contributions to the conversation are usually brief compared with hers. This is not surprising since my aim was to get *her* talking about her work and beliefs, rather than to have debates with her. However, the subjects for conversation were often ones which I raised initially. I recall one occasion when I waited for her to begin a conversation and she started to speak, then hesitated, paused for a moment and then asked me to start her off with something. This means that I had power in suggesting where conversations began.

It is obvious that in analysing data I shall place emphasis, and that much of this will be on the events which I have found striking for whatever reason. These will include events which have come to my attention because Clare talked of them. They will include many that were of significance because I (implicitly?) wanted them to be significant.

These remarks on significance raise the important question of 'significant for whom?' Where it has been possible to share my significances with Clare, or with pupils, and include their comments it is perhaps less important that I chose them initially.

Data Item 4.2: Diary Extract (16.3.88)

I now see my concern here, with attribution of significance, as indication of a growing awareness of the need to recognise in how far theory guided observation and analysis. It was not my aim *just* to describe the classrooms which I observed. I wanted to describe aspects in terms of an investigative approach, and indeed could not describe except through my own frame of reference. However, I had to be careful to avoid what Furlong and Edwards (1977) identify as "too much prior theorizing, the observer simply having to select the right example to fit his preconceived ideas"

THEORETICAL PERSPECTIVES

In Phase 1, I was overtly aware of my lack of objectivity, and less overtly of my adherence to personal constructs of 'an investigative approach'⁷ Striving to become more objective, as I moved into Phase 2, had implications not only for my relationship with the teachers and pupils involved, but also for my awareness of theory which influenced salience as I saw it. While at no time did I pretend to objectivity, its impossibility dawned on me only slowly, and I found it difficult to separate describing *what I saw* from describing *what I saw as significant because it related to my personal theory*. In Phase 1, I wanted to be more objective while veering towards the extreme of 'following a theory too closely'. As my awareness of this tendency grew, it became incumbent on me to question my conjectures or conclusions as honestly as I was able.

Having identified the need to justify attributed significance, it became more important to address its nature when becoming aware of significant events rather than retrospectively. I began to question the relationship between attribution of significance and an investigative approach, and subsequently a constructivist philosophy. This was simultaneous with testing out the teaching triad at the beginning of the Phase 3 field work. I had to decide whether simply to observe the Phase 3 teaching for incidences or manifestations of aspects of the teaching triad, or whether to take the teacher into my confidence in overtly seeking verification of the triad. The following diary entry is important to my thinking at this stage.

Thoughts in preparation for starting third phase of research at Ben West's school

What is this phase about?

1. Correcting inadequacies in the methodology of Phase 2

For example, getting the pupil's perspective. In Phase 2, I often wished, too late, that I had been able to ask pupils about certain classroom events – e.g. Virginia⁸ with her hand up in Clare's class.

This will involve being alert to significant events so that I can pursue them immediately with pupils – there are practical difficulties here about when it will be possible to talk to pupils.

There are also more fundamental difficulties to do with significance, which is another consideration for Phase 3.

⁷ This is discussed in more detail in Chapter 5. See page 90

⁸ The episode with Virginia to which this refers was described in Jaworski (1988c).

2. What is of significance, when, and to whom?

The question for Virginia was never asked because I didn't think about it until it was much too late to expect Virginia to remember. The comment from Clare came quite soon after the lesson. At what point did the significance of this event arise? It had *local* significance because we discussed it at the time. However, its *global* significance did not emerge until I was doing analysis of the data at a much later stage.

What does this have to say about my methodology? Perhaps that I must be prepared to do one stage of analysis immediately after an event. A question to explore here concerns what issues arise at different stages of analysis.

3. How does the time at which analysis is done influence the significances, or the issues which arise?

I need to be more disciplined in relating significance to my current line of thinking. In the classroom there is an immediacy which cuts out deeper reflection, e.g. I notice Virginia with her hand up, but I do not attribute further significance to this. Clare comments on what I noticed, and here is the opportunity to recognise that it is an event worth pursuing further, and of talking to the pupil concerned. Is it however already too late to talk with the pupil?

Am I pushing for something which is unrealistic? Is all that I can do to recognise what happened in the Virginia event, and possibly to see if any sharpening of awareness which arises from this causes me to act differently in Phase 3?

4. Trying out some of the categories from Clare, Mike, Felicity on Ben & Co. Which of them make sense in the new context? Which do not seem relevant?

I can't decide how explicit I want to be about this with Ben. If, for example, I introduce the idea of 'management of learning', in order to probe what this means for Ben, am I likely to pre-empt what I might get? If Ben has never thought in those terms, might it nevertheless influence his subsequent thinking and action?

On the other hand, if the observations from my research are to prove helpful, it is important to find out what teachers might find useful from it. If Ben was to say that he didn't find 'management of learning' a helpful categorisation, it might be very fruitful to explore how he perceives what it encompasses. Do I want to discuss this with him – at the level of considering ML, or indeed at the meta-level of this discussion?

I recognise now that inevitably there were different significances for different stages of analysis. What emerged from transcripts as being significant was not always something which I could have noticed and questioned in the classroom or during an interview. What is striking, however, is that I see here the origins of my perception of the importance of levels of reflection, which began in embryo in the levels of operation which I recognised in Phase 1 (see Appendix 3). To paraphrase my thinking then, I seemed to be saying that if I could recognise significance *in the moment* then I should be in a better position to ask questions about it. This is very closely related to what Mason calls *noticing in the moment* (see for example Mason and Davis, 1988; Davis et al, 1989), and what Schön (1983; 1987) calls *reflection in action*. Von Glasersfeld (1987a) speaks of *reflection* 'in the sense that it was originally introduced by Locke, i.e. for the ability of the mind to observe its own operations.' I recognise that I began here to see how reflection might be used in a disciplined way to enhance awareness and hence to enable development. Mason develops this notion in his 'Discipline of noticing' (see for example Davis and Mason, 1989) as does Schön (1987) in his 'Educating the reflective practitioner'. In Chapter 8, I have shown how this recognition led to implications of my study for teacher development.

VERIFICATION

Ultimately the validity of my conclusions rests in my ability to justify them in terms of my whole research design. I can often not point to one *single* justifying factor. Ball (1982) points out that:

... verification is intrinsic to the process of the research itself. Verification is constantly to the forefront of participant observation and cannot be separated out from the collection and analysis of and theorising from data.

This was the case in my study, and there were a number of techniques which I employed throughout the research.

The first was *triangulation* which was used in comparing data from a number of sources, for example my own account with those of the teacher and a pupil or pupils. In one stage of Phase 1, I had secondary observation from my supervisor, Christine Shiu, and, in Phase 2, I was able to employ another colleague, Sheila Hirst, to perform secondary observation of Mike's lessons. Both of their accounts contributed to triangulation of data. As Ball points out, "Triangulation is not a recipe for producing ultimately 'truthful' accounts, it is rather that the different accounts can add to the perspectives which contribute to the emergent story." A major

contribution to this triangulation was the stimulus-recall work with teachers and with pupils. The video sequences had the power to trigger associations and aid the reflections of participants.

Secondly, much of what I wrote at various stages throughout the research was referred to the teachers for their comments. My use of this technique of *respondent validation*, is aptly described in the remarks of Edwards and Furlong (1985):

The five teachers were able to read early drafts of our account of their work, and were encouraged both to amend it in detail and to challenge it more broadly if they felt themselves misrepresented.

They comment on:

the power enjoyed by a writer over an 'associate' reader, especially where the reader may have neither the time nor (unless blatantly misrepresented) the motivation to rewrite the account. Indeed the researchers' account may express such different concerns and priorities from those of the participants as not to invite a general challenge at all.

My experience is both similar and different to this. In the main the teachers with whom I worked were sufficiently interested in what I wrote to comment on it either verbally or in writing at some level, and I gained some very useful material in this way. However, it was undeniable that often my focus was different to their focus, and so, particularly where I was moving towards theory, I often was not offered much response. Some remarks from Clare, which came after reading a late draft of Chapter 6, were illuminating in this. She claimed that it was her own developing thinking which enabled her to comment on my analysis in a way which had not been possible when she read a much earlier draft. Some of Mike's later comments certainly seemed to owe much to his developing thinking since the field work.

Finally it is in my research biography that verification ultimately resides. Ball (1982) says of this:

The research biography is in effect a representation of the research process both in terms of an account of the internal validity of research methods, standing as an autobiographical presentation of the experience of doing the research, and in itself a commentary upon these methods it stands as a source of external validity, as a critical biography, a retrospective examination of biases and weaknesses. The

research biography also represents what Denzin, 1975⁹, calls sophisticated rigour, a commitment to making data, data elicitation and explanatory schemes as visible as possible, thus opening up the possibility of replication or the generation of alternative interpretations of data.

The three chapters which follow form part of this research biography. It has been impossible to separate my reporting of the research from the methodological issues associated with it, so methodological detail permeates these chapters. Whereas this chapter has looked at methodology from a general perspective, the following chapters present the associated practical manifestations and issues.

TERMINOLOGY

I end this chapter with reference to two items of terminology which will be used in a particular way in the following chapters. The first is *manifestation*, and the second *characterisation*.

An important aspect of this research has been the linking of theory and practice, for example, theoretical notions of an investigative approach with their practical *manifestations* in the classroom. The reader may ask why I have used the word *manifestations* when the simpler word 'examples' would do. My usage is deliberate. Often the term *example* carries with it a notion of genericity. The *example* in some way *stands for* the rule. However, often it has been the case, particularly in terms of emergent theory, that some theoretical idea might be manifested in different ways, none of which would provide a generic example. Indeed to base the generality on the specific manifestation would be to deny much of the richness and subtlety of the idea. Thus, incidents from the practice of teaching may be seen as *manifestations* of particular theoretical aspects of teaching.

In linking theory with practice, it has been important to look for patterns in the teaching studied. The term *characterisation* is used rather than the simpler 'description' to indicate the expression of the richness of pattern in a teacher's practice. The *characterisation* will include descriptions of aspects of that practice, but will embody some sense of generality which the word 'description' alone does not necessarily imply.

⁹ Ball's bibliography does not include a reference to Denzin 1975. It includes two other references to Denzin, viz: Denzin, N, 1970 (ed.) *Sociological methods: A handbook*. Aldine Pub. Co. and Denzin, N. 1971, 'The logic of naturalistic enquiry', in *Social forces* L.2.

CHAPTER 5

THE PHASE ONE RESEARCH

Of what might an investigative approach to mathematics teaching consist?

Introduction

A local mathematics advisor, aware of my interest in investigative work, invited me to join a group of teachers who were setting out to write some LEA materials to help teachers in beginning investigations in their classrooms. After working with the group for some time, two of the teachers involved, Jane and Felicity, invited me to their school, Amberley, to take part in some investigation lessons with their first and second year pupils. They knew that I was interested in going beyond investigations and wanted me to discuss with them what this might mean and how it might be possible.

Pupils in the first two years were broadly set for mathematics, according to records of previous achievement. The teachers used the SMP scheme for two lessons per week (the *booklets* lessons), and had one lesson, the one which I regularly attended, on some class topic involving investigational work (the *classwork* lesson).

From the beginning, I saw the work at Amberley as a pilot study from which I should clarify my research questions and methodology in exploring the nature of an investigative approach.

Stages of involvement in the Phase 1 work

The phase fell into three stages as follows:

1. THE INTRODUCTORY WORK

This took place over a year (1985) during which I periodically spent time with the teachers, in their classrooms or talking about teaching. I kept very brief lesson notes, usually written just after a lesson, and wrote a diary in which I recorded those aspects of a lesson which had struck me in some way.

At this stage I was treated by the pupils as a second teacher in the classroom, and by the teachers as a colleague or advisor. The teachers and I spent considerable time talking through the lessons in which we had taken part.

2. TEACHING LESSONS MYSELF

I felt uncomfortable with the deference which I was paid by the teachers, as if I was some kind of expert. I asked if I might take some lessons myself to have opportunity to try out some of the ideas regarding investigative teaching about which I had been writing (Jaworski, 1985b), but I also wanted us to have the opportunity to discuss some of *my* lessons, to put us on a rather more equal footing. We agreed that I should teach parallel first year classes, one from each teacher, in succession on a Tuesday morning, for half a term. Each teacher observed me teaching her class. My supervisor, Christine Shiu, observed several of these lessons and wrote field notes. I kept a record of the significant events which I recalled from my teaching of the lessons. The two teachers, Christine and I met where possible at the end of each Tuesday, to discuss the lessons. I continued to keep a diary of significant events and my own reflections on them. While doing my own teaching I continued to observe lessons of the two teachers, mainly Felicity as this was possible on the day I visited the school.

3. PAIRS OF LESSONS OF THE TWO TEACHERS

One effect of my teaching which the teachers had valued, was that their classes were taught in parallel. It was thus possible to contrast pairs of lessons, to note similarities and differences, and to learn from pupils' responses to similar teaching. (I have avoided saying that these classes received *the same lessons*, because in any pair of lessons the differences were remarkable, and an important aspect of our discussion concerned how these differences related to the different pupils in the classes.) As a result of this, in the next half term, the teachers planned a series of lessons which they would teach in parallel and later discuss. I would observe both lessons in each pair. They decided to take a syllabus topic, in the event *tessellations*, and design investigative work related to the topic

In these lessons I made field notes, and, at the end of the day, sat with the two teachers to talk through the lessons, recording the conversation on audio tape. From the tape, I wrote a summary of the main points of the discussion, highlighting significant episodes and issues which had arisen, and posted this to the teachers so that they could read it before we met the

following week. In this way I mixed data collection with analysis, analysis from one week's lessons feeding into the next. The recorded conversations were later transcribed and used to substantiate and validate interpretations at this stage. I also continued my diary – a separate, more detailed, record of significant anecdotes and issues. At this stage I did not define *significant*, I simply kept a record of what particularly struck me from the lessons and discussions. However I now believe that this work led to my subsequent focus on *significance*, discussed in Chapter 4.

DATA COLLECTION AND ANALYSIS ACROSS THE THREE STAGES

Data collection varied from diary entries made soon after an event, through field notes written during lessons to audio recordings and subsequent transcripts of conversations. Much of the data which fed the *immediate* analysis of Phase 1 work was my own memory of events which went unrecorded. Analysis, which took place soon after collection, mostly took the form of written accounts of lessons and conversations with questions which were asked and issues which were raised.

Analysis and reflection

STAGE 1 - INITIAL OBSERVATIONS

The early work coincided with the writing of (Jaworski, 1985b) and preparation of the video pack (Open University, 1985). Looking back to my written remarks, in field notes and diary, I can see that my focus was highly influenced by my thinking in producing these publications. For example as a result of watching a pair of parallel lessons on 'lines and regions' I wrote:

Use of my own terminology

Some *pretty aimless work* in lessons 1/2. Many pupils not at all sure how to count lines and regions, so not surprising that no patterns emerged. Very few had any idea what they were looking for – they were *specialising, but very unsystematically*, and only a small number made any attempt at a generalisation. They clearly need guidance in identifying processes and questions to ask. *Cannot just ask children to investigate without giving them some framework for it.*

Data item 5.1: Diary extract (18.1.85)

The words in italics were underlined (in red!) in my notes. *Systematic specialisation* leading to *generalisation* had been processes which I had written of as being of value for development of mathematical thinking.

Looking back on these notes, I particularly notice that I was judgmental, with no overt recognition of the interpretive nature of my remarks. With reference to afternoon lessons the same day I wrote more approvingly that, “pupils readily appreciated what was involved”, and “they knew they were being successful and motivation was high”. I made little attempt to say what aspects of what I saw contributed to what I perceived as *ready appreciation, knowledge of success, and motivation*.

I wrote brief notes on a subsequent lesson, taught by Felicity, in which a third year group were working on combinations of rotations and reflections of simple shapes.

An early significant event

Felicity’s third year — doing combinations of transformations — in fact symmetry transformations of a square. She asked them to try out ‘a few’ combinations of their choice and enter the results in a pre-drawn Cayley table. After working through a few mechanically they then started to predict. Felicity said, “If you notice something happening, *write it down*” One girl predicted that the whole main diagonal would contain the identity transformation. Another challenged this. The first girl tested it and realised her mistake. Felicity said, “What have you learned from that?” The girl said, “Not to be satisfied with a prediction without testing it first”. Very nice interchange — would like to capture on film.

By the end of the lesson the group were generalising and explaining to each other successfully.

Data item 5.2: Diary extract (28.2.85)

This very brief description goes some way to characterising this lesson in terms of aspects which I found significant to an investigative approach. For example,

- Trying out ‘a few’ combinations of their choice — specialising using their own examples.
- “If you notice something, *write it down*” I emphasise the teacher’s words here which seemed to offer important advice

- The anecdote of the girl predicting, and the subsequent testing revealing a mistake in her prediction. The teacher's emphasis of the process, making it explicit to the pupil.

I notice here a difference in the quality of my interpretation between Phase 1 and Phase 3. The final statement in data item 5.2 is highly subjective. In Phase 3 I should have been very concerned to say what aspects of *generalising and explaining* I saw, and what had seemed to me to be *successful* about it. A major difference between the two phases is my own emphasis. In Phase 1 it was on *processes* that I valued which were clear to me and which I did not need to spell out in detail. I simply noted my recognition of their occurrence. In Phase 3 I should have been more careful to identify the particular *manifestations* of these processes that seemed evident, and to justify conclusions which I made. This is one indication of the shift in my thinking across the three phases.

However, at this time I was striving to be more precise about what I saw intuitively as an investigative approach. An example of this is shown in the following extract from my diary:

Current thinking

What is an investigative approach? Encouraging pupils to sort out ideas for themselves and make the ideas their own. Pupils talking and listening to each other – do/talk/record, see/say/record (Open University, 1982). Teacher talking *with* pupils not *at* them. Always encouraging questions of 'what if'; prediction and testing out prediction. Question posing – how much from teacher, how much from pupil? Being clear what you want pupils to do – if you genuinely want them to have freedom to choose or decide for themselves, don't then *impose* ideas onto them or direct in a particular way.

Data item 5.3: Diary extract (19.3.85)

STAGE 2 – TEACHING LESSONS MYSELF, AND OBSERVING OTHERS.

1: A SIGNIFICANT LESSON OBSERVED

This stage actually began in the Spring term of 1986, but in spirit it began right at the end of the 1985 Autumn term with a third-year lesson given by Felicity since,

1. It was the first lesson in which I started to make field notes and so marks an important methodological stage in my work. All reporting of Stage I above was of descriptions written from memory after an event.
2. An important issue arose for the first time – that of the difference between a *classwork lesson* and the *SMP 'booklets' lessons*. I see now that it bears a strong relationship to issues which arose in later phases to do with the demands of a syllabus versus the objectives of investigative work.

In this lesson pupils were working on the SMP 11-16 booklets. Most were working independently and the teacher visited individual pupils and talked with them about their work. I wrote that she talked to maybe a quarter or a third of the class during the lesson. My field notes were in the form of jottings in bubbles on a page of A4. I did not try to sequence the remarks. In terms of later language of significance, I was, very briefly, recording my own significances from this lesson. Some of them were as follows:

Observations in an early lesson

Pupils asking for help. T. "What would happen if ..." Leading questions? What do *you* think? Go and try it. Why do you think?

Most pupils working quietly

Boy with hand up talking to next boy. No work happening. – Quick question and answer.

What does helping involve?

"What do I do for that then?" Fel, "Well what does the question ask you?" ... Fel, "So, you've answered your own questions then!" Smiles. Girl pleased.

Who talks most?

Data item 5.4: Excerpts from Amberley field notes (5.12.85)

I talked with Felicity after the lesson, and, while we talked, made notes which I later summarised. My summary says that we had agreed to contrast ways of working (a) with booklets (b) in investigations. I recorded the following comments from Felicity:

Felicity's comments

1. Pupils work better when working at their own pace (i.e. from booklets) and on own topic than when working as a class from the blackboard. *My Qu:* what is *better* in this context?
2. Difficulty is that they don't talk together about their individual problems. They only see things in their own way, not in different ways. Working with others might provide for broader views. However, for one girl, quiet by nature, – in a group too quiet, might not interact – *needs* one-one with teacher. Teacher's decision because of knowing pupils.
3. Booklets all inclusive – leave teacher with less scope for working in own way. Booklets include investigations but too closed and directed. Disappointing pupil response – no room for own thought, pupils treated them as *typical book exercises*.

My qu: What is a 'typical book exercise' and what is a pupil's response to such?

Data item 5.5: Summary of conversation (5.12.85)

The teacher had referred to pupils *enjoying* their work on booklets. I jotted down further questions, asking myself why this was; how important it was for individuals to work at their own pace, and how a blinkered perception of mathematics could be avoided.

I noted that pupils' response to the teacher seemed 'always brief and uncertain' and asked how a teacher could encourage more contribution from pupils, e.g. by waiting longer for a response. I saw little group work or pupil interaction and asked how important this was and how it could be incorporated.

I quote below from a letter to Felicity as a result of the lesson described above and questions which I had raised, and her response to it. This letter and response are indicative (a) of my own thinking at this time, of questions which I started to raise which were precursors of later thinking; (b) of the teacher's thinking in this phase.

As part of the letter I had asked the following questions:

My questions to Felicity

- What are the objectives of a 'booklets' lesson? Can deficiencies be met by complementing booklets in some way in other lessons? What does 'working investigatively' mean in a booklets lesson?
- How can a teacher get a personal measure of what each pupil understands of a particular topic? How does a teacher know what sense the pupils are making of the mathematics they meet?

Data item 5.6: Extract from letter to Felicity (5.12.85)

Among Felicity's responses to the letter were the following remarks:

Felicity's response

- Objectives of a booklets lesson – allow pupils to cover and understand material at own pace and level and perhaps to discuss with me or other pupils any points of difficulty.

Working investigatively in a booklets lesson – (a) solve a puzzle or a problem (b) arrive at a solution to a question by trying different approaches e.g. trial and error, deduction, elimination. When working investigatively with the booklets I get the feel that the author usually has some idea as to the end point which the pupil should arrive at. Therefore the questions are usually quite directed and are not open-ended. I would try to counteract this *rather narrow approach* by perhaps throwing in some rather more abstract problems which may hopefully call on some of the methods which they have had to employ previously.
- Sort out booklets which would perhaps promote discussion about certain ideas and ask a group of pupils to work on some booklet at the same time.

Test. Not just recall facts. Give a specific problem which calls for application of facts acquired, which may require certain processes being used.

Data item 5.7: Extract from Felicity's response to my letter of 5.12.85

Here, I was struggling to make links between teaching mathematical topics and working investigatively. I feel that Felicity saw working investigatively to be closely linked with open-ended tasks, and to be trying to relate this to mathematical topics. The third stage of this phase, where I observed pairs of the teachers' lessons, highlighted issues which this linking raised for the teachers. I was also beginning to struggle with notions of 'sense-making' and how a teacher can learn about pupil construal. I felt at the time that Felicity's response, in terms of *testing*,

was a pragmatic one, whereas I had meant my question more in terms of a teacher's gaining of awareness.

The writing of this letter emphasises the *immediacy* of data-analysis in Phase 1. It was an advantage of the type of data that analysis and follow-up could be so instant. I wrote the letter because I did not wish to wait for my next visit to ask Felicity the particular questions. This had the advantage that *I could gain access to her thinking* while events were fresh in her mind. It also meant that I was less able to distance myself from the events and issues concerned than I was in the later analysis of transcripts in Phases 2 and 3.

Levels of interpretation and validation are important here. In trying to 'gain access to her thinking', I recognise the fruit metaphor (Chapter 3, p 42). I was here trying to make sense of Felicity's thinking by raising issues related to comments which she had made. These levels of commenting enhance the account which I can give of this event. I recognise my own attempts to reconcile my views with those of the teacher. My focus was very strongly with *my* concerns, but this was not overt at the time.

2: MY OWN LESSONS

My chief aim for these lessons was to try out myself some aspects which I felt to be important to an investigative approach to teaching mathematics. I wrote the following list of aims for the first lesson, but also noted, "It will probably need a whole series of lessons to achieve *all* of this!"

Aims for Lesson 1

1. Getting to know the group, putting them at ease, setting the scene for a way of working.
2. Encouraging all pupils to talk, but emphasising the importance of listening too.
3. Getting a sense of negotiation of understanding. If I don't understand what you say we have to negotiate.
4. Respecting each others' explanations and ideas.
5. Importance of images to understanding. We may *think* we are talking about the same thing but our images may be different.

Data item 5.8: Aims for the lessons which I would teach. (Jan, 1986)

I recognise now that these aims concern much of what I have come to regard as being essential elements of two strands of *the teaching triad*, i.e. *management of learning* and *sensitivity to students*. These notions are introduced formally in Chapter 6, and explored further in Chapter 7. I notice that I did not declare mathematical objectives, although my lesson plan involved mathematical activities. For the first lesson I set up activities which, although mathematical in nature, were designed to allow realisation of the above aims. (See Appendix 3 for details of these activities and others in this series of lessons.) In analysing to what extent my aims had been satisfied in this first lesson I indicated a number of issues which had arisen. For example, I wrote,

Reflecting on my first lesson

Most pupils did talk. Harder to measure listening – if a boy is quiet, is he listening? Many pupils were unwilling to listen. – evidence of this when one person was contributing to whole class discussion and others were avidly sharing their own ideas in pairs – syndrome of ‘I want to share my ideas, not listen to hers.’

Data item 5.9: Excerpt from my reflection on lesson 1 (28.1.86)

I indicated, with examples, that there had been quite overt negotiation at times, but was disappointed with the lack of respect shown, in some cases where pupils openly laughed at the remarks of others, or made disrespectful remarks like, ‘because he’s stupid!’. I wrote that it was my intention to work hard at developing respect.

This suggests that my focus with regard to investigative teaching was very firmly on the creation of an ethos in which investigative work could take place. However, in lesson three, questions of ethos and of mathematical thinking and development came into conflict, and the mathematics ‘won’.

I began this lesson by writing on the board;

1. $2 + 3 = 5$
2. $4 + 6 = 10$
- 3.

and I invited pupils to suggest what I might write against 3. There were many suggestions and I spent time asking pupils to explain and justify their own suggestions, or comment on the suggestions offered. Subsequently I asked which of the suggestions the class would prefer and why? Many hands went up and pupils indicated their choice, sometimes

able to articulate, albeit not very precisely, their reasons. The lesson was well advanced. I perceived a tension in choosing how to proceed at this point. Two directions offered important but conflicting purposes. A quick decision on which statement to choose would allow us to reach some mathematical conclusions, for example, a generalisation for whatever sequence emerged. However, further discussion, perhaps in fours, would reinforce my objectives with regard to negotiation. Two reasons led to my choice of the first of these. Firstly, I recognised that I should like pupils to experience a mathematical outcome from the lesson, as no follow-up was possible; secondly, I felt that too much discussion and negotiation might be counter-productive if pupils became bored or lost a sense of purpose in the lesson. So I asked for a vote, one statement was chosen, and we proceeded to investigate the particular sequence.

I know now that my own experiencing and recognition of such issues was a part of my development as a teacher, and the resulting awareness which I gained played an important part in my recognition of significant moments in the lessons of other teachers (see, for example, Appendix 2, section 2.4). I was able to discuss my thinking on these issues with colleagues and this enabled me to become clearer myself of objectives and teaching practices. This is an example of what I discuss in Chapter 8 of stages in teacher development resulting from *reflection* on a lesson and *accounting for* perceptions of the lesson.

I spoke of the moment of decision, in this lesson as a 'decision point'¹, when talking of it with the teachers afterwards, and they took up this notion, recognising that there were often such points of rather crucial decision in their own lessons. In a subsequent lesson in the third stage of this phase, where I observed Felicity teaching, she indicated to me that she had noticed a decision point. She stopped herself, at the point of interrupting what pupils were doing, in order to tell me of her choices and then to make a choice overtly. This incident also has significance in terms of teacher development, and I shall refer to it again in Chapter 8.

The lessons which followed had objectives related both to mathematics and to ethos. My awareness, which I have referred to above, continued to develop, and at the end of the series of lessons I was able to articulate the main issues which had arisen from this teaching, including important

¹ Calderhead (1984) reviews the research on teachers' classroom decision making. My own labelling of 'decision point' came close to Cooney's (1988) description. In his terms the decision here could be classified as both *cognitive* and *managerial*. This has parallels with *Management of Learning* and *Mathematical Challenge* which I discuss in Chapter 6.

decision points which had occurred during the lessons. I see this as the stage of *critical analysis* which I discuss in Chapter 8. This notion of decision points permeated my awareness in future observations and was a source of questions to the teachers whom I observed. I felt it led to encouraging the teachers to inspect their actions rather more intensely than they might otherwise have done, and I discuss implications of this in terms of *distancing* in Chapter 8.

One important issue, which emerged through my discussions with Christine Shiu who had observed the lessons, was that of the differing focuses and perceptions of the teacher and the observer in a lesson. Not only did Christine see incidents to which I did not have access, because I was attending to something else at the time, she was able to see my teaching in a way that was not accessible to me because of her particular focus.

I found her comments very valuable in alerting me to aspects of my teaching of which I might not have been overtly aware, and in providing a *distancing* role. This again became important to my own work with later teachers, and contributed to validity both in providing a further perspective at the time, and influencing my gaining a deeper perspective on future occasions.

The levels at which I was operating during this period seemed extremely complex, and I made some effort to unravel them². Briefly they involved *being a teacher, being aware of being a teacher, raising issues* and relating them to teaching more generally, *relating my perceptions* of teaching arising from my own lessons to other lessons which I observed, and *distilling at an abstract level* elements of teaching and learning which contributed to what I regarded as an investigative approach. I wrote an account, at this time, of my awareness of these levels with examples from the series of lessons, and I include this account in Appendix 3, section 2.5.

In teaching the pairs of lessons in the second stage, I worked with the same objectives and tried to present the same tasks or activities to each of the two classes. However, the classes were different and, perhaps inevitably, each lesson was different in some respects from its pair. I saw the main reason for this being in the responsiveness of the pupils to what I asked of them.

² Antaki and Lewis (1986) provide an account of such levels of meta-cognitive awareness which accords strongly with this experience.

One class seemed very much more lively than the other and this influenced my own levels of challenge and response³. After a pair of lessons the teachers, who had each observed me teaching her own class, talked with me about our perceptions of the lessons. This was a most valuable experience for me, reinforcing what I said of Christine Shiu above. In many years of teaching, I had not had chance to work with others and plan together in this way, reflecting jointly on outcomes and considering the issues involved. The teachers also claimed to find these conversations valuable. They felt that they learned from the differences perceived in the pairs of lessons. They therefore decided, for the second half term, that they themselves would offer pairs of lessons which they would jointly plan. They wanted these to be based on some syllabus area, but wanted to try to make the lessons investigative in style.

STAGE 3 — PAIRS OF LESSONS TAUGHT BY FELICITY AND JANE

They settled on the topic of *tessellation*, as this would allow them to do work on shape and angle, and together we talked about mathematical objectives and discussed what activities would bring the pupils up against the mathematical ideas which the teachers wanted to address. I observed each lesson in the pair and made field notes. After each lesson, I used my field notes to write an account of the lesson as I saw it. At this stage I had not begun to think overtly of other perceptions of the lesson. I saw my account as being in some sense 'accurate' in so far as it could only include what I noticed and what I did not notice usually went unregarded. That whatever I did notice implied some level of significance for me was not a recognition which I recall from this time. I was only just beginning to realise that what I saw was no more than my perception of what happened. I recorded conversations with the teachers and used these to support my accounts of our conversations. The issue of *planning* versus *outcome* cropped up again and again as it had in my own lessons. I was fascinated by the teachers' different interpretations of what we had jointly planned, and the different ways in which the groups of children reacted to what they were offered or asked to take part in.

This observation and analysis was very important with regard to my developing sense of characterising an investigative approach and finding an appropriate methodology in which to work, as was my growing awareness of my own perceptions. Also, many of the issues which arose with these teachers were precursors of issues which permeated my later

³ Foreshadowing considerations of *Mathematical Challenge and Sensitivity to Students*. See Chapter 6.

research. My focus in reporting on this third stage of Phase 1, will therefore be one of using what occurred, my reflections on this and the issues which arose, to set the scene for the work of Phases 2 and 3.

I shall use one lesson, *Tessellations*, from this stage to illustrate

- my records of the teaching observed;
- issues which emerged during subsequent discussion with the teachers;
- my analysis of the data collected in field notes and recordings.

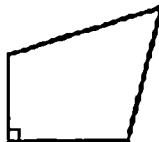
3: A LESSON ON TESSELLATIONS

The account which follows is of the first 50 minutes of the lesson. I wrote it later the same day from field notes and memory. The numbers throughout the account relate to the comments which follow it.

Tessellations

[9.10 am]

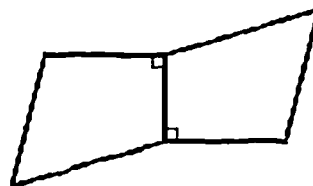
Pupils grouped around a table in a circle, sitting, standing. Outer ones could not see very well. Jane asks, "Suppose I want to tile my kitchen floor with tiles of this shape. (She shows them a cardboard cut out shape and places it on the table in front of them.) Can I do it?"



A number of pupils say 'yes'.

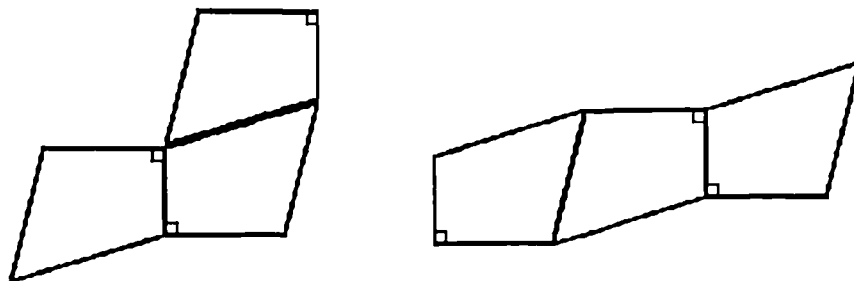
Jane invites Susan to take a tile, then "How could Susan fit her tile to the one on the table?"

A few pupils offer suggestions, not very clear. Susan makes attempts at placing the tile. It is placed.



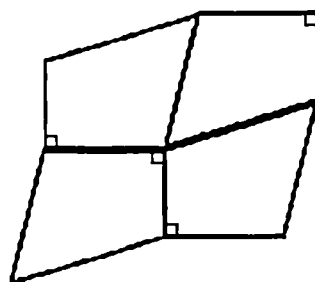
Jane – “Can someone tell Alison where to put the next one?”

Two possibilities emerge:



There has been a lot of shuffling, with noises of agreement and disagreement about possible positions. No one is articulating very well.

Jane – “Emma, can you put another one there? (She does) Is that right?”



A number of pupils say ‘yes’.

Simon – “Roughly”, then, “Miss, why don’t you have a carpet in your kitchen? [1]

Pupils are getting very restless on edge of group. Vicky, next to me, is not listening at all.[2]

Shain – “I know how to do it miss!”

Jane doesn’t hear. [2] It’s hard to sustain everyone’s interest.[3]

A pattern is building up on the table. No one is being critical.[2]

Jane seems to decide to move things on. She labels the corners of a tile A,B,C,D and prompts the group to notice the arrangement of four tiles shown above. What angles meet at a point? [4] One of each of A, B, C, D. Asks about angles of a quadrilateral. They add up to 360 degrees. So why do these tiles tessellate?

Ashley – “All different sizes.

Simon – Where they meet, angles add up to 360.

Deelip – All different angles meet at centre.

Lindsay – Two sides have to be both the same length.

Jane cuts some of the tiles, but they are still quadrilaterals. Will it still work? [4] Pupils at table start fitting the new tiles, but the outsiders are now very restless. It's been half an hour here.

[9.40 am]

Jane – “Go back to seats. Write instructions for laying tiles – what has to meet at a point? Compare instructions”

I go to watch Vicky. She hasn't a clue. She has to start fitting tiles. Julie has a vague idea. I get her to explain, but Vicky is lost. Is this usual? Some pupils have certainly got the idea – ‘you’ve got to have one of each angle’, seems to be generally accepted.[5]

[10.00 am]

Jane invites contributions. [6]

[The lesson continued for another 20 minutes]

Data item 5.10: Lesson account (29.4.86)

The comments which follow are current reflections on the above account, relating principally to methodology. The numbers label statements of the account:

- [1] I did not record Jane's response to Simon in my field notes, so I am not able to say more here. Thus it is not possible to ask questions about Simon's focus of attention or Jane's awareness of it. In Phase 2, I became more aware of such deficiencies and tried to take steps to remedy this. However, here I did not think of asking these questions, either at the point of recording Simon's remark in my field notes, or later in writing my account. I remember seeing it as no more than an amusing aside. Now I should want to ask questions about Simon's construal.
- [2] How do I know this? It is a high level of interpretation which I should now want to justify.

- [3] This remark is a result of recognising that I have been in such situations myself and I speak from my own experience
- [4] Whose question is this? It might have come directly from the teacher. It might be a question that I held to be of current concern, even if no one uttered it. The style of this account makes it difficult in retrospect to perceive what happened.
- [5] I have a vivid memory of my conversation with these girls, which the words above evoke, despite their scant nature. So, although some of this report leaves me asking questions about what actually occurred, who raised certain questions, what evidence I had for certain interpretations; in other places the words are highly evocative. Although I can no longer recall what the girls said, I can remember Vicky's state of unknowing and Julie's tentative attempt to explain. I can make assertions with some confidence about their construal of the activity of the lesson
- [6] I recognise now the value of a good audio recording of a lesson such as this. A transcript of the lesson would back up incidents, such as that of Julie and Vicki, which I recorded as significant in my field notes, substantiating and validating the perceived significance.

4: MY IMMEDIATE ACCOUNT OF REFLECTIONS ON THE *PAIR* OF TESSELLATIONS LESSONS

Jane's lesson described above was one of a pair, the other taught by Felicity. At the end of the day the three of us sat together in an empty classroom, with tape recorder running, and discussed the two lessons. On my way home I replayed the tape, and as soon as I arrived home I wrote an account of our conversation. I include extracts, below.

Conversations with teachers

Jane had been ill the day before and still was not feeling too well, and was aware that her lesson had been influenced by this. She had been unable to think ahead of a situation and anticipate what strategies might be most appropriate. The group around the table had suffered from the outer people not being able to see clearly what was happening, and not feeling a part of the activity, so that it was easy for them to be distracted and not a part of the discussion. * This sort of group situation is very hard to handle under the best of circumstances.

We talked of Alison and her comments. She is clearly intelligent, but switches off when not being challenged directly. * Jane wants to try getting her asking questions herself, and also interacting more with others in the group. The activity for next half term may help her with this.

Felicity felt that her lesson had been more 'successful' than her previous one – possibly because she had been 'better prepared' * and so had felt more confident in handling the pupils suggestions. She was trying to deflect comments and questions rather than re-phrasing them herself and was noticing some success in this. *

She noticed that the whole class discussion after the group work did not throw up *all* the ideas that had come from the groups. (In particular she had worked with Nicky's group and I had worked with Howard's group – both had ideas about *how* shapes tessellate, which had not come out in the general discussion.) This raises questions about 'how *does* the teacher get a sense of what went on in groups which she was *not* able to visit? * She may have to rely on what they have written, and this does not always do justice to their ideas.

She suggested the possibility of forming one or two groups of pupils to work investigatively during a 'booklets' lesson where the rest of the class work independently on SMP booklets. This would enable her to attend to the groups and have a chance of getting a sense of their thinking, and being able to spend more time with them herself. * An experiment on this is planned for next half term.

I felt very stimulated by this discussion. I felt that important issues were starting to emerge on which we want to work, and that we are identifying ways of working on them. It is as if our feelings of there being vague things to attend to are sharpening up into specific issues and forms of action.

* NB The *'s point to issues or ideas which we can try to keep track of, follow up, notice, think about etc.

Data item 5.11: Account of conversations with teachers (29.4.86)

I promptly posted off copies of this to the two teachers, and my recollection is that it was written as much for them as for me. I had been invited initially because they were in the process of learning about what investigational work might mean for them. Our collaboration was very much a two-way one and it seemed important to feed back to the teachers as much as I could in response to their welcoming me into their classrooms. Towards the end of our work together, they said that I had been a valuable catalyst in getting them to discuss and reflect on their work, and to share planning for and learning from lessons. This mirrored my own feelings after the lessons which I taught. They hoped to be able to

continue their own work together next term when I should be no longer a regular visitor. I kept in touch with them for some time after this Phase ended, and they told me that sadly, various pressures meant that they very rarely found the time to talk in this way. My presence had made them find the time, but no doubt other aspects of their work, or free time, suffered in consequence. With the teachers in Phases 2 and 3, I gave little feedback as there was less of an overt learning situation on the part of the teachers. However, all of the teachers said that my presence had allowed them to reflect on their teaching in a way that might not otherwise have occurred. I conjecture reasons for this in terms of the *teacher-researcher relationship* which I elaborate in Chapter 8.

5: LATER REFLECTIONS ON THIS ACCOUNT

The above account refers very briefly to concerns which we discussed at length. I wrote the account for the people concerned, expecting my words to evoke our discussion, and, I recognise now, homing in on the issues which seemed salient to me. In our conversation, despite an express wish to let the teachers lead the conversation, I recognise my own channelling comments. However, at the beginning of this stage of paired lessons I had written as part of my objectives for the role I wanted to play, "Try to remain neutral until I have *their* reaction and comments – only then satisfy my own queries." For consistency with the different stages and levels of thinking which occurred, I shall now reflect on the above account with reference to a transcript of the conversation which I subsequently obtained.

At the beginning of the account, I referred to Felicity's feeling that her lesson had been more *successful* than her previous one, which she had despondently referred to as being very 'flat'. I now quote from the transcript:

Use of transcript for verification (1)	
Fel	... but I felt that it was more positive today.
BJ	When you make a general statement, 'it was more positive today' – I know what you mean there, and I agree with you – but if you're trying to pinpoint <i>today</i> what made it more positive, as opposed to last week when you felt it wasn't so positive; what was it?
Fel	I was better prepared today.
BJ	Well, was it just that?

- | | |
|-----|--|
| Fel | I think it was. |
| BJ | Or was it simply your feeling of guilt, that you weren't well enough prepared last week? I didn't notice that you were ill-prepared last week. |
| Fel | I didn't feel ill-prepared last week. After the lesson I felt that I hadn't thought about it enough then. That's how I felt afterwards, although I didn't feel like that before I went in. I felt that I had given it enough thought, but I don't think last week that I had thought of all the avenues of thought that they might think about. Whereas this week, I spent time with the shapes and did it myself to get some idea of how they might feel, and I felt I was better equipped for some of their responses. |

Data item 5.12: Extract from transcript (29.4.86)

This is all the transcript gives me directly to support my word 'successful'. In fact Felicity's word had been 'positive'. However, I also had my own response to the lesson, which had been one of exhilaration, to back up my interpretation. I had seen a bubbly lesson, with many lively contributions by pupils to whole-group discussion and a buzz of activity and discussion in the groups. The atmosphere had been very different to that in Jane's lesson which I reported above, and indeed to Felicity's previous lesson on tessellations. So my interpretive word 'successful' drew on my own experience of what I had deemed to be a successful lesson, as well as on Felicity's own reflection on it.

At the end of the first paragraph I had written, 'This sort of group situation is very hard to handle under the best of circumstances.', which the reader could construe as a very patronising comment. However, the comment relates to my note at [3] above (p 102). When I was teaching the pairs of lessons, I recall occasions where I had been in a situation of recognising that pupils might not be fully attending, yet having reason to continue the whole-group activity, and recognising questions related to how to get pupils more involved. Of course I cannot assume that either of the teachers would have realised this from what I wrote. My awareness of differing perceptions makes this now an obvious remark, but when I wrote the sentence I was too bound up in my own meaning to consider others' construal of it.

In the second paragraph I referred to a pupil, Alison, whom we had talked about at length. This now reminds me of some of the extended conversations with Clare in Phase 2 about particular pupils, from which I

developed the classification heading of 'Sensitivity to Students'. Much of the conversations with Jane and Felicity revolved around particular pupils or groups of pupils, and their particular responses or needs.

Both teachers reflected at length on what had happened in their lesson, and how this fitted in with their planning and satisfied their objectives. My account above does not do justice to this. The following excerpts from the transcript are first from Felicity and then, separately, from Jane:

Use of transcript for verification (2)

Fel My aim was to follow one of the questions that was asked at the end of the last lesson, which was, 'why are some tessellating and some not?'. Because some people had got to the stage where they saw hexagons tessellating, and quadrilaterals, but they found pentagons didn't, nor did octagons. And some of them were asking, 'Why aren't they tessellating?'. And so the aim of my lesson today was to try to find out why some shapes tessellated and why some didn't. And the other aim was to work on the children explaining more fully when they were discussing things. I don't know whether I achieved any of the first one. ...

Because we didn't get on as far as we might get on, but I wasn't unduly concerned about that, because I felt that the point we had got to and what we had done up to that point was really very valuable. ...

The group work that they did, I'm not sure that it worked exactly as I'd hoped it would work and that they actually focussed on the angles meeting at a point as I'd hoped that they might, but I did think it gave the opportunity to discuss in smaller sections some of the points.

Jane I think that if I were doing the same thing again with a different group ... I would have cut out a lot more [shapes] and I would have put them out and taken groups of six together; and rather than having *that* [whole group around the table] I would have had more shapes available, and perhaps not even the same shapes ... and even asked the group to cut them up ...

Because if you put it on paper and you make a mistake, you can rub it out, you can actually turn it round and turn it over – it's something *there*. If I was starting at the same point and wanting to get the same message across ...

Both teachers were very concerned about what they wanted to ‘get across’, and this kept coming up in differing circumstances and differing language. I now interpret this as a forerunner of my noticing what Clare, in Phase 2, referred to as *prodding and guiding* – in how far this was justified and in how far it was part of her responsibility as a teacher, (see Interlude B, p 193)), and of the *investigative* versus *didactic* approach which Ben talked about in Phase 3, of the *mathematical topic* versus *investigation* dilemma which I later referred to as *didactic/constructivist tension* (see Chapter 7 p 245)).

The analysis in this section has been constructed as part of this chapter. I have made the links between accounts and transcripts as a post hoc analysis of my analysis at the time in the Phase 1 work. However, I did one further piece of writing, beyond the accounts of lessons and conversations which I have indicated in Phase 1 which was a precursor of my later use of transcript material. This was in response to a paper (Underhill, 1986) which I encountered shortly after the Phase 1 field-work ended. In it I used Phase 1 transcript material to write a response (Jaworski, 1986) to Underhill on the topic of a *constructivist approach to teacher development*⁴. Both the subject content of these papers, and my use of transcript data to provide manifestations of theoretical ideas were highly significant to future work. I discuss my own thinking in this respect in Interlude A which follows.

Conclusions

In this chapter I have discussed my pilot study as I view it looking back after research in, and analysis of, the two subsequent phases of my study. I have tried to present both analysis as it occurred at the time and my meta-comments on this analysis. I have been very aware of places in the earlier analysis where I seem to have been interpretive and judgmental without overt justification. However, setting this in context, I can recognise my own progression from raising issues in which I was myself deeply engaged to a much later ability to recognise issues but to regard them from without. In the early stages of Phase 1, my attention was *in* the issues and this prevented me from a more critical appraisal of the conclusions which I reached. As I became more aware of the act of raising issues and the analysis of this, I was able to be more distant from the issues themselves,

⁴ The papers Underhill (1986) and Jaworski (1986) have subsequently been published together as Underhill and Jaworski (1991).

and therefore more able to reflect *on* them rather than *in* them. It is painful to look back on this earlier research and perceive its limitations. However, a strand of this thesis is to chart the development of my own thinking (See Chapter 8 for an overview of this), so it has been necessary to try to look at Phase 1 as honestly as possible. I now see very positively the beginnings of issues significant to an investigative approach to teaching mathematics and the beginnings of methodology, in that I began a triangulation of perceptions on the teaching and learning which I observed. A re-analysis of the Phase 1 data might be done to link my own perceptions with those of the teachers recorded in conversations and those of the pupils recorded in interviews. I had a sense of these three sources supporting each other at the time, but I made no formal attempt to document this.

One aspect of my work in Phase 1 which concerned me then and to some extent in Phase 2 was what I saw as a lack of *objectivity* in the research. I recognised my closeness to the teachers and the teaching and felt that much of what I was thinking and writing was too subjective to be of value in research terms. One of my chief aims in *starting the Phase 2 work* was to endeavour to be more objective – in this case by becoming less involved in the teaching, and by trying to keep my own views to myself as much as possible. In retrospect I put a subtly different interpretation on this lack of objectivity. Then I saw it as my being bound up in the action which I was trying to observe and analyse. Now I perceive it as the inability to reflect on the *raising* of issues, although I could reflect on the issues themselves. Thus, what I called a lack of objectivity, might now be seen as a more limited awareness. An increased awareness allows me to handle the impossibility of being objective and count this as one of the considerations in making an analysis of an event. Thus Phase 1 was very important in this development. I needed to tackle the questions of objectivity in order to develop higher levels of awareness.

INTERLUDE A

THE MOVE FROM PHASE ONE TO PHASE TWO

From theory-validation to theory-construction, and from objectivity to inter-subjectivity.

When I began this study I had some clear ideas of what investigational work involved in terms of classroom practices which had evolved from my own teaching, my own mathematical studies, and from discussions with colleagues. These included the overt use of mathematical processes, the value of group work and of pupil discussion, the importance of trust, the dangers of teacher-lust, etc (Jaworski, 1985). I made no attempt to define an investigative approach, but rather talked theoretically about what it might involve, with some attempt to relate this to practices embodied in a videotape of classroom excerpts.

In my early work at Amberley, I observed lessons with my focus directly influenced by this theoretical view. Thus, I remarked in my diary that children were 'specialising, but not very systematically' and were 'quick to spot patterns', wrote 'Pupils now need to be encouraged to write down findings coherently', and commented that the teacher said to a pupil, "If you notice something happening, write it down". In retrospect, what I was doing implicitly was seeing manifestations of aspects of an investigative approach as I had theorised it.

It was an important realisation for me that any theory is a generalisation, and that classroom manifestations of such theory always include nuances or particularities that the general theory cannot predict. Thus the theory might claim that 'writing down findings' is valuable for various reasons. In practice there are many other facets of the classroom environment to consider – for example the pupil's particular focus, time factors and social norms – and the teacher's actions must be related to all of these, not just to the isolated bit of theory, despite its seeming importance. The issue of *planning* versus *outcome* speaks directly to this theory-practice dichotomy. Planning, although directed at what will be done, involves *theorising*, whereas the outcome – the *practice* – may involve aspects for which the theory did not account.

I became aware that the classroom manifestations which I observed, of an investigative approach to teaching mathematics, carried the essence of how practice and theory might be related, although I did not articulate this awareness in these terms at the time. I was aware that to theorise about, for example, the contribution which classroom discussion could make to the learning of mathematics (see for example DES, 1982, para 246), was valuable in terms of general considerations about classroom practice, but that in terms of what a teacher actually might *do* to instigate discussion, or what the subsequent discussion would look like, it was less than adequate. What I set out to do was to identify instances of classroom practice where theoretical aspects could be seen to be manifested, and somehow to characterise the theoretical aspects in terms of the practice. In doing this I expected to raise issues of importance to the practitioner, which might contribute to teaching knowledge more widely.

The 'fit' with radical constructivism

The teaching issue which arose most strongly from Phase 1 work was that needing to 'get across' mathematical ideas did not seem to *fit*¹ with encouraging pupils to investigate. This was strongly highlighted when I first formerly encountered notions of radical constructivism in Underhill (1986) and an associated presentation at PME X². My excitement with the ideas involved was due to my introduction to a theory which seemed to *fit* my thinking regarding an investigative approach to teaching mathematics. I was inspired to write a response to the Underhill paper (Jaworski, 1986) which set experiences from the Phase 1 work into a constructivist perspective related to teacher development. The main ideas in this paper are seminal to my developing thinking at this time, so I will summarise them here.

Underhill had written,

New learning is not something which the teacher-educators will 'give' to the teacher. Nor is new learning something which the teacher will 'give' to the learner.

Implicit in a constructivist belief was the recognition that knowledge could only exist in the mind of the knower, and that any objective reality was unknowable, even if it should exist (von Glasersfeld, 1983 – see p 15

¹ My use of the word 'fit' is technical in the sense used by von Glasersfeld (1984)

² The tenth meeting of the International Group for the Psychology of Mathematics Education, held in London, in July 1985.

above). Yet I recognised that belief in a transmission process made teaching easier or more bearable for the teacher, and this was witnessed by the two teachers in Phase 1 (See Example 1 in Jaworski, 1986). I also recognised that teachers were encouraged in perceptions of teaching-as-transmission by a popular belief in the handing over of knowledge, and in particular by their pupils' sharing this belief. One of the teachers wryly commented on the sort of approach she felt her pupils would prefer, and in consequence the difficulty of sustaining one which encouraged them to make constructions of their own rather than to memorise given information. (See Example 2, *ibid*) I recognised that even though one might espouse a constructivist philosophy, it was nevertheless hard to break away from long encouraged belief in the objectivity of knowledge and the possibility of giving this knowledge to another person.

I produced examples from the Phase 1 research which illustrated both teachers' and pupils' belief in some form of objective reality of knowledge, both mathematical and pedagogic, and pointed out ways in which this influenced classroom interactions and approaches to teaching. (Examples 3 and 4, *ibid*). This seemed to contradict the teachers' developing beliefs in an investigative approach, and resulted in a tension which arose from a desire for pupils to discover particular mathematical 'facts', and the reluctance of the teachers to 'tell' these facts when it seemed that the pupils were not going to discover them in quite the form which the teacher desired. It seemed that the telling of the facts would in some way invalidate the pupils' construction of them. I realise now that I was only just coming to terms myself with the implications of this. The issue of 'what and when to tell' had exercised me during Phase 1 as it had the two teachers with whom I was working. I felt that *telling* indicated a lapse back into 'transmission teaching', and was therefore something to be avoided. Yet the realisation that expecting pupils to discover everything for themselves was equally a nonsense, created a real tension for me at that time. It was becoming clearer that pupils' construal of mathematics included making sense of what teachers told them just as it included making sense of their own discoveries, and that learning involved learners in processing information regardless of where it originated. Underhill had written,

Even if we get learners to believe [in constructivism] what assurances will we have that they will behave as we wish?

This seemed to me to be a part of the whole dilemma of trying to teach within a constructivist philosophy, that despite trying to behave like

constructivists ourselves we nevertheless find ourselves trapped within our own expectations regarding the outcomes of our teaching.

The struggle with notions of ‘what is’ and ‘how you can know it’ was made manifest in my perception of pupils’ behaviour and beliefs at the time. In introducing investigational work the teachers offered pupils the two sorts of lessons which I have characterised as *booklets lessons* and *classwork lessons* (see Chapter 5). Interviews with pupils revealed that they perceived these two types of lessons quite differently. They differed on which types of lesson they preferred, but all felt that the classwork lessons made them think harder than the booklets lessons, whereas the booklets lessons were easier to cope with. One pupil said that the booklets lessons offered ‘little questions’ to which the answers were easy to find and could be checked off on an answer sheet, so you knew when you were right or wrong. The classwork lessons had big questions which often made your brain hurt, and to which you were never sure of the answers. I now see this as a distinction between existing comfortably within a given knowledge base, compared to being confronted with a possible lack of ontology.

RELATING CONSTRUCTIVISM TO TEACHER-DEVELOPMENT

The Underhill paper had focused on teacher education and development, and this was part of my own concern at the time in relation to the work for which I was employed. I therefore could not avoid also thinking in terms of teacher development, although my study of the Phase 1 teachers was not overtly about their development. I found myself drawing parallels between the pupils’ learning of mathematics and the teachers’ learning of pedagogical practices. One teacher had raised the issue of how you get pupils to be critical of what they perceive as the status quo, for example, *how* to get them to ask questions like ‘*why* do quadrilaterals tessellate?’ when the pupils seem happy to accept that ‘they just do’. I compared this to asking, from a teacher-educator’s perspective, ‘how do you get teachers to *notice* things like ‘decision points’, so that they can subsequently work on them for greater awareness of options in teaching’. From a constructivist point of view, teachers will make their own constructions of *the need to ask questions*, or the value of *noticing*. A teacher-educator can draw attention to such notions which will influence but not determine the constructions made. This became one of the most important issues which I addressed, and to differing degrees saw the teachers addressing, throughout my study.

Implications for Phase 2

I can summarise the above discussion in the two major implications for Phase 2:

1. My focus changed subtly from:
looking out, implicitly, for certain theoretical aspects of an investigative approach which I had myself identified, i.e. starting from a strong concept of investigative teaching and looking for it;
to
regarding what took place in the classroom and trying to characterise that, i.e. looking at what was there and then telling the story of it in terms of investigative teaching.
2. Seeing an investigative approach in terms of constructivism, made me start to question my desire for objectivity and move towards a recognition that *inter-subjectivity* was what I needed to aim for.

CHAPTER 6

THE PHASE TWO RESEARCH

The characterisation of an investigative approach through the practice of two teachers

INTRODUCTION

Phase 2 constitutes the first part of my main study. My chief aim for this phase was the characterising of the mathematics teaching which I observed in terms of an investigative approach. However, an important new focus was to explore how this might fit with a constructivist philosophy of knowledge and learning in terms of designing teaching to facilitate pupils' construal of mathematical concepts. The pilot study had been valuable in establishing a methodology of observation and informal interview, the details of which could now be modified. I had become aware of the importance of raising issues – the teachers' issues and my own – and of exploring these issues and their possible implications with the teachers. The Phase 1 teachers were only beginning to explore an investigative approach to teaching mathematics. I hoped to find teachers more experienced in this for the Phase 2 study. This was difficult without personal knowledge of such teachers. I met Clare at an opportune time. Our conversations about teaching led me to be interested in her approach, so that when she invited me to observe some of her lessons, I accepted. I was not committed to using this observation as part of Phase 2, and could have chosen other teachers elsewhere. However, the more I saw of Clare's teaching, the more convinced I became that she was a suitable Phase 2 subject.

Her school, Beacham, had been set up and developed with a *progressive* ideology (Edwards and Mercer 1987, p 35ff) in which students were overtly respected and treated as individuals, and students and staff were on first-name terms. Clare had been teaching for five years and was recognised in her school as being competent and successful. She was a pastoral team leader for a group of staff having care of a number of classes within the school. Her care for, and interest in, students' social as well as academic well-being was apparent in her relations with students in mathematics lessons. Classes in the school were mixed-ability, and students in a class stayed together for most of their lessons. Clare invited

me to observe a fourth year class whose students she had taught in their third year. Thus at the beginning of their fourth year she already knew them quite well.

I often encountered Mike, Clare's head of department in mathematics, when I went into the school and he showed interest in my work with Clare. The mathematics department was quite a close-knit group who worked as a team, any one teacher often using materials prepared by others. The start of Phase 2 coincided with the introduction of the GCSE examination, and GCSE coursework was an important focus of the work of the department at that time. I attended a department meeting where the teachers discussed issues related to grading such coursework, and started to become aware of some of the principles to which the department worked. As my observation of Clare's classes progressed, discussions with Mike became more frequent and subsequently I was invited to observe one of Mike's classes, a third year group which was his own class for pastoral care. In this way Mike also became a subject in my Phase 2 study.

The department used the KMP (Kent Mathematics Project) individualised mathematics scheme. This consisted of sets of linked work cards through which a route was designed, by the teacher, for each student, according to their particular needs, so they could progress at their own pace. The classes I observed were familiar with this scheme and it operated smoothly with students taking and replacing cards, marking their own work, checking it with the teacher and periodically doing review tests. In both classes the teacher's operation in KMP lessons was chiefly in talking with individual students or groups of students. There was little or no teaching 'from the front'. However, these lessons were only about half of the diet of the class. The other half of their lessons were known as 'project' lessons. In these, students did extended pieces of work, 'projects', which usually had a common starting point introduced by the teacher. From this point, students diverged according to their own interests and abilities. The finished projects then contributed to students' coursework for the GCSE examination and were assessed by their teacher.

A sentence with which I became familiar during my observation was, "It's only a KMP lesson today". This was said on occasion by both teachers, not I think to undervalue the KMP lessons, but to imply that I should find them less interesting to observe than the project lessons. In one respect this was true. A project lesson, especially in its early stages, often involved more obvious energy and stimulation than a KMP lesson. This is not surprising in that it usually involved the teacher in attracting students'

interest and creating motivation for involvement. Thus a lot of effort went into the introduction of a project and this was very visible. However, once project lessons were under way, students worked at their own pace and on their own ideas, and the atmosphere was not very different to KMP. In both classes, students sat around tables in groups, mostly of their own choosing. One difference between KMP and *project* work was that in KMP lessons students often worked on their own with only social interaction with others around them. In project work, active group cooperation and joint involvement was encouraged, and students would be more likely to work together on their mathematics. However, these ways of working were not exclusive. Some students worked together on KMP cards, and some students worked singly on projects. My own interest, primarily in the *teaching*, was just as much in the interactions between teacher and students in the small group situations, as in the whole class, 'teacher-up-front' situation. Thus I could, in theory, learn as much from a KMP interaction as from a project interaction. In the event, I believe that I found more salient moments in the project lessons than the KMP lessons. The common focus created a certain stimulus and a feeling of shared purpose. There were occasional whole class periods where the teacher encouraged sharing of ideas. The nature of project work was that it challenged students to think beyond given starting points and very diverse questions could be tackled. KMP was rather more contained. One goal was the end of the current work card, or the end of a sequence of cards. The distinction was very much what I had observed between SMP lessons and classwork lessons at Amberley. However, I felt that the teachers' ways of working *with* pupils in their groups in KMP lessons was very similar to that in projects. In both types of lessons pupils were expected to think for themselves and to be able to justify any results they presented.

At Beacham, pupils were referred to as 'students', and I shall follow this usage now throughout the chapter.

METHODOLOGY

DATA COLLECTION

I set out in Phase 2 to be seen rather as an interested, sympathetic observer than a second teacher in the lessons. This did not mean that I avoided eye contact with students or conversations when these arose. I wanted to be able to ask questions in an atmosphere of trust and respect. However, I did not seek to get involved in the regular progress of a lesson.

In the first term of observation of Clare's lessons, I gathered data only through field notes. I did no recording on audio- or video-tape. This was mainly due to my desire to be unobtrusive, and to affect the course of a lesson as little as possible. It was also in part due to being unsure of Clare's own preferences and being reluctant to intrude until I knew her better. After the first term she agreed to the use of recorders and I recorded most lessons on audio-tape and some on video-tape. I recorded most of Mike's lessons on one or both media. Typically, the teacher carried one audio-recorder, a second recorder being used when appropriate to gather data from selected groups within the class. Thus ultimately my classroom data consisted of field notes, audio- and video-recordings, and transcripts of these.

I also interviewed the teachers informally, both before and after their lessons, whenever possible. Many of these interviews were extensive, and, as in Phase 1, often seemed more like *conversations* or *discussions* than interviews. I sought to know the teachers' thinking regarding the planning and execution of lessons, their reflections on those lessons, and their overall objectives and philosophy which influenced planning and operation. I tried in the early stages of Phase 2 to avoid expressing my own opinions or being drawn into the discussions. I was more successful in this with Clare than with Mike where, often, interview turned into discussion and it was hard to resist getting involved in the issues which arose¹. My aim was to influence the teachers' talk as little as possible. However, later reflection revealed the impossibility of avoiding influence, firstly in that subsequent responses depended on every question which I asked, and secondly in that I could only view what I saw and heard through my own perspective and interpretation.

When Clare read the above paragraphs, she responded with the words, "you also, by initiating, influenced the process of reflection". This supports what I will say in greater detail later about the teacher-researcher relationship, and teacher reflection.

I triangulated data in three ways. The first was by employing secondary observation in Mike's classes. The second was by seeking student comments in a number of ways to which I shall refer during the chapter. The third was through the use of video-tape from Clare's and Mike's lessons for stimulated recall with Clare and Mike together, in meetings of

¹ An example of this is provided, in transcripts of typical conversations, in Appendix 4.

the whole mathematics department, and, on one occasion, with Mike and a group of his students.

DATA ANALYSIS

Most of my formal analysis of the data collected was done after observation had ended, and thus could not influence observation as the study progressed. However, it meant that a substantial amount of data was available for study at the later stage. I began by analysing the early Clare data and testing out this analysis on the later lessons. Analysis of the Mike data came later, and was undertaken in two stages. The first stage was a broad identification of significant events, practices, and issues arising from Mike's teaching. The second stage, which was done after analysis of the Phase 3 data, was an attempt to test out a descriptive framework which arose as a result of the Clare analysis, and was supported by the Phase 3 analysis. This chronology is important to my accounts. Figure 6.1 illustrates it diagrammatically.

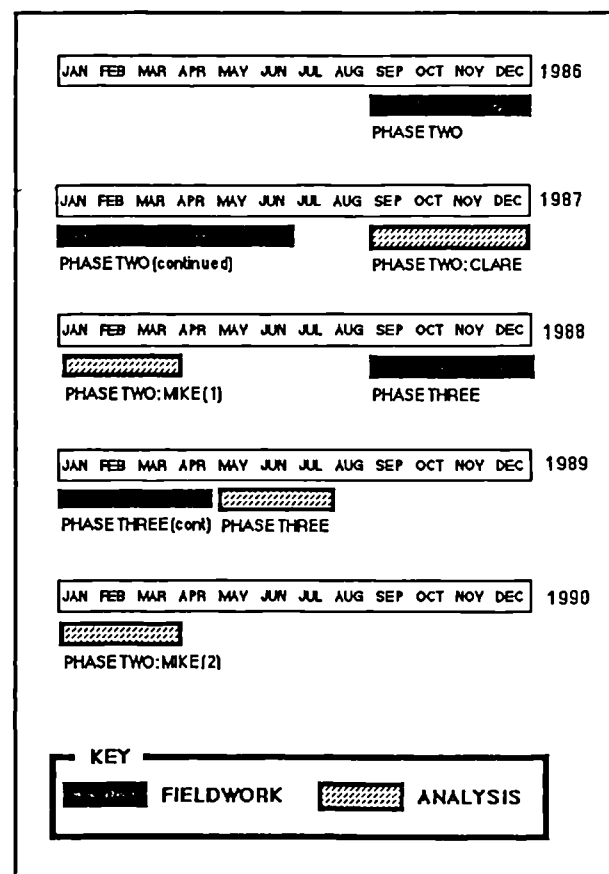


Figure 6.1: Research chronology

The study of Clare's teaching

INTRODUCTION

I began to observe Clare teaching her fourth-year class in September 1986. My observation fell chiefly into two parts – that in the Autumn term of 1986, and that in the Spring term of 1987. In the Summer term of 1987, I continued observation briefly, including one other class. I interviewed students in her fourth-year class, and I collected questionnaire data from all her classes.

The different modes of collecting data in Clare's classroom influenced my analysis of the data. I worked first on my field notes from the Autumn term, looking closely at what I had deemed to be of significance, and trying to categorise it in some way. As a result of this I tested out my categorisation using the transcripts from the Spring and Summer term recordings. Finally, I triangulated previous analysis with data obtained from students in the Summer term. This is very much a methodological overview, details of which are important to conclusions drawn and to my subsequent study, so I shall elaborate on this methodology as I discuss my data analysis.

THE LESSONS FROM WHICH DATA WERE COLLECTED

Figure 6.2 gives lessons observed over the three terms. Those not labelled KMP were 'project' lessons. I have included all lessons observed, for chronology and completeness, but some will not be referred to in further discussion.

Autumn Term 1986	Spring Term 1987	Summer Term 1987
1 Lamp, child and shadow	9 Knots 1	15 Packaging
2 KMP	10 Knots 2	16 KMP
3 Fractions 1	11 KMP	17 Circles in rectangles*
4 Fractions 2	12 KMP	
5 KMP	13 KMP	
6 KMP	14 Lines crossing	
7 Statistics booklets		
8 Statistics presentation		* with third year class

Figure 6.2: Lessons observed with Clare

In the KMP lessons, students followed their own work card programme set by the teacher. Sometimes they worked on cards individually, sometimes together.

ANALYSIS OF THE AUTUMN TERM LESSONS – CLASSIFYING ATTRIBUTES FROM FIELD NOTES AND RECOLLECTIONS

This analysis resulted from working on my field notes, from the seven lessons which I observed, in an attempt to identify what I had regarded as significant and in some way to categorise it. The process is outlined in Chapter 4, and given in more detail in Appendix 4.

This detailed scanning of field notes and related categorisation led to a tentative characterisation of Clare's Autumn term lessons in three main categories which I called, Management of Learning (ML), Sensitivity to Students (SS) and Mathematical Challenge (MC)². These resulted from the integration of many other categories as may be seen by referring to Appendix 4.

1: MANAGEMENT OF LEARNING

I shall look first at attributes and categories from the early analysis which combined under the more global ML heading, and go on to discuss further manifestations of ML in Clare's teaching.

1.1: ATTRIBUTES AND CATEGORIES WHICH ML SUBSUMED

I shall begin with the data from the Autumn term lessons and draw from the early analysis particular manifestations of attributes and categories which ultimately contributed to a classification as ML.

I regarded as similar a number of Clare's questions or instructions which seemed to be trying to find out students' *intentions*, or to make explicit to students that she expected them to have intentions. She seemed to indicate that they had choice and were expected to make decisions. Particular forms of words noted were:

² These are strongly paralleled by Cooney's (1989) triadic scheme of classifying decisions, as managerial, affective or cognitive.

Some of Clare's words from lesson 1		
(3) ³	Tell me what you're going to do.	ML
(5)	So, what're you going to do?	Q
(7)	What're you going to find out if that's true?	Q
(10)	What're you going to do next?	Q
(13)	What're you going to do with that?	Q

Data Item 6.1: Extract from field notes (26.9.86)

The coding on the right indicates the categories into which I placed these items at the time. I observe that Q, for *questioning*, was not a very helpful category – it did not discriminate between types of question. I eventually abandoned it in favour of noting other attributes of the item than merely that it was a question. The above questions seemed to be to do with encouraging in pupils an attitude towards their work which would ultimately influence their learning. They thus seemed to come under the ML heading.

Certain items seemed to me to indicate *strategies* which the teacher used. These were

More of Clare's words from lesson 1		
(6)	You try to convince – that you're right. (Twice)	S
(9)	Telling two groups to tell each other about what they've been doing.	S
(16)	Spend the last few minutes recapping what you've been doing in this lesson.	S

Data Item 6.2: Extract from field notes (26.9.86)

In number (6), Clare urges two girls to convince each other that statements which they have made are correct. In number (9), she urges something similar on two separate groups. In number (16), she urges them to think through what they have been doing in the lesson. All of these seem like

³ Whereas in *transcripts* the figures to the left simply number statements, in these extracts from *fieldnotes*, the numbering was part of the subsequent analysis, where they identified *significant* items in sequence – an example of this can be found in Appendix 4, sequence 1, together with details of the coding categories on the right.

particular strategies to foster and reinforce *thinking*; in (6) and (9) also to encourage *expression* of ideas, which might reinforce the ideas or encourage modification and refining; in (16) also to encourage *reflection*. In all cases these instructions seem to reinforce ways of working which the teacher valued, and thus implicitly to support learning. Hence they provide further manifestations of ML.

In this first lesson, I had noted aspects of ‘working in groups or pairs’ and ‘pupils moving freely around the room’, which I had categorised as *classroom atmosphere*, A. These seemed also to be related to Clare’s overall classroom management and creation of an atmosphere for learning. The working in groups was a feature of most of her lessons, where students moved freely about the room. This movement was mainly purposeful, and there was usually a good working atmosphere. There were times when Clare remonstrated with students when she was unhappy with their activity or behaviour. I noted her remarks on the *noise* in the room, asking students to work more quietly. There were also times when she directed students towards particular places in the room or to particular groups or questioned aspects of their activity. For example, she spoke to two students who had been out of the room for a time as part of their activity, and said “Your reason for going outside was 75% novelty and 25% to do with shadows – am I right?”. This indicated, albeit in a friendly way, that she was aware of *what* they had done, and possibly of *why*, and I saw it as a reminder to them that she was monitoring their activity. In interviews in the Summer term I asked students, “What do you think about the way Clare runs the lessons – about the organisation, about the things she expects you to do or not do?” Responses to this supported the above analysis, for example,

Well, she’s basically very strict. It’s a funny sort of strictness because it’s not sit down and quietness and this, because she allows a certain amount of leeway. So I mean she will let you sit with your friends when you start off, and chat, but sooner or later she decides, you know, if it’s good for you.

I think it’s more controlled, nobody actually, people talk, but nobody really blatantly mucks around. (Kim, 7.6.87)

Finally, at the very end of the lesson, Clare asked students to prepare their *Lamp, child and shadow* projects to hand in for assessment. She acknowledged that some students might have needed a little more class time for this, but her remark was, “You need to come to the next lesson with a programme of work – to convince me of why you need more time”. This remark was typical of many others which encouraged students

actively to take responsibility for their work. It was managerial in making her expectations clear and fostering their own ability to think for and organise themselves.

1.2: MANIFESTATIONS OF ML

In the section above, I have shown that many of my initial attempts at classification became subsequently seen as facets of Management of Learning. I shall now look at further situations from the Autumn term's observation for particular manifestations which I see, together, as characterising ML.

Explicitly recorded as ML at the time were items from the two *Fractions* lessons as shown in Data item 6.3, which consists of significant items from my field notes.

Statements made by Clare during the two <i>Fractions</i> lessons	
1	Can we have a hands down think. I did $\frac{1}{2}$ – I want <i>you</i> to <i>think</i> what <i>you</i> might do next.
2	Anyone who's ahead of this, try to think how to explain repetition in $\frac{1}{7}$
3	While you're doing this, with another bit of your brain do what Vicki did last week – look for other recurring patterns.
4	I want you to decide what you think about the $\frac{1}{7}$ -ths – before opening the booklet on fractions to look for other patterns.
5	All groups, pool what you've found – think of what questions to ask next.
6	In about three minutes I want some feedback from you – just think what you're going to say.

Data item 6.3: Extracts from *Fractions* lessons (10/14.10.86)

Clare had introduced this lesson to the whole class, from the front, with a mixture of what I described in my field notes as 'exposition with leading questions'. She had invoked their imagery by asking them to "Imagine you have two pizzas, and you're sharing them equally between three people. Circular diagrams representing 'pizzas' with shaded sectors to represent *pieces of pizza* had been used to illustrate particular fractions such as $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$. Clare had said, "A third is like something divided by

three.” Then she pointed to $\frac{1}{2}$ and said, “This little line, in the middle of the fraction, is telling me to divide”. She asked one girl, Katy, “What is one divided by two?”. Katy said “two”. Clare asked Katy to work out on her calculator $1 \div 2$. When Katy replied, “Nought point five”, she asked “Surprised?”, then, “If you have one thing shared between two people, how much does each get?” Katy looked blank. It was at this point that Clare instructed the class as in Statement (1) above, “Can we have a hands down think. I did $\frac{1}{2}$. I want *you* to *think* what *you* might do next” then, leaving them to attend to her instruction, she went to talk with Katy.

I saw this as being a manifestation of a complex set of reasoning on the part of the teacher. Specific to ML was first of all, ‘hands down think’. This was a form of words used frequently by Clare to emphasise that she wanted them to think about something, but without the instantaneous hand waving that might occur with quick superficial thinking. Then she indicated *what* she wanted them to think about. I interpret this as follows – ‘You have seen me do something with $\frac{1}{2}$. This was just an example. What *else* might we do this to? How? What might we get?’ I recognise that my interpretation is consistent with my experience as a mathematics teacher and what *I* would have been hoping for if I had made such a statement. I also realise that the students’ interpretations might have been very different. However it is the teaching intention that I am seeking here. I feel that it points towards a management of the learning situation, indicating to the students what she would expect of them at this instant – valuing their considered thinking, and giving a pointer to what to think about.

However, it seems that the motivation for this instruction was to give Clare space to go and talk with Katy who seemed to be having difficulty either with the notion of fraction as an operation of division and as parts of a whole, or with the language involved. Thus, this overtly embodies management of the classroom – keeping the class productively occupied while allowing the teacher to give individual attention where and when this was required.

Finally the manifestation embodies aspects of my other two major categories – being sensitive to individual students’ needs, and offering Mathematical Challenge. In the first case, Katy seemed to need individual support, and Clare wanted to give it then and there, not when the moment had passed. Secondly, she wanted the class to think themselves about the link between the fraction and the division operation, and saw the

opportunity to offer this challenge. Thus, although I offer statement (1) above as a (multiple) manifestation of Management of Learning, it also carries with it elements of the other two categories, which I shall subsequently consider.

The four paragraphs above indicate the complexity of the analysis in which I have been engaged, and its communication. From observation of a lesson and pages of field notes, I distil a statement such as (1) above which manifests particular characteristics of teaching. Then I need four paragraphs to set it in context and explain its significance. This initiates a pattern for my presentation of analysis in Chapters 6 and 7, and explains their length.

Statement (2), 'Anyone who's ahead of this, try to think how to explain repetition in $\frac{1}{7}$ ' came at a time when Clare was working with the whole class on dividing 1.0000 by 7, i.e. $7 \overline{)1.00000}$. She seemed to realise that whereas some students needed more time with this, others were ready to move on – and could themselves work on an explanation for the repeating pattern in the decimal representation of $\frac{1}{7}$ – i.e. 0.142857 142857 142 ... Thus, again, she managed the classroom, catering to the needs of two sets of students, and encouraging them to decide which group they wanted to join. They could continue to take part with the whole class in working on the division of 1 by 7, or they could opt out of this to consider the repeating patterns in $\frac{1}{7}$, as a decimal. She did not instruct anyone as to which activity they should participate in. This seemed to indicate that she respected their willingness and ability to choose wisely, but at the same time meant that some may not make the best choice. As I gained further experience with Clare I realised that in such a situation she would be monitoring students' activity and if she felt anyone was making poor choices, she would not hesitate to recommend, or insist on, another course of action.⁴

⁴I am making interpretations and judgements continually as I comment on particular situations and the word 'seemed' will crop up frequently. However, sometimes it will be omitted since to say, over and over, "it seemed to me" interrupts and is unnecessarily repetitive; especially since everything I write is of something *as it seems to me*. It is up to readers to reflect on what I have written and to consider this against what is quoted and what interpretation their own experience leads them to make. I see my task as that of characterising an investigative approach. As I offer manifestations of this, I am not offering the situation in any absolute sense - I can only offer what I saw, and it is *what I saw*, not any independent event, which is the manifestation. 'What I saw', includes my selection of what to quote as well as my interpretation of it. Moreover, this text has been read by the teachers themselves (respondent validation) and where they have disagreed with my interpretations, or added to them, I have made appropriate modifications.

In the second lesson on fractions, the following week, the start of the lesson was spent in recalling examples of recurring patterns in decimals of fractions, for example in $\frac{1}{7}$, and in $\frac{1}{11}$. She then set students the task of recording the decimal representations of all fractions up to $\frac{1}{7}$, i.e. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{1}{5}$ As part of this task, she gave the instruction in statement (3), “While you’re doing this, with another bit of your brain do what Vicki did last week – look for other recurring patterns.” This could be seen as Mathematical Challenge – ‘look for recurring patterns’, or as Sensitivity to Students, valuing ‘what Vicki did last week’, but it is the ‘with another bit of your brain’ which I feel is a manifestation of Management of Learning. It seems to say, “you have brains – use them”, and also, “you can often do more than one task at the same time”, thus encouraging them to develop effective ways of working.

Following her setting of this task, she moved around the classroom, interacting with various groups offering different comments to each. My field notes recorded the following:

Excerpt from Field Notes in Fractions 2

C talks to group 1. – “did you notice anything?” They talk about $\frac{1}{7}$, $\frac{2}{7}$, etc. patterns – C getting them to describe precisely. C – “Do you think that what Martin said was right Joanne?”

“I want you to decide what you think about the $\frac{1}{7}$ ths – before opening the booklet on fractions to look for other patterns.”

Three girls talk – Martin works alone – then joins them. Martin – “I see what’s happening – it’s shifting to the right”

Data item 6.4: Extract from field notes (14.10.86)

Statement (4) is seen in this as an instruction from Clare to the group in particular response to the way she has seen them working. It focuses their attention, and there is evidence of them tackling her instruction and coming up with ideas about the patterns. Again this seems overtly ML.

While the students were working in groups, she interrupted their work to give the instruction at statement (5), “All groups, pool what you’ve found – think of what questions to ask next.”. Some students had been working individually, despite sitting together as a group. At this instruction, many

of them did start to share their thoughts, although I recorded in my notes that some still continued to work alone. Again, this seemed to be a managing instruction, concerning the way in which Clare preferred them to work while leaving the choice to them.

Towards the end of the lesson, she issued the instruction at statement (6), “In about three minutes I want some feedback from you – just think what you’re going to say”, indicating that she wanted some more global sharing of thoughts and inviting them to prepare for this, “just think what you’re going to say”, emphasising again the importance of thinking. In responding to my words above, Clare added, “Also as a warning, allowing preparation so more people can contribute to the group”.

In Jaworski (1988c) I offered further manifestations of Management of Learning from Clare’s lessons, and indeed, selecting any of the first seven lessons, I could offer many others.

2: SENSITIVITY TO STUDENTS

In my discussion above, I have been able to point to brief phrases – questions and instructions – which have in some way manifested aspects of ML, and which allow me to draw a characterisation of what I see as ML. It is less easy to find brief phrases to similarly characterise Clare’s Sensitivity to Students.

Often my perceptions of sensitivity lay within the *way* in which Clare talked to a student in a lesson, rather than the actual words. Often it was in a lengthy exchange with a student that her sensitivity became apparent.

Because I did not record the Autumn term lessons I do not have transcripts from such interchanges. However, in a few cases my field notes had enough detail to carry the essence of the exchange. One case of this was in lesson 2, a KMP lesson, where Clare spoke with a boy, Nigel about some work on fractions and percentages.

Talking with Nigel in Lesson 2

Clare It might be a good idea to copy this out into a table. How might you do it?

Nigel $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}$

Clare $\frac{1}{10}$ might be useful – put $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}$ in if you like (brief pause)
How did you find out what they were as percentages?

There were further exchanges here which I did not note, then:

Clare What're you going to do then?

Nigel (Hesitates, thinking) I'll try, well I know that $4, \frac{1}{4}$ is 25. It's lower than 25. I'll work down to a lower number

Clare Well that's one way you could do it, (pause) remember that ...

Data item 6.5: Extract from field notes (7.10.86)

I wrote in my notes, 'Discussion now between both. T gently suggesting, student thinking aloud. T struggling to help student to see without telling him. She then decides to give him some instructions as to how to proceed.' I labelled this as DP – decision point. I felt that I could discern her probing and trying to decide how much input to give, whether an explanation was appropriate, then deciding to give some explanation. When the exchange came to an end, she said to Nigel, "When you said $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}$ in one sense my heart sank, but ..." She went on to acknowledge Nigel's contribution, and her tone seemed to convey respect for his thinking and encouragement to continue. At the same time she implied a level of collaboration, sharing her own feelings about what he was doing.

I saw Clare consistently 'tailor' interactions to the particular student. Kim, a particularly quick student, challenged her quite aggressively in the first *fractions* lessons, saying, "I've done this before!", seeming to imply, "Why should I do this?". Clare replied, quite sharply, "I don't ask you to waste your time – don't treat it like that." In the second KMP lesson, while students were working on their particular cards, she talked with individuals about project work which she had assessed. To Katy she said, "I was quite surprised by this. In the third year you didn't make much effort – but now that it matters ... I didn't know you could write as well as this.", and to Ann who struggled a lot but who had got a better grade than

usual, “This is very good for *you*, // would you like me to suggest one or two things you could do to improve it?” The remarks, even the one to Kim, were encouraging and supportive. To Kim she seemed to be saying, ‘I’m aware of your ability, and I wouldn’t ask you to spend time doing something that I didn’t consider to be valuable’. There was acknowledgement of Ann’s difficulties and of Katy’s former lack of effort, but praise for evident progress in both cases.

However, my awareness of Clare’s intense individual caring for students came from our informal conversations before and after lessons, where she would typically talk for fifteen minutes at a time about characteristics of a particular student. I reported on some of these in Jaworski (1988), and shall include later a manifestation with supporting transcript evidence. Appendix 4 contains further evidence. However, one of my first experiences of this was after one of the fractions lessons when Clare referred to a number of the students with whom she had interacted, in the lesson.

Remarks on students after <i>Fractions I</i>	
Rebecca:	“Rebecca is very bright. But she couldn’t divide 6 by $\frac{4}{5}$! I wasn’t going to tell her! But I couldn’t think of how to tell her <i>how</i> to divide fractions”;
Kevin:	“He has a very interesting background ...”;
Amy:	“She had a hip replaced. She has <i>such</i> a lot of difficulties ...”

Data item 6.6: Extracts from field notes (10.10.86)

She referred to her approach to Rebecca – “trying to take the student through some thought processes – very rarely tell something straight off”. She talked extensively about Kevin’s background, how she felt it influenced his rather extrovert behaviour in the classroom. She talked of how frustrated she felt in her ability really to help Amy, with whom she felt she was not succeeding as a teacher.

3: MATHEMATICAL CHALLENGE

As I look back to my analysis of the Autumn term lessons I realise that my recognition of Mathematical Challenge at that stage was mainly implicit. None of my coding symbols seemed to be explicitly MC, yet I recognised as significant various situations which seemed to defy coding as I allocated

no symbols to them in my analysis. These included statements like, “Spotting patterns at a simple level is going to help you later on.” and “I don’t want to give you specific instructions. *You* have to find patterns. You *can’t* be wrong.”, which I shall discuss further shortly. The lack of detailed transcript is a problem here too. There are situations which are potentially significant in terms of Mathematical Challenge, but which are difficult to analyse because too little data was captured.

The reference to Rebecca above, embodies a degree of MC. Clare regarded Rebecca as ‘bright’. An implication was that Rebecca should have been able to divide 6 by $\frac{4}{5}$. Clare wanted her to try to figure out a method of doing this herself, yet could not see what help it was appropriate to give to start her off. So there was a question here of how to make the challenge realistic.

In the reference to Nigel above, Clare was not too happy with the way he was beginning his work on fractions and percentages. For a student who is not very sure of the process of converting fractions into percentages, looking at fractions like $\frac{1}{6}$ or $\frac{1}{7}$ might be confusing rather than helpful or revealing. Yet Nigel was using a strategy of trying out special cases systematically, which was a process which Clare might have encouraged under other circumstances, as in an example below. Her degree of sensitivity to Nigel possibly also inhibited any direct challenge to his initial idea. However, further probing and discussion led to Nigel changing direction in a way of which she approved, and so she was able to say that she had been worried by his initial approach. This was a meta-comment, in that it referred to her initial commenting. Clare often shared, with students, thoughts about her comments to them.

Both of these manifestations of Mathematical Challenge are closely linked to Clare’s sensitivity to the student – in Rebecca’s case believing some strong challenge to be necessary, in Nigel’s case being gentler and more supportive until she could influence him in the way she felt appropriate. In the second KMP lesson where Clare was discussing with students her assessment of their projects, I wrote in my field notes, ‘Clare’s encouraging tone of voice supports students, respects, yet she doesn’t hesitate to point out mistakes and deficiencies.’ I felt that Clare was uncompromising where the mathematics was concerned. Where she felt that a student was not progressing appropriately she had to find a way to make some change. In ‘It’s a cuboid’ from the *packaging* lesson in the next section, I shall analyse a situation which embodies a high degree of

challenge. I felt that one of her reasons for despair where Amy was concerned, was that she was unable to find any approaches which worked for Amy, and so Amy remained unchallenged.

I shall offer three further manifestations of Mathematical Challenge as I saw it at this time. They each arose from the second *fractions* lesson. In the first case, the class had been working on the repeating decimal patterns of $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, ... and had written down particular decimal representations. Clare wrote on the board results of their calculations:

$$\frac{1}{7} = 0.142857$$

$$\frac{2}{7} = 0.28714$$

$$\frac{3}{7} = 0.428571$$

and asked what they noticed. Nigel responded with, “it’s going round like a circle”, to which Clare responded, “Right, let’s put them round in a circle. She drew the diagram

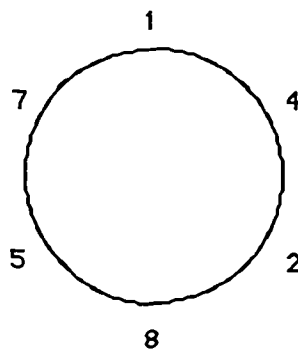


Figure 6.3: Clare’s diagram

and asked, “Is Nigel right? (There were nods.) Yes!” Then, “We can say it’s *cyclic*.” She ended the whole-class part of the lesson with the words, “Spotting patterns at a simple level is going to help you later on.”

There seemed to be important messages in this, for example, ‘patterns are important’, ‘even things like fractions have patterns associated with them’, ‘you can spot these patterns easily yourself’, ‘overtly looking for patterns is a useful thing to do’. These messages are characteristic of ML, but, at the same time, they challenge students to consider aspects of fractions which are mathematically sophisticated, and overtly encourage

mathematical generalisation. By inviting students' comments, taking them up and putting emphasis on them she valued their thinking which is characteristic of SS. These few moments of classroom interaction offer a manifestation of the interrelationship of ML, MC and SS.

A boy, Martin was exploring 'rounding off' and associated computer representations of certain decimals. He noticed that recurring decimals often 'changed at the end'. For example the decimal equivalent of $\frac{2}{3}$ is 0.6 recurring, and this might be written as 0.667, or 0.66667, or 0.66666667, none of which were exact representations. She said later of this that she had wanted him to talk about *approximation* although he had not used this word, and she was not quite sure just how much he understood. Should she leave him where he was, hoping that he understood, or should she push him further, perhaps getting him to compare 0.67 with $\frac{2}{3}$, or more provocatively 0.9-recurring with 1? This might bring him up against the notion of approximation, but also, it might be too much for him to cope with at this stage. Where that lesson was concerned she left him without further probing. My field notes were very sketchy here, and this relies much on personal memory.

It was the end of the lesson, and she sent the class away with the words, "I don't want to give you specific instructions. *You* have to find patterns. You *can't* be wrong." Again there was the overt challenge to find and express their own patterns. And this time the words "You *can't* be wrong.", perhaps attempting to remove the inhibiting stigma of many perceptions of mathematics, that it was too easy to be wrong, that maybe it was better to do nothing than to be wrong.

4: THEORETICAL CONSIDERATIONS

What I have written (in 1990) in the three sections above has been a synthesis of the analysis that I did in 1987, with reference to data collected in 1986. This synthesis is inevitably influenced by aspects of my more recent thinking, and in particular my now more developed view of the three categories – Management of Learning, Sensitivity to Students and Mathematical Challenge – which I began to call the *Teaching Triad*, and which has since become a central theoretical construct in this study. However, I have tried to be as faithful as possible to my thinking as it was in 1987.

At that time ML was starting to encompass many of the other coding categories which I had used originally to describe items of significance. I

recognise now that this is not surprising, as the nature of Management of Learning is such that it is easier to identify particular manifestations of it, such as classroom strategies, forms of questioning, organisation etc. in terms of words or phrases, or classroom activity. Both SS and MC are more subtle. They depend on tone of voice, on nuances of inflection, on the development of ideas, the exchange of remarks, on personal characteristics and relationships. Manifestations of these are much less easy to offer in simple coding, and more detailed data seems necessary to validate analyses made. Looking back to my field notes in the Autumn term lessons, I can much more readily justify interpretations related to ML than those related to SS or MC as the latter depend very much on my own memories of details of which I have no other record.

I chose to present the data above from lessons on *Fractions* rather than the other early lessons (see Figure 6.2) for two reasons. I wanted to choose lessons where the work was not overtly investigational (i.e. involving investigations), and *fractions* is not a mathematical topic usually associated with investigative work. Certainly lessons 1, 6 and 7 might be seen to provide more opportunity for investigations. The activities in the KMP lessons were very diverse, and I should have needed a greater volume of explanation of the different contexts. However, most of the characteristics which I have distilled from the fractions lessons could have come from any of lessons 1 to 7. Before going on to the next section of analysis, I shall briefly relate what I have discussed here to my theory chapters in terms of investigative work, mathematical concept formation and classroom practice.

Indicative of an investigative approach were phrases from the teacher such as, “I want you to think what you might do next”, “look for other recurring patterns”, “think of what questions to ask next”. Students were being asked to enter into the world of fractions by looking for patterns and asking their own questions. The activity of recording the decimal representations of all fractions up to $\frac{1}{7}$, i.e. $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}$ and seeking recurring patterns was overtly investigative and, I suggest, designed to foster concept construction. In the case of recurring patterns in *sevenths*, the teacher’s valuing of Nigel’s image of the numbers going round in a circle lent respectability to an individual’s own images and seemed to encourage linking of schema. Martin, exploring different ways of writing $\frac{2}{3}$, i.e. 0.6-recurring, as 0.667, or 0.66667, or 0.666666667, seemed to be operating at the limits of his experience, relating what he was doing here to his experiences with computers, and the teacher had to decide how far to push him towards ideas of approximation. Manifest in this situation

seemed to be the notion of Martin's ZPD (Vygotsky, 1978) and its limits. I suggest that there were *high levels of thinking* (Desforges and Cockburn, 1987) involved for these students. I can say little about their actual construal, and therefore their learning, but I saw students undertaking the teacher's tasks and I saw evidence of their involvement. Contrary to Doyle's (1986) findings, these students were mostly not resisting 'higher level cognitive demands'. The atmosphere of the classroom was conducive to this work. This is not to say that it was never noisy or that there were never disruptions. When these occurred the teacher dealt with them as I indicated earlier (See also item 12 in Insert 6.2 in Appendix 4). However, the ethos of the school was to engender mutual respect and this was overt in Clare's classroom.

ANALYSIS OF RECORDED LESSONS – CAN REFERENCE TO VIDEO AND AUDIO MATERIAL SUPPORT THE TRIAD?

My relationship with Clare developed over the Autumn term, so that I felt more able to be obtrusive in her classroom without fear of getting in the way of what she was trying to do. I realised that she would continue with her teaching despite my presence or movements and that, unlike the teachers at Amberley, she would not try to draw me into her thinking or decision-making during a lesson. I also realised that I was often missing much of each lesson by not being close enough to hear what was said, and certainly by being unable to keep a detailed enough record of it for later study. I asked Clare's permission to audio-record the lessons, and she agreed to wear a recorder and microphone, and possibly to leave another recorder with some selected group. I also moved about the room rather more than I had before. This meant that I collected much more data from subsequent lessons. Some of these lessons were also recorded using a video camera.

In this section I shall choose situations from certain lessons which were recorded on video and/or audio tape, and use transcripts to analyse their significance in terms of the three categories of the Teaching Triad, in an attempt to justify my belief in the power of this triad to enable characterisation of teaching situations.

1: THE PACKAGING LESSON

The first situation to which I shall refer consists of about five minutes from the *packaging* lesson which was recorded on video tape. In the lesson previous to this, which I had not observed, Clare had organised a brainstorming session with the class concerning questions which might be

explored with regard to packaging of various kinds. A set of twenty questions had resulted, and Clare had photocopied a sheet of these for each member of the class.

This was a new project for the class, and students were encouraged to start with a question of their choice. A number of girls sitting at a table together had chosen to work on the questions, 'Which shapes are scaled down versions of other shapes? How can you tell? How can you check?' They had identified three variables, *Volume*, *Surface Area* and *Shape*, which they were trying to relate, and they had decided that they needed to fix one of these variables in order to explore the other two. The one they decided to fix was *shape*, and they decided to make it a *cuboid*.

I shall analyse this situation with reference to the teaching triad. Does the teaching triad help a gaining of insight into the teaching situation and its contribution to the thinking and learning of the students?

It's a cuboid

This involves a group of girls, including Rebecca and Diana, who were working on questions relating to volume and surface area using a large collection of packets from commercially produced products. The teacher, Clare, listened to their conversation for some moments, and then interjected:

- (1) Clare We're saying, volume, surface area and shape, three, sort of variables, *variables*. And *you're* saying, you've fixed the shape – it's a *cuboid*. And I'm going to say to you / hm,⁵

She pauses and looks around

Clare I'll be back in a minute

but she continues talking

Clare *That* is a cuboid.

She picks up a tea packet.

Clare *That* is a cuboid.

She picks up an electric light bulb packet

- (5) Clare and ...

⁵Transcript conventions were adapted from those used in Edwards and Mercer (1987) See Appendix 1 for details.

She goes away – then returns with a metre rule

Clare *This is a cuboid.*

She looks around their faces. Some are grinning

Clare *And you're telling me that those are all the same shape?*
[Everyone grins]

8 Reb *Well, no-o. They've all got six separate sides though.*

Clare *They've all got six sides. But I wouldn't say that that is the same shape as that.*

She compares the metre rule with the bulb box

(10) Reb *No-o*

Clare *Why not?*

Di *Yes you would ...*

There is an inaudible exchange between the girls D and R

Clare *What's different?*

There are some very hard to hear responses here. They include the words size and longer

Clare *Different in size, yes.*

Clare reached out for yet another box, a large cereal packet, which she held alongside the small cereal packet

(15) Clare *Would you say that those two are different shapes?*

Reb *They're similar.*

Clare *What does similar mean?*

Reb *Same shape, different sizes. [They all laugh]*

During the last four exchanges there was hesitancy, a lot of eye contact, giggles, each person looking at others in the group, the teacher seeming to monitor the energy in the group.

Clare *Same shape but different sizes. / That's going round in circles isn't it? [R nods exaggeratedly. Others laugh. Teacher laughs.]. We still don't know what you mean by shape. / What d'you mean by shape?*

She gathers three objects, the two cereal packets and the metre rule. She places the rule alongside the small cereal packet

(20) Clare This and this are different shapes, but they're both cuboids.

She now puts the cereal packets side by side

Clare This and this are the same shape and different sizes. What makes them the *same* shape?

One girl refers to a scaled down version. Another to measuring the sides – to see if they're in the same ratio . Clare picks up their words and emphasises them

(22) Clare Right. So it's about *ratio* and about *scale*.

Data item 6.7: Cuboid transcript (May 1987)

I feel the episode splits into three stages: statements 1–7; statements 8–14; and statements 15–22. I shall take each stage in turn.

I see statements 1–7 as the teacher's challenge. She has listened to the students and made a decision to intervene. From her point of view the problem seemed to be that a cuboid would not do, and she wanted somehow to draw their attention to this. She did it by setting up quite a dramatic little scene in which she asked them to compare three different cuboids. Her departure to get the metre rule, although I think not planned, added to the drama because there was a pause between her pointing to the two boxes and then returning with the rule. Her tone was provocative. The girls' attention was captured. There were half-smiles, almost as if they were asking, 'What is she up to?' When she produced the rule they grinned. It seemed obvious that the rule was not the same shape as either of the boxes. The situation here has similarities with Piaget's rods experiment, (Inhelder and Piaget, 1958) designed to get students to consider relationships between variables. In this the researchers asked only neutral questions, not trying to teach. However, here there is a teaching act to be considered and this might be seen rather in terms of Vygotsky's (1978) ZPD – judging students' potential for making progress and providing the necessary scaffolding.

One interpretation is that the teacher was aggressive. She was denying them the chance to formulate for themselves this notion of 'shape' being too imprecise a variable. She was forcing onto them her perspective, forcing the pace of their thinking, directing them mathematically. Asking

more neutral questions, corresponding to the Piagetian situation, would have left the girls to move forward only as their own thinking allowed.

An alternative interpretation is that the teacher saw the girls' thinking as being fuzzy and not seeming to be making progress. She could see an opportunity to focus their thinking in a way that would lead into some 'useful' mathematics. She had to decide whether to push them in this direction. Having made the decision (for whatever reason), rather than offering an explanation of why cuboids would be too imprecise, she set up a provocative situation and challenged them with an apparent contradiction, capturing their interest and attention (cf the Shell Centre work on *conflict discussion* e.g. Bell and Bassford, 1989) This could be seen as providing scaffolding to enable progress. There was a relaxed and friendly atmosphere, but at the same time a build-up of tension as the contradiction became apparent.

There seemed to be here a manifestation of a high degree of Mathematical Challenge. There was considerable risk involved. The girls might not take up the challenge. It might be inappropriate. They might not be able to cope with it. They might lose their own, perhaps precarious, thinking and possibly their confidence. The teacher in having somehow to salvage the situation, might increase students' dependency on her.

However, with Clare, Mathematical Challenge seemed always to be allied to Sensitivity to Students. This teacher knew these students well. I had much evidence of this. The three girls concerned had demonstrated ability to think well mathematically. She believed that they had high mathematical potential. She also had a very good relationship with them. The risks which she took were allied to this knowledge.

In the next stage in the episode, statements 8–14, there was a lessening of the tension as the girls began to think through what had been offered. It is almost as if they are thinking aloud, rather than participating in discussion. The shapes all have six sides. However, the metre rule and the bulb box are not the *same* shape. How are they different? Well, they *are* the same in some respects. Here the teacher was less intrusive, but her remarks were still focusing. "*What* is different?" There was space for them to think, to internalise the problem. But the teacher was still pushing.

A situation like this depends very greatly on the teacher's sensitivity to students' perceptions, both mathematical and social. In analysing why I felt that this episode was successful with regard to the teacher's objectives

and the students' gain, I put it down to the decision-making which had to take place at various points. Clearly the teacher had to take the initial decision to intervene and to do so as provocatively as she did. However, there was another crucial decision hovering in the middle stage. Were the students able to take up the challenge? Could they make progress? What else should she offer? I see it being in the making of an appropriate decision here that sensitivity to the students is most crucial. The success or otherwise of such episodes is very rarely just chance, but involves a high degree of vital decision-making (Cooney, 1989; Calderhead, 1984).

In the final stage, statements 15–22, the teacher seemed to judge that she could push further. She chose two cereal packets of different size but the same shape, and asked if they were the same. She was rewarded instantly as one girl offered the crucial word, 'similar'. So she pushed harder, "What does similar mean?". The reply is not helpful. They are going round in circles. She diffused the tension by acknowledging this and laughing, and they all laughed with her. However, she persevered, and in the interchanges which followed she was given further appropriate language – *ratio* and *scale*.

She could of course have gone on to ask, 'What does *ratio* mean?' However, she chose to leave it there. Her emphasis on ratio and scale, picking up the girls' own words, was probably sufficient to provide a new starting point. It seemed that the girls had entered into her thinking, as she had initially entered into theirs. She seemed to be convinced that they were involved sufficiently to be able to make progress. Thus the episode had a successful outcome.

My interpretation *successful outcome* lies in the girls producing evidence that their thinking became more focused as a result of the exchange with the teacher. They moved from vague articulations of shape to much more precise ones involving similarity, ratio and scale. With a correspondingly more precise conceptual foundation, they could be more likely to make progress in relating their original variables. Ultimately some assessment could be made of the episode in terms of what the girls did next and where their thinking eventually led. However, judging only this episode, there seemed to be an effective balance between challenge and sensitivity. The teaching situation seemed to be effective in terms of what the girls gained from it, and what the teacher might have hoped to achieve.

So far, I have said nothing of Management of Learning, and in the situation itself, there seems overtly to be little of this. However, the

scenario was only possible because of the way the activity had been set up. The teacher had created a situation in which students could engage in meaningful and potentially productive work. They had a set of questions on which to start. The students had been instrumental in devising these questions, so they were meaningful to them. They could choose whichever interested them most, which increased motivation. They were encouraged to work in groups, to articulate and to share ideas. Thus what the students were engaged in in Situation 1 owed much to the teacher's overall management.

2: THE LINES CROSSING LESSON

Data item 6.8 comes from Lesson 13: *Lines crossing*, which was recorded on video tape. In this Clare joins a girl, Rebecca, who is working alone at Clare's request. Rebecca and Diana, who were two of the girls in Data item 6.7, usually worked together. Clare had decided that they had developed such a good working relationship that it was hard for her to distinguish their individual thinking, and therefore difficult to make individual assessments. She told me also that she was not sure whether they were sharing jointly in the thinking, or whether perhaps Diana was leaning rather too much on Rebecca. So, for the *Lines crossing* project she had asked them to work separately, explaining her reasons to them. This is a manifestation of ML.

Clare had set up the lesson by asking the class to imagine a line, then another line crossing it, and another, and so on. As she invited them to bring in another line she asked them to count the crossings which they could visualise.

When the number of lines became too difficult to imagine, she stopped and asked them to contribute some of their images. Some students had lines parallel to each other, some had lines all crossing at the same point, some had maximised the number of crossings. Clare drew some situations on the board and asked what different numbers of crossings were possible for any given number of lines, for example for three lines. Students contributed particular examples, and Clare asked, "What is the *maximum* number of crossings you can get for three lines?" She invited the class to explore further themselves. This introduction is a manifestation of MC Through invoking students' imagery and asking questions, she challenged them to take on the problem.

When she came to Rebecca, Rebecca had been drawing line patterns and counting crossings.

Lines Crossing

Clare joins Rebecca and sits down with her.

(1) Reb I've decided that the maximum number is six crossings.

Clare For how many lines?

Reb Four.

Clare Why?

(5) Reb Because I've done four of them and I can't get any more than that. And each time I do one line, and I cross it like that, and the next time I make sure I cross both of them, and the next time I make sure I'm crossing three –

Clare Can you just run through that again – you draw one line

Reb Then draw two, – that's crossing the first line –

Clare So, the second line has got to cross the first line, yes –

Reb Cos that makes the first cross. This one must cross because that's the maximum number it can cross. It can't bend round –

(10) Clare So the third line has to cross –

Reb two lines.

Clare Right

Reb And the fourth line, to get the maximum number, has to cross three.

Clare The fourth line's got to cross three, yes

(15) Reb And that's all you can do.

Clare Cos you're doing four lines?

Reb Yes.

Clare OK.

Reb And that makes 1, 2, 3, 4, 5, 6 ... and eleven regions.

- (20) Clare Right. / What're you going to do next?
- Reb I started on five, and doing that, the most so far – I've only done two – is ten crossings because I'm making sure I cross the first line, the second line I'm crossing one, the third line I'm crossing two – all the way up.
- Clare Are you saying then that there is a method for doing this, and you're following a method?
- Reb I'm making sure I cross all of them.
- Clare Yes, one by one. Have you looked at regions for this one?
- (25) Reb Seventeen.
- Clare seventeen regions. / What are you going to do with your results?
- Reb Trying to find the maximum first.
- Clare What're you doing with them when you've found them?
- Reb When I've found all up to seven I'll make a chart.
- (30) Clare OK. What'll you show in your chart?
- Reb The number of rods, or lines, the number of crossings and the number of regions.
- Clare Right. Now you have described to me a method for making sure that you got the maximum number of crossings, do you think that you can explain that method more concisely?
- Reb Not yet.
- (34) Clare Not yet. It's something you might like to bear in mind, OK?

Data item 6.8: Crossings transcript (6.3.87)

When I talked later to Clare about this interaction, she was rather dismissive of it. She said,

I don't know how much use that conversation was to either of us really. It was just a version of 'how're you getting on, all right? OK'. A more long drawn out version. I don't know how much use as a task it was. (Clare, Sept 1987)

She then relented slightly and said that it had probably helped Rebecca in 'organising her thinking', and 'it helped me in getting into the problem'. However, she felt that there had been few decisions to make and that this was why she had not made a stronger input. Contrasting with 'It's a cuboid', she felt that there had been decisions to make *there* to do with pushing the students in a particular direction.

I was interested in the value which she put on the *decisions*. It was almost as if the making of decisions in some way tested her as a teacher. She felt happy with the cuboids interaction, and perhaps this was satisfaction in feeling that the decisions had been the right ones in that case. She frequently agonised over certain decisions which she had taken (Jaworski, 1991), but in this case felt that they had been justified. So, an implication was that if no decisions had to be taken, then the teaching event was less valuable. I cannot agree with this. I saw the event with Rebecca, and other similar events, as being an important part of Clare's work with students.

The first point which I would make is that there is a difference for the teacher between approaching a group of students working together and approaching a single student. In Data item 6.7, 'It's a cuboid', she was able to hover with the group for a time and listen to their discussion. As a result of this, when she eventually intervened, she had worked out what stage they were at, and had decided that intervention on her part was necessary⁶.

In Rebecca's case there was no conversation to listen to. She could look over and see what Rebecca was doing, but little more. I see the first few statements being a genuine attempt on the part of the teacher to enter into the student's thinking. The request at statement 6 seemed genuinely one of enquiry. However, it served to get Rebecca to reiterate her thought process, and perhaps this helped her to refine or reinforce what she knew implicitly. I see the difference in her approach to the two groups as a further manifestation of SS in her trying to enter the students' thinking.

By statement 19, I suspect that the teacher not only had a very clear idea of what Rebecca was doing and thinking, but also felt quite satisfied with it. I say this because my experience of Clare leads me to believe that she would have made more forceful interventions otherwise. Instead she asked one of her frequent questions, "What're you going to do next?"

⁶ In responding to this writing, Clare commented, "This is so obvious, but I never realised it before, and it affects me every day in the classroom".

Rebecca was very articulate, and this, potentially, is another reason why Clare needed to say so little. However, at statement 22, Clare's focusing is apparent. What Rebecca has just said does more than describe a particular case. It sounds like a generic statement of how her process works generally. Clare calls it her 'method'. Rebecca seems not to have identified it yet as a method, but her response indicates that it is as the teacher suspected. She asked if this also extended to regions and Rebecca indicated that it did. I recognised that had I been the teacher I should have been very pleased with this thinking, and so my interpretation of Clare's low-profile comments to Rebecca were in line with this. There was no need for heavy intervention as the student seemed to be doing very well without it. However, both monitoring and assessment were taking place implicitly and the interchange served these purposes very well.

Statements 26 and 28 involve Clare's enquiry of how Rebecca will present what she had found, and again this seems satisfactory. So Clare ends with a challenge, at statement 32. She comes back to the notion of 'method', acknowledging that Rebecca indeed had a method, she asked if that method could be expressed more concisely. I believe that she meant by that, could Rebecca perhaps find some succinct way of expressing what she had found, perhaps some form of symbols, perhaps a formula. Rebecca's response, "Not yet!", seems particularly mature. In few words it seemed to say, 'I know what you mean. I'm not there yet. But I will be'. This indicated to me a degree of confidence with which as teacher I should have been extremely pleased.

It is tempting to say that there was little of Mathematical Challenge in this interaction. Indeed the challenge seemed minimal, but yet it was there. It seemed to say, 'I know you have a method. I expect you to express that method concisely'. That more words were not needed seems to indicate that there was a high level of common understanding of what the teacher meant by this. I had seen many other occasions where precisely this need to express concisely had been part of the teaching/learning agenda. It pointed to Clare's Management of Learning. Having set up her expectations of what students should aim for in mathematical exploration, she had then only to make brief mention of it in order for the students to understand what she was after.

I felt there was also a high degree of sensitivity. There was no need for heavy handedness here. Rebecca was quietly confident. Clare knew her well and was aware of what she was capable. As I shall say over and over again, meeting a student with an appropriate degree of challenge, requires

a corresponding degree of sensitivity. I believe that the keener the sensitivity, the better the challenge in enabling the students to make progress.

I therefore felt that Clare's revised assessment of the interaction was more accurate. The articulation of her 'method' was likely to have helped Rebecca make it more explicit for herself, and therefore put her in a better position to generalise it in some concise way. For Clare, the interchange had allowed her insight into Rebecca's thinking concerning this project. I believe that Clare's assessment of these projects, examples of which I mentioned earlier, was very dependent on such interaction with the students. It was only by being intensely aware of what thinking had taken place, that she was able to judge the written version when it was presented to her. If the written version did not do justice to this thinking, then students would be advised on what ways in the future they could best capitalise on their thinking for achieving a grade which was commensurate with it. One student spontaneously attested to this during an interview (see p 152).

I shall refer to one further situation from the *Lines crossing* lesson. It concerns Clare's interaction with a boy, Jaime. Clare had talked to me extensively of Jaime in the past, and was very excited by what had occurred in this lesson. Jaime came from a family whose language at home was not English, although he seemed to understand English and to communicate with his peers in English. However, he did not do very well in mathematics lessons, seeming rather lethargic, uncaring and not eager to get involved.

In the Lines crossing lesson, Jaime had actually got very involved with the problem and had, according to Clare, done some interesting and valuable work, which, taking into account his usual attitude, was very exciting for her. Unfortunately the interaction had not been recorded, but Clare described it to me afterwards.

Jaime

Very early on, and you haven't got this on tape, I had a look at what he was doing. He called me over after working for about five minutes and I thought that perhaps he hadn't understood what he was supposed to be doing.

And when I went over to see what he was doing, he knew *exactly* what he was supposed to be doing and he had practically gone straight to the heart of the problem. He was saying, I think, "Look I've got these (lines) and I know what is going to happen. This one is going to cross this many, and that one is going to cross that many." And *it was wonderful*.

Then later on, *not much later on*, he actually *got up out of his seat* and came across to me, which is again rather unheard of for him, and showed me these beautiful drawings which he had done, in which he had drawn the lines in a way such that they made a curve.

So that his first two lines had been at right angles to each other, and his third line had been – if you imagine those as axes [Clare moved her hands to indicate what she meant] his third line had had a very steep negative gradient, so that it cut the vertical axis very high up, and the horizontal axis a little way along.

So his second line had just been a little less steep, and his third line a bit less steep, and so they had made a curve as well. And as well as getting the numbers of the crossing *quite easily*, he also discovered that these made a curve. ...

After that lesson, I mentioned to him how very exciting I found that and *he was obviously very pleased as well*, because he was *smiling*, and he was *working*, and *he really felt he was getting somewhere*.

Data item 6.9: Jaime (6.3.87)

This situation has a number of features which were typical of Clare's approach to working with students. The first is her intense interest in and caring for students. The above quotations comprise only a few excerpts from what she said of Jaime. She talked extensively of what she knew of him as a person, as a student in her lessons, and of his mathematical achievement in this particular case. Jaime was just one of many students of whom she spoke in this way. Secondly, I feel that her words above are revealing of her own approach to mathematics and the mathematical thinking of her students. She believed that mathematics was exciting, and her enthusiasm came across in the way she spoke. Perhaps for Jaime, seeing her excitement in what he had done raised his self esteem and motivated him to tackle more challenging work. In contrast with Rebecca earlier, Jaime's thinking was in quite another direction. Rebecca had focused on the numbers of lines and regions, whereas Jaime's focus was on the curve which seemed to arise from the intersections. It was typical of Clare's operation that both of these directions were respected and supported. The creating of a task open enough for students to choose and tackle diverse directions was manifested in these two situations. In the

case of Jaime, I see a strong manifestation of Clare's sensitivity, with a degree of challenge as Jaime's own ideas were nurtured and encouraged.

STUDENTS' AND TEACHER'S VIEWS

I shall end my characterisation of Clare's teaching with some references to views expressed by her students and by herself, which support the previous analysis. I solicited students' views in three ways, by talking to individuals informally in lessons, by interviewing pairs of students semi-formally towards the end of the Summer term, and by a questionnaire which was given to all of Clare's classes at the end of the Summer term.

Informal conversations between Clare's students and myself were rare in lessons. I had deliberately set out to be unintrusive in her lessons as a contrast to Amberley, and so I did not seek the involvement of student conversations. However, there were just a few occasions when this occurred. On one, a boy, Kevin, drew me into conversation. It was during one of the KMP lessons. Kevin was sitting close to me, and at one point he made some remark to me about the card on which he was working. I asked him how KMP compared with work in the project lessons, recalling similar conversations with pupils at Amberley. His response was that in a KMP lesson you could work at your own pace, there was not as much competition, and you didn't need to worry about getting ahead as in the case of projects. On the other hand, projects were good too as they encouraged your own ideas and invention, and made you feel good about your achievement. I was impressed by his articulate and well reasoned response. Later in the lesson, he spoke to me again of his own accord to tell me about the answers on the back of the card. He said that some people cheated, but it was not worth it, because of the tests. However, sometimes the answers would give you a clue as to what to do. Also, writing down just the answer was not much use, since if you looked back, later on, it may not mean anything. You needed to write some explanation. I wondered, on reflection, how much of this was due to his own thinking and in how far he was reflecting messages which he had picked up from the teacher. When I mentioned this to Clare later, as well as giving a lengthy history of Kevin himself, she said that she was pleased with his response because she was in accord with most of these views. This was supported by remarks she made, after one of the KMP lessons, about strategies which she encouraged, "explicitly, and implicitly, depending on who they are", when students were stuck:

I find people usually get stuck about two stages further on from when they really stopped understanding, so to go back is one [strategy]. ... I have talked to all of them about using the answers, and how it is not cheating to look at the answer if you're stuck. // I talk to them about talking to each other about maths // and I talk to them also about trying to explain things to someone else, because that is certainly something that I have found helped me with my maths. (Clare 10.2.87)

This fostering of effective learning strategies supports my perception of Clare's Management of Learning, and it is echoed by the perceptions expressed by Kevin, above.

The semi-formal interviews involved me in sitting with a pair of students in a teacher's office and recording their responses to a number of questions which I asked. I chose the pairs in negotiation with Clare, and Clare excused them from her lesson for 15 minutes to talk with me. I asked them about particular incidents that I had remembered from various lessons, about their perceptions of lessons and of Clare's teaching. I shall quote from some of their remarks where I feel it relates to aspects of Clare's teaching and her objectives for their learning.

One boy, Kim, with whom Clare had remonstrated as I quoted earlier (see p 131), compared work in Clare's KMP lessons with that which he had experienced two years ago, in the 'Foundation' year.

when you've finished a card or something, Clare asks you questions on it ... but ... in the Foundation year, if you had done a card, he just kind of marked it off, and didn't ask you any questions on it, you know. But Clare doesn't trust anyone – I suppose it's pretty good because like she kind of, she asks you questions to make sure you understand it, and if you haven't she makes you do it again. (Kim 7.6.87)

I asked how he felt about that, and he replied:

Well you get annoyed at times because she makes it hard for you, but in the end, you know, / I'm glad she does, I suppose. (Kim, 7.6.87)

Clare herself had said after one of the KMP lessons:

I hope I have said the same thing to them so many times they now do things without having to come and ask me first. For instance, if they get to the end of a work card they now know that I am going to ask them questions about that card, or I am going to make suggestions about things they might have thought about. Or I might say, 'did you do this that and the other?' And so I think more of them do that without coming up to me first. Now that saves a visit to the teacher, so that's training isn't it? (Clare 10.2.87)

Kim went on to make the remark to which I referred earlier (see p 125) about Clare's 'funny sort of strictness'. It may have been this 'training' of Clare's to which he was referring. He said a little later:

She seems to be pushing you along, you know, because I think she sees your capabilities more than you do. (Kim, 7.6.87)

I asked one of the girls how she felt maths lessons compared with lessons in other subjects, and her response accorded with what Kim had said.

I think in maths, especially with Clare, people do more work in the class as a whole. She is much of a stricter teacher, and she really pushes you forward, to get your goal, to the height of your ability in maths. I think a lot of people are doing quite well in maths because she is always there to give you that extra push and makes you go further. (Vicki, 7.6.87)

I put it to Clare that the students saw her as being strict, and sought her response which was:

That was wonderful really because I don't mind being thought of as strict if they understand why I'm like that. But it was two things wasn't it ... it wasn't just being strict in order to have quietness, but also realising that it was for them as well. (Clare, 12.6.87)

I asked another girl how she felt about the assessment of projects. She said that she found it helpful sometimes, and off-putting some times:

when you know you've really tried hard on something, and you keep getting stuck. Like, I don't mind doing a project, it's writing it up afterwards, because I sort of get stuck, and when you get a really low grade, when you've really tried your best on something, it's a bit off-putting. (Tandy, 7.6.87)

I asked what they would have to do to get a better grade another time, and she replied,

After each project Clare will tell us some of the things we missed / to show us some of the things we should have. It's really helpful. (Tandy, 7.6.87)

When I asked this girl if there was anything she had not liked in Maths recently, she said:

Oh yes, I don't like it when Clare sort of is doing something on the board and then she sort of says 'Tandy do this', say the answer, and you don't really know. I don't like that. (Tandy, 7.6.87)

And her partner agreed strongly with this:

No, I don't like that. It sort of makes me really nervous, when she says, Ann what's the answer, and you don't know. It makes you feel really bad. (Ann, 7.6.87)

I found these remarks quite salutary, that despite Clare's sensitivity there were still occasions where students felt threatened. Such negative feelings had not been obvious to me in the lessons. However, research suggests that this often happens (e.g. Hoyles, 1982). Clare recently commented on Ann's words, offering alternative construals:

These remarks are quite a surprise, because I thought I hardly ever asked for answers, only for ideas, suggestions, contributions. Two things might be happening: (i) they are so conditioned to expect to give 'answers' that they haven't recognised a different situation of open contribution; (ii) I ask for answers more than I think I do. (Clare, March, 1991)

The remarks above relate to, and in the main support, characteristics which I have claimed of Clare's teaching. Apart from the negative response immediately above, there were no remarks relating to Clare which painted a different picture. I felt that the students were quite frank with me within the obvious constraints.

The questionnaire data was rather different. Firstly it was collected from *all* her classes, not just the 4th year class. I had wanted to leave scope for students to express themselves freely while suggesting areas of consideration. I gave my set of questions to Clare, and she read them out to the students guaranteeing the anonymity of their responses. (No names were given. Students put their papers into an envelope which Clare sealed in front of them.) The questions were:

- 1 What use is KMP work in helping you learn mathematics?
- 2 What use is project work in helping you learn mathematics?
- 3 What mathematics have you learned most recently?
- 4 What have you enjoyed most/least in working on mathematics recently? Can you say why?
- 5 What mathematics do you find easy/hard?
- 6 What is the most useful help your maths teacher can/does give you?
- 7 What do you think learning maths involves? How do you do it?

In analysing the responses from Clare's students, I looked particularly for spontaneous statements regarding views of mathematics and of teaching which seemed to relate to characteristics which I had identified of Clare's

teaching, and I quote a few responses below. The number of the question to which response is given are appended in brackets in each case.

The following five comments seem implicitly to support Clare's teaching:

- 1 Project work seems to me to be more helpful in learning maths because you are doing an individual piece of work which is yours. It also helps you to work and solve problems on your own. (2)
- 2 Project work – not accepting knowledge, searching for it. (2)
- 3 The maths teacher can and does help us find our own answers. She does it so that we find the answer while she just puts us on the right track. (6)
- 4 I think learning mathematics involves trying to work out a problem on your own and understanding what a teacher says to you. I learn by trying to sort out things on my own and then asking if I really need help. (7)
- 5 I learn mathematics easily when giving other people help and working in groups. It involves being able to talk and share ideas rather than keeping them to yourself. (7)

I feel that comment 6, although supportive of the teacher seems to suggest a dependency which Clare might not wish to encourage.

- 6 The most useful help a maths teacher can give me is to show and make me understand something. My maths teacher does give me a lot of help of this kind.(6)

Comment 7, on the other hand, seems almost to be a reverse of 6, and it could be being critical of the teacher.

- 7 I usually get the help I need, but I have to ask or explain before help is given. (6)

Comments 8 and 9 both presented interesting perspectives of mathematics and of teaching, and could be construed as having developed from Clare's approaches.

- 8 Re-explaining of problems that are not clear in the text – this is sometimes given, but often I get told to work it out for myself. I suppose that being told to work it out for myself is the most useful help given, but I think a little more guidance would be helpful at times. (6)
- 9 I feel that learning maths involves showing diagrams, keeping to one part of maths at a time, using 3-D models and demonstrations, being willing to learn, – the last being the most important. (7)

Comment 10, on the other hand, reflects a view which many other comments support, that being told what to learn is easier to cope with than having freedom to go in directions that are unclear to you.

10 I prefer to have clear-cut formulas that I can learn. (5)

Although having responses expressed in students' own words in this way is more enlightening of their views than if they had ticked preselected statements, I nevertheless felt much less happy in making interpretations from comments here than on those made in the interviews. It could be said that the anonymity allows students to be more honest, but it also prevents any degree of clarification of complex or ambiguous statements, which are often the more interesting responses. I felt that these questionnaires had not added a great deal to my knowledge of students' views. I regretted not having asked more direct questions of students throughout the observation, rather than trying to gather this information retrospectively.

In later reflection (1990), I realise that seeking students' views is very much more complex than I had started to realise at this stage, and still cannot claim to know the best ways of monitoring their responses, explicit or implicit, to the teaching they experience.

CONCLUDING MY CHARACTERISATION OF CLARE'S TEACHING

In my earlier analysis, I attempted to show how the teaching triad arose, and in the later analysis, I attempted to support its applicability. Where Clare's teaching was concerned I saw three strongly linked categories, elements or domains (Figure 6.4, left). My image of them was very much that of a picture of interlocking circles (right):

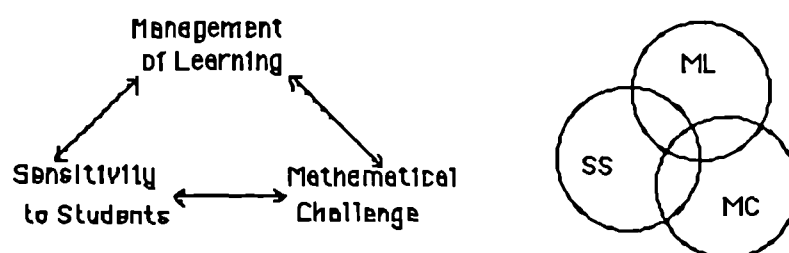


Figure 6.4: The Teaching Triad

Management of Learning encompasses a set of teaching strategies and beliefs about teaching which influence the prevailing classroom atmosphere and the way in which lessons are conducted. Sensitivity to

Students involves the teacher-student relationship and the teacher's knowledge of individual students and influences the way in which the teacher interacts with and challenges students. Mathematical Challenge involves the teacher's own epistemological standpoint and the way in which she offers mathematics to her pupils depending on their individual needs and levels of progress. The three are closely interrelated, yet individual in identity, and have potential to describe the complex classroom environment. Although they are interrelated in that there were situations in which aspects of all three were present, I saw them as being mainly distinct. Where Clare was concerned, I felt it was usually possible to describe what I saw to be significant in terms of at least one of the categories, and I feel that the three categories are valuable for characterising her teaching. It was a disappointment to me that when I discussed this initially with Clare herself, she was not very interested in their descriptive power, not seeing the need to categorise, or indeed wanting to do so. However, in responding to the above writing, a considerable time after her initial response to the triad, Clare presented a very different view which I shall include in my final remarks below. I shall talk, in Chapter 7, of Ben's response to the triad which provided me with greater insights and strengthened my belief in its power to describe.

The Teaching Triad is an example of theory arising from data. I looked at Clare's teaching as an example of an investigative approach from my own constructivist perspective. However, it is not obvious that the triad is a consequence of this perspective and it remains to link the triad to it.

My inclusion of Clare's own remarks about her teaching objectives and general philosophy of teaching and learning has been limited to those directly related to situations to which I have made reference. These are far from being a representative sample of such remarks. Two most important characteristics of Clare were the degree of independent reflection in which I saw her engage, and her correspondingly well-developed philosophies linking theory with practice. I feel that, during the time that I worked with Clare, the opportunity to articulate her thoughts contributed to a higher degree of awareness of her own philosophy. In the course of our work, she wrote some reflective remarks for me about her experience of it, and I quote the following paragraph.

I found that I had to dredge up ideas from my subconscious to justify some of what I did, and discovered that much of my practices result from ancient decisions and intentional changes which have become habits through repeated application. I could still justify many of these habits on ideological grounds and would make the same decisions again, but the process of trawling my memory and asking 'Why do I let X and Y sit together?' and 'Why do I feel awkward if only boys answer my questions?' is a valuable one and I will initiate it for myself from time to time. Two thoughts struck me at about this time: How on earth can teachers be expected to function correctly on so many different levels at the same time? (No wonder I'm always so tired!) and what is going to happen when Barbara hits on something I can no longer justify? (Clare, April 1987)

I shall discuss this philosophy further when I refer to 'the reflective teacher' in Chapter 8. I provide further evidence of Clare's philosophy, in particular her *Sensitivity to Students*, at Appendix 4, since it takes the form of fairly lengthy passages which would have unbalanced this text. Two other pieces of writing, (Jaworski, 1988 and 1991) provide evidence of Clare's operation with respect to the teaching triad, the latter focusing on the *Knots* lessons which I have not mentioned here.

During my work with Clare there was little discussion about an *investigative approach* per se, since Clare rarely used the words investigation, or investigative, and I did not try to impose my vocabulary. However, in terms discussed in Chapter 1, '*Lines Crossing*' might be called an *investigation*, and pupils here were encouraged to develop mathematical processes and strategies. The work on *packaging* was directly related to particular mathematical concepts e.g. area and scale. It was nevertheless investigative in style, requiring students to ask and explore their own questions. Both of these lessons were introduced through Clare's direct invoking of students' imagery, asking them to imagine situations which she described (see Appendix 4 for more detail of this.).

I made no explicit effort at the time to interpret what I was seeing in constructivist terms. At this time constructivist ideas were still very new to me, and I was still trying to sort out in my own mind just how they might relate to the classroom. It was not until I became clearer about what I understood by constructivism that I started to re-interpret what I saw in these lessons. In Jaworski (1991), I reworked some of my writings on Clare in constructivist terms with the overt intention of relating significant events from Clare's classroom to a constructivist philosophy.

In my analysis of recorded lessons, I have provided further examples of high-level thinking processes. In *'It's a cuboid'*, the girls were grappling with ideas of ratio and scale. They were not being asked to memorise facts, or to work from standardised exercises. In Edwards and Mercer's (1987) terms, the teacher's 'cues' might be seen to influence the girls' constructions, with consequent ritualised knowledge.

I have no doubt that the teacher's intervention, which I might describe as 'strong' did influence construction – indeed can it ever not? It seems reasonable to suggest that links were being made or challenged and existing schemas modified. It seems crucial that the girls entered into the thinking which I believe is evidenced in the excerpt.

In *'Lines Crossing'*, Rebecca was tackling the problem systematically with evidence of original thought. Here, the intervention seemed much milder. However, there was emphasis on having a method and explaining it concisely. Although, Rebecca could have ignored this, it had potential to influence her own future high level cognitive operation.

The students' own comments provided evidence of Clare's higher level cognitive demands. In particular Kim ruefully acknowledged their value, as did the student who made comment 8 from the questionnaire. Clare had agonised over 'prodding and directing' (Jaworski, 1991) and this spoke directly to Edwards and Mercer's (1987) 'teacher's dilemma' which is elaborated further in the next chapter.

Students recognised Clare's propensity to push them, as in "she really pushes you forward, to get your goal, to the height of your ability in maths" and "she is always there to give you that extra push and makes you go further", and this speaks to her 'scaffolding' to promote movement across the ZPD (see p 45). However, her sensitivity came through to me continually. When Kim returned to the classroom after his interview with me, he asked Clare, "How does she know so much about us?" I do not know to what he was referring, as I asked very open questions relating to lessons which I had seen. However, I felt that I did know very much about some of the students, Kim included, because Clare had talked about their work and progress extensively.

That she was very aware of gender issues in the classroom was evidenced by an episode which I described in Jaworski (1991), and to which she referred obliquely in the extract from her writing which I quoted above (p 157), noticing how only boys responded to a particular question she

asked, overtly asking herself why this was the case, and trying to learn from the sort of question asked.

Also, in response to listening to my audio tape of the episode, she wrote:

Do I discriminate negatively for some students while positively for others? Bright boys are never given their head, for example.
(Clare, Jan 1987)

I am convinced that the higher level cognitive demands were received well by students *because* of her sensitivity of operation, within a school which valued this sort of approach. However, her overt management of the learning situation engendered an ethos in the classroom, a framework for operation, in which expectations were understood. I believe that without these levels of SS and ML the cognitive demands could not have been met with the degree of success which I postulate.

I end this discussion of Clare's teaching with a quotation from her remarks written in response to reading this text. Again she offers alternative construals, this time of her own motives, emphasising her operation as a reflective practitioner (Schön, 1983):

I think your analysis is a good reflection of my motives at any time. I remember my very clear resistance and discomfort at the idea of the triad – a feeling that if I got involved in analysing what I did, the whole edifice might fall apart OR my responses would be less spontaneous, the responses to students might be more stereotyped, the repertoire of techniques which I carry near my subconscious might be less accessible etc. I also felt that what I do was working, so don't tip up the apple cart. It's partly to do with stress, and that it was your agenda, not mine. (Clare, March, 1991)

She continued to speak of her work with student-teachers in her current post as Head of Mathematics, and suggested that she could now identify elements of the triad in her work with the students, helping them to develop as teachers. I felt that this was a retrospective validating of my characterisation in terms of the teaching triad. It seemed as if the ideas presented here resonated with her wider confidence and experience, whereas they had been somewhat threatening before.

The study of Mike's teaching

INTRODUCTION

Analysis of data from Mike's lessons initially took the form of recording significant episodes without the detailed seeking for descriptors that formed the essence of the early Clare analysis. A number of issues had arisen from Mike's teaching, and I recorded these in some detail.

After analysis of data from Ben's teaching, in Phase 3, where the power of the teaching triad to describe the teaching situation was reinforced, I decided to go back and re-analyse the Mike data explicitly from the perspective of the teaching triad. Ben's image of the teaching triad had been of SS and MC firmly within the 'umbrella' of ML, as I shall discuss in Chapter 7, and I found it hard to avoid this perspective. However, for my analysis here, I do no more than consider ML first. In the conclusion to my reporting on Mike I shall comment on the relative status of the three elements as I have seen them here.

Data collection from Mike's lessons included that from secondary observation done by Sheila Hirst, and I offer her comments where relevant.

LESSONS FROM WHICH DATA WERE COLLECTED

Autumn Term 1986	Spring Term 1987	Summer Term 1987
1 Strange Billiard Table 2 Billiard presentations 3 KMP	4 KMP 5 Pythagoras 1 6 Pythagoras 2 7 Pythagoras 3 8 Pythagoras Posters 9 KMP 10 KMP	11 Racetrack 12 Circles 13 Body maths

Figure 6.5: Lessons observed with Mike

MANAGEMENT OF LEARNING

Mike's Management of Learning seemed quite overt to me in many respects. In my initial analysis I used the word 'control' to described certain aspects of his lessons. In Data item 6.10, which includes excerpts from Mike's first lesson on *Billiards*, I offer manifestations of what I saw as Mike's 'control' in the learning situation..

Introduction to Billiards

The class were given a sheet on paper on which 'The strange billiard table' was introduced (see Appendix 1). Mike asked them to read the sheet, and then said

- (1) "Run through, in your mind, what happens – silently – *don't* put hands up yet"

When the class had had some thinking time, Mike asked,

- (2) "Everyone got something?"

and, when there were nods asked for their contributions. After a number of contributions had been made, he asked,

- (3) "Anyone going to say anything different?"

After initial discussion of what the strange billiard table was about and what they might explore he set a task:

- (4) "In groups, decide on a different thing to try, and ask, 'What happens?'"

Then he said,

- (5) "While you're doing it ... – *what am I going to ask you to do ?*"

There was a pause between these words. He started giving an instruction, seemed to think better of it, and instead asked the class what instruction he had been about to give. One response from the class was, "Keep quiet", which he acknowledged with a nod, but other hands were up and he took another response which was, "Ask questions". His reply was "YES!" Other hands went down. It seemed to me that others had been about to offer this response too.

While the students were working in groups, Mike circulated, talking with students and with groups. After some discussion with one group, he left them saying,

- (6) "Can I just give you two minutes – then come back and talk about it?"

Data item 6.10: Extracts from beginning of 'Strange Billiard Table' lesson (7.11.86)

In (1) Mike asked students to consider the sheet before going further – to enter mentally into what was contained. The instruction seemed to carry more than just *what* they should do. It seemed to emphasise the mental process, "in your mind", and doing it yourself, "silently", "Don't put hands up yet". In this it seemed to convey a philosophy for working, to

emphasise the thinking process. With the question, “Everyone got something?” (2), he not only ascertained that the class was ready, he also indicated that each person was supposed to have achieved something during the thinking time. Now the hands went up and he invited students to contribute their ideas, after which he checked again, (3), “Anyone going to say anything different?”. This offered opportunity for further contribution, but acknowledged that there might be people who had comments similar to what had been offered and thus who had done their thinking but had nothing else to add. I saw it *respecting* the students’ involvement – perhaps implicitly saying, ‘I know you *all* had something to contribute, but you may feel that someone else has said adequately what you would have said, and so are not attempting to repeat that.’ It may also have signalled his own expectations of students by saying, ‘If you did *not* have any thoughts to contribute, I won’t embarrass you by asking you directly, but it’s worth realising that I hoped you *would* have something.’

In all of Mike’s project work lessons which I saw, pupils worked in groups, mostly of their own choosing, although in KMP lessons they often worked singly. It was a regular feature of project lessons that, after introducing an activity to the whole class, Mike then asked them to work in their groups on some aspect of the activity. In this case the paper had described a scenario. As a result of student contributions and Mike’s comments on them, possible questions to explore had been suggested. He then set them a task (4) – told them what he wanted them to *do* – in groups, to try different examples of what they had found on the paper. He then started to say, (5), “While you are doing it (*I want you to ...*)”. The words in brackets were never uttered. Instead he asked them, “What am I going to ask you to do?” This seemed to be blatantly, “Guess what’s in my mind”, but it appeared that most of the class knew the answer – “(*You’re going to ask us to*) Ask questions!” As I hadn’t known what he wanted them to do, I was very struck by this. A part of his classroom rubric was that the students should ask their own questions. He acknowledged later that he was always asking them to ask questions, hence they knew that this is what he expected of them, and knew what he wanted without his having to spell it out. When I subsequently offered him this text to read, for respondent validation, he further said, “I believe I did this deliberately to stress the ‘you *can* get into my head, and *do*’ I had not had them long, remember”.

What I have described above seemed to be an advanced form of ML, which I might call ‘cued strategy’. It involved recognition by the teacher of a valued aspect of working mathematically, *asking their own questions*,

communication of this to the students, recognition of it by the students, its becoming a part of their way of working, and recognition by the students of it being something which the teacher expected of them and which they knew they should do without it being spelled out each time. Perhaps the ultimate stage would be when the teacher saw no need even to refer to it, because he could be sure that it would happen as a natural part of the class's working.

Clare's 'hands down think' was another example of the phenomenon, *cued strategy*. When she uttered these words herself, students readily went into *hands-down-think* mode. I could have envisaged her asking, "what do I want you to do", with the response, "hands down think". In Chapter 7, I point to Ben's "what question am I going to ask?", with response, "*is there a pattern?*" on cue. I wonder which of the ways of working which I have observed to be already established might have been achieved through cued strategy. This is a recent question on my part and so I have not explored it with the teachers.

I recognise here a significant part of my research process. I speak of a phenomenon to which I have now given a name, i.e. cued strategy. The naming of the phenomenon abstracts it and makes it available for further discussion or research. This occurred with the naming of the essential elements of Clare's practice, which led to the teaching triad. It occurred with Mike in his identification of 'cognitive density' to which I shall subsequently refer. It seems an important stage in the identification of significance and seeking for generality. Once a phenomenon has been abstracted in this way it is possible to seek out further manifestations of it and explore its more general significance. This research process might similarly be valuably named, e.g. as 'abstraction by naming'. It relates closely to the 'discipline of noticing' (Mason 1984), and the view of teacher development which I present in Chapter 8.'

Cued strategy might be seen as one element of Mike's *control*. Control is an emotive word, often used negatively to suggest that a teacher is not giving students any freedom to develop their own thoughts, but channelling their thoughts in very particular ways. I use the word deliberately of Mike because I saw him trying heavily to influence the way his students thought, while at the same time leaving the 'content' of such thoughts (e.g.. what aspects of 'billiards' they tackled) up to them. (He endorsed this strongly in responding to this writing.) In (6), "Can I just give you two minutes – then come back and talk about it?", he seemed to ask a question, but its effect was more like an instruction. It signalled to

the group that they had two minutes in which to think about something, and he would then expect them to be able to talk to him about it. At the same time it gave them some space for this thinking, without the pressure of his continued presence. I wrote in my field notes at the time, "Nice movement between groups. Mike is controlling whilst giving time and encouraging thinking".

The emphasis on *thinking* was prevalent in all Mike's lessons that I saw. This often involved the use of some technique to get students mentally involved in, and creating their own images of a situation. In Data item 6.10, 'Introduction to billiards', they had to envisage the scenario described. At other times Mike attempted deliberately to invoke their mental imagery by saying, "Imagine ..." and following this with a description of a situation. We discussed at length his objectives in this, and one of his remarks was,

"Over a few years of teaching I've become more aware of situations where I have a particular picture in my mind, and the students might have one in theirs, and very often they're not the same one."
(Mike, May 1987)

In this, I suggest, he recognised implicitly that *coming to know* is related to experience, so that students' perceptions are likely to be different from his own. He put a lot of effort into encouraging the class to create mental images and to share these images with others to make perceptions more public, differences in perception respectable, and to gain more understanding himself of their perceptions. I saw this encouraging in students a value for their own thoughts as well as for the respectability of differing thoughts. This contrasts strongly with a view of mathematics as rigid with particular rights and wrongs with little scope for negotiation, and it seemed an important characteristic of working consistently with a constructivist philosophy⁷.

Mike's Management of Learning included his management of the lessons and the activity in the classroom. He typically broke up lessons into periods of differing action and varying energy, which contributed to keeping students interested, motivated and on task, as in Data item 6.11.

⁷ Mike's respondent remark to this sentence was as follows: "Oh, so I was a constructivist before I knew what one was! Does that mean that I constructed constructivism?" I point this out to emphasise that remarks on constructivism are *my* interpretations of what I observe. None of the teachers had claimed to be constructivist. However, if Mike comes to believe that he is a constructivist, this must be his own construction!

Continuing with Billiards

Towards the end of the first lesson on Billiards, Mike said to the class,

(1) “Right! Can you stop what you’re doing.

(Pause for students to quieten and settle)

Can someone give us some reports on what you’re doing? Not every group, there won’t be time.

About four students responded with descriptions of their activity, on which Mike made brief comments, then he asked,

(2) “I want you to try asking some questions now – let’s have your attention while we get some questions on the board”

He then asked them to contribute questions,

“... things that occurred to you while *doing* and while *listening*.”

At the end of the lesson he asked students to think about what they had done during the lesson, and then said,

(3) “Don’t forget that you’ve got your red books to write things down in”

Data item 6.11: Extract from ‘Strange Billiard Table’ lesson (7.11.86)

There are examples here of his managing the *action* in the lessons – asking them to stop, in (1), pausing while this happened, and requiring their attention, in (2) (SH wrote ‘Waits for quiet, all attentive – great control!’). In many of the activities, there was some time in the lesson spent on ‘reporting back’, and in (1) he asked for reports from some of the groups. It was a time for feedback and for sharing of ideas and methods. The reporting back enabled groups to hear about the questions which others had tackled and perhaps to gain a broader perspective of the task than just their own thinking allowed. It also enabled Mike to pull the class back together, pointing out the similarities and differences in what they had done. In this case, the class had been invited to explore the billiard table and think of questions to ask. Groups first reported on what they had done, and Mike then asked for some of the questions which had resulted.

Two types of activity seemed to be valued – you can learn *while you are doing things yourself*, or *while you are listening to the reports from others*.

Implicit in this seemed to be the importance of listening carefully to what others had to say. Evidence that they did indeed listen was provided when, during a number of 'reporting back' sessions, I observed that the group who were reporting were questioned by other students about what they had done and why. Most students seemed to be actively involved in reporting, listening or questioning without embarrassment at contributing, and it seemed to be a productive use of time. Teacher's remarks valued what had been offered and invited comments on it, and other students took it seriously by asking questions which gave evidence of relating to their own thinking. It seemed a very *productive* activity, although I realise that there are occasions where reporting back can be unproductive (e.g. Pimm, in press).

After the lesson I queried Mike's remark about 'red books' (3). He explained that the red books were 'thinking books'. He encouraged the students to write down their thoughts and ideas about a piece of work so that they would have a record of them for future occasions. There were subsequent lessons where he began a lesson with an instruction to 'recall in silence what we were thinking about last lesson', and he indicated that the red books might help them in this. Here I saw a deliberate device to value thinking, and to encourage reflection on their work. Students were often asked to *recall* at the beginning of a lesson, to *talk in pairs* as preparation for some written task, to *report back* as a result of some group working, to *think – no hands* before offering answers to questions, and to *come up with their own questions* related to some activity. In most of the situations which I offer to manifest other aspects of Mike's teaching, there are likely to be further instances embodying this level of control, indicating what he expected from students.

I have to question my own valuing of this control over the way students worked in his classroom, when I should not value such control over mathematical content of a lesson. For example in the teaching of *ratio*, a teacher can tell students exactly what she want them to know about ratio, they can even write it down and memorise it, but what will they *actually* know about ratio? I feel that this control limits by creating instrumental rather than relational understanding (see p 39). Mike's control served to create an environment in which mathematical thinking could be fostered. In what ways did it constrain or inhibit?

SENSITIVITY TO STUDENTS

I can point to manifestations of Mike's *Sensitivity to Students* in the episodes which I described in the last section. For instance, in Data item 6.10, 'Introduction to billiards', at (2) and (3), he invited responses from students regarding what sense they had made of the 'Billiards' sheet. After some remarks had been offered, he encouraged further responses, yet gave students space not to offer a response, for whatever reason, if they did not wish to. While his way of doing this emphasised how much he valued their thinking and responses, it yet seemed not embarrass those who had nothing to offer or who did not wish to contribute. I suggest that such an approach encourages the trust of the students, and diminishes the threat of feeling foolish because your contribution might be regarded as silly, or wrong. It thus shows sensitivity to students' feelings and emotions, and their respect for their own thinking. It also assumes that students will take seriously the nature of the activity, and not abuse the teacher's trust in allowing them to choose whether to think or to offer contributions. This assumption in itself encourages students to take responsibility for their learning. These levels of trust are not automatic; they have to be nurtured. Where this trust is not present, the teaching situation becomes very different. This was the case with Simon in Phase 3, and I shall address this in Chapter 7. I shall say more about trust in Mike's classroom later in this chapter.

The next three data items include extracts from the transcript of Mike's first Pythagoras lesson (see Figure 6.5). Each involves students working on one of a pair of tasks. It is in the nature of Mike's intervention that I suggest his sensitivity is manifested.

In 'But what do you do?', Data item 6.12, the girls were working on the 'Square sums' task. They had been given no information beyond a piece of paper on which was written the following:

Square Sums

$$1^2 + 2^2 = 5$$

What other numbers can be made by
adding square numbers together?

Investigate

But what do you do?

Sara called on Mike to ask him a question about the square sums task, and it appeared that she and her partner Emma needed clarification as to what they were supposed to do with it.

- (1) Sara On this, do you just have to add up square numbers?
Loads and loads of square numbers?

Mike Hm, / Well, what does it *tell* you to do?

Emm It says (Reads) What other numbers can be made by adding square numbers? Course it's adding – it says *adding* square numbers. So, is that what you do?

Sara You just add up hundreds and hundreds of square numbers?

- (5) Mike Well, you see that last word ..

Sara Investigate ..

Mike What does that *mean* to you?

Emm Look up (inaudible)

Mike all right –

- (10) Sara Yeah, but, check up what ? Check up on adding square numbers? Check up what?

Mike Well, to me, when I try something like that, when I wrote that down, it's er, it made me ask the question, it made me ask this question – what other numbers can I make? Now that's just the first question which came to me when I was writing it. So I think I just played around with some numbers to see what other numbers I could make up. Perhaps another question would be – can I make *every* number up? I can make 5 – can I make 7, by adding these square numbers together? Can I make 56? Then all sorts of questions start coming out. What numbers *can't* I make up. Is there something special about these three numbers, 1, 2, and 5? And just investigate, to me – means just do some – just play around – play around with some numbers. And if anything comes to you – any ideas you have, any thoughts you have, write them down and try and work on them.

S1 Right.

- (13) Mike OK?

Mike had set the task, so it could be presumed that he knew what he wanted students to do. So why did he not *tell* them what to do? Mike himself in later reflecting on this, wrote,

The first few lines are interesting – I believe (now) that neither they nor I actually believed that I was asking them to add up lots of numbers. So, in a way I felt they were not clear about an interpretation of “investigate”. I think they said, “Look it up in the dictionary” (statement 8) (Some help that was!) – which is where “check up” could have come from. For me the important word here is the final “what” in Sara’s fourth intervention (Statement 10). I feel it was asking something like, “what are the rules of this investigation game?”. That is, “what do I investigate?” Hence my intervention. I was clearly thinking on my feet and coming up with questions. The intervention is quite long. I think I often did that so that I left them with so many words all they had to go on once I had left was a “sense of” what I had been talking about. If you read it, all I’ve done is swap “investigate” with “play around”. (Mike, February 1990)

Mike had chosen to respond in terms of his own experience – *what I did*, *what I might ask* – rather than saying *do this*, or *ask that*. By doing it in this way, potentially he offered them questions with which to start, but left the situation open enough for them to reinterpret it in their own terms and possibly to come up with other questions. I feel there is an important distinction here which is related to the girls own needs and to the issue of control. Mike could quite simply have told them what to do. In the short term the girls might have been happier with this. It might have involved less struggle for them. But, in the same situation another time, they would still be as dependent on his telling them what to do. So, for their own development of problem solving ability, it is valuable for them to think out an approach for themselves. However, if they never get started, nothing is gained and they may moreover lose interest and motivation. Mike’s compromise is to present some ideas in the context of his own experience. By providing a scenario in which he described his approach to investigation he potentially enabled them to make a start in thinking how they might tackle their own investigating. They could choose to follow what he had said, or they could use his description to trigger their own exploration.

The reader might choose to see this as if Mike *did* tell them what to do, in relating his own experience. Perhaps he would be interpreted by the girls implicitly as saying, ‘If I would do it this way, then so should you’. However, Mike himself saw it as offering too much for them to remember and recreate, so he hoped they would get a sense of what they might do, and then reinterpret this in their own terms. Ultimately, what seemed

important was that they should think through for themselves what they wanted to do, and why, rather than just following instructions from the teacher. This seems to be related to Desforges and Cockburn's (1987) remarks on students resisting higher cognitive demands, and teachers' collusion. Mike seems to resist collusion and uphold his cognitive demands on students' thinking.

Mike said to me when I asked him about such reference to his own experience, that he has wide experience of mathematics and of problem solving and that he saw part of his responsibility as a teacher to enable students to benefit from his experience. However, he wanted students to think things out for themselves and, in the following statement about 'telling' students 'the answer', he acknowledged a tension:

I'm conscious often of having at the back of my mind the desire not to tell an answer, and I will often ask so many questions that in the end I have more or less said "what is 2 and 2" just to get them to say a word. Because you feel that once they have said an answer then that is it. I'm conscious of that at the back of my mind, but I don't think there is anything wrong in sometimes admitting they've reached a stage where I've got to tell them something. (Mike, 30.1.87)

Sometimes it is silly not to tell. The question is when is it appropriate and when not? This is related to the 'teacher's dilemma' (Edwards and Mercer, 1987) which I discuss in some detail in Chapter 7.

Related to questions of 'when to tell', is the question, 'when is it appropriate for the teacher to join a group of students and intervene?' In Situation 6, Mike's intervention was in response to a question from one of the girls.

In the same lesson, I saw him come to a group as if he would intervene, but after hovering for a few moments he went away again. When I asked him about this he said,

Yes, I walked up because I was walking around anyway, looking at where intervention would be necessary and I think I said to myself, 'do I need to intervene?' And I decided that I could not see a reason why I should intervene without inventing one, a sort of artificial teacher intervention – I want to interfere.

That happened on a couple of occasions when I walked round and thought well, I could stop and say 'what are you doing – what is going on?', but they didn't seem to be at that stage – they seemed to be at a stage where something was going on, but I didn't see a reason to intervene. (Mike, 30.1.87)

Yet there were times when he did come in to a group and ask them, 'what are you doing?', as there were also times when he gave an answer or an explanation.

In Data item 6.13, Mike joined a group of three boys, not this time at *their* request as with the girls above, but presumably because he had some reason for joining them, in the light of his comment above. The boys seemed to have worked out an approach to the *triangle lengths* task, which was:

Triangle Lengths

Draw a triangle with a right-angle.

Measure accurately all 3 sides.

Can you find any relationship between the three lengths?

Mike did not comment on their approach directly, but he did remark on one aspect of it. One student (Richard) speaks for the group.

Is it accurate?

(1) Mike You three, what's going on?

Rich Right, well, we're doing the – em, the triangle problem. And we thought that we (inaudible) looking for a pattern, so we – Robert's doing one to five, I'm doing five to ten, and Wayne's doing ten to fifteen. One by one, two by two, three by three, – then we'll draw a graph, and see if we can spot any pattern in that.

Mike You haven't got triangles – does it matter? [They had drawn figures as below]



Rich Oh, we're measuring from there to there. [i.e. from A to B, my labelling]

- (5) Mike There's an important word on there – that's *accurate*. Can I just point that out – bring that to your attention? That word *accurate*, when you're measuring.
- Rich We're not sure, because we keep thinking it's (inaudible) then we'd be point one out, but we've got the ruler exactly on. We're not sure whether the ruler's wrong or – the paper, because we're measuring that.
- Mike Yes. Well, that's the thing you have to decide. Are you saying you think you might have got a pattern, but it's point one out? So only if it was point one better – I've got a nice pattern. Is that what you're saying?
- Rich (Inaudible)
- (9) Mike Well perhaps, I'll leave that with you to decide, whether you're going to stick to it being as you measure it, or whether you might allow a little bit of tolerance – one way or the other, how much you're gonna allow. That's the thing you might want to decide as a group.

Data item 6.15: Extract (2) from Pythagoras lesson 1 (30.1.87)

He did not tell them what to do or how to do it, but he did emphasise 'accurate'. Did this imply to the boys that they were not being accurate enough? Whatever its implications, the teacher seemed to make the intervention just to emphasis that one word, so it must have been important. Mike himself wrote, in reflection on Situation 7:

I think this reflects some aspect of practical work I never came to grips with. I don't think here I really knew what to do or which way to go. I was struggling with the notion of just how would they learn from this. If they were inaccurate and still found 'almost a pattern', what would that say to them? I would like to think now that at the time I wanted them to feel this tension. Hence the stressing on the word 'accurate'. My reason for leaving them with that was to have them consider the two levels on which they were working – on paper and in their head – and that there is a distinction there that is important to recognise. (Mike, Feb 1990)

Here Mike acknowledges that an appropriate intervention is not easy or straightforward – what can *he* do to help *them* to learn from the situation?

In the final situation in this section, Mike seems rather more directive than in the two above. He was talking with a boy, Phil, who was working on 'Square Sums'.

Phil 1

- (1) Mike What other numbers can be made by the square numbers together? Can you make every number?
- Phil I'm not sure – I haven't done every number.
- Mike Well, you've got 25, and you've got 39 – I'm thinking, 26, 27, 28, – can I make any of those as square numbers – by adding square numbers together?
- Phil Well, I'll work on numbers up to 30 – so probably – that'd be (inaudible)
- (5) Mike So how are you going to tackle it? How're you going to work through?
- Phil I'll have to try all different kinds of numbers. I found out that's 25 and that's 29, so I have to try all the numbers between there.
- (7) Mike That's right. Then work your way up. Good.

Data item 6.14: Extract (3) from Pythagoras lesson 1 (30.1.87)

In this case he seems not only to suggest to Phil precisely what he might try, but also to push him quite hard to say himself what he will do. When I asked Mike to respond to this piece of transcript he wrote the following:

I will take it from you that this was with Phil. That I feel is significant, for my response will depend on who I was working with. I can well believe the first comment, "I haven't tried every number" (statement 2) to be a classic Phil comment! I feel the word "Well" (statement 3) is significant in my response; it's a "come on this is serious now". It seems as if I am trying to have him experience the systematic searching/attacking. I seem to have identified *that* as what I want him to work on there. I suppose my "So how are you going ..." (statement 5) is to have him articulate what *he* sees as his way forward. So it's a closing down around what I believed to be his position on the zone of proximal development. "Then work your way up" seems redundant unless I'm referring to going beyond 29 or 30 (I can't quite make out what's going on!) It is a way of me legitimating 'his' suggestion – i.e. how he will work on mine! (Mike, Feb 1990)

Again, Mike points to the importance of his knowledge of the student concerned in making his response. Having observed Phil to some extent

myself, I understand why Mike would push him rather more than some other students. He was not easily impressed by the teacher, and would go his own way unless some strong directing took place. Thus he was less likely to be inhibited by the teacher's words than some other students. Hence the teacher could be, or sometimes had to be more directive with Phil if he wanted to influence Phil's thinking. Statement 5 is indicative of this and consistent with Mike's comment above about 'closing down'.

I was very interested in Mike's spontaneous use of Vygotsky's ZPD here. I have suggested (Jaworski, 1990) that many of the interventions which I have observed could be regarded as examples of teachers 'scaffolding'⁸ students' learning across their ZPD. In each of the situations in this section there seem to be elements of this, and perhaps Sensitivity to Students could be tied in directly with notions of ZPD.

MATHEMATICAL CHALLENGE

I have indicated that there is often a tension between two of the elements of the teaching triad – *Sensitivity to Students*, and *Mathematical Challenge*. They are also closely bound to each other. I shall look back to the situations which I quoted in the last section, offered from the point of view of the teacher's sensitivity to the students concerned in the nature of his intervention but I shall now focus on degrees and forms of Mathematical Challenge.

The Pythagorean relationship is embodied in both scenarios, 'Square Sums' and 'Triangle Lengths', on which this lesson was based. Mike hoped that, as a result of working on these scenarios, students would start to be aware of relationships which would lead to the introduction of Pythagoras' theorem. This can be stated as, 'the sum of the squares on/of the (lengths of the) two shorter sides of a right angled triangle is equal to the square on/of the (length of the) hypotenuse'. The on/of distinction leads to the two scenarios which Mike offered. The theorem can be seen to relate properties of numbers, or properties of triangles, or properties of both. A good sense of the Pythagorean relationship would involve awareness of all of these related properties.

In setting the activity for this lesson Mike had been quite explicit in his instructions. They were to work in groups of four, but in two pairs within the group. Each group was given two pieces of paper, one for each pair –

⁸ This term is discussed in Chapter 3 (see p 45) and it will be considered further in the next chapter.

one containing 'Square Sums', one containing 'Triangle Lengths'. Pairs were told to work from their own piece of paper, but to talk to each other about what they were doing. Mike hoped that there would be cross-fertilization, and that groups would gain an inkling of a relationship between the two scenarios. The class's work on Pythagoras extended beyond this first lesson, and here students were simply trying to come to terms with the scenario with which they had chosen to begin.

In Situation 6, the girls were only just starting, and they needed help with how to start. The 'Square Sums' statement itself posed a high degree of challenge, but the girls could not take up that challenge without actually *doing* something, and they could not believe what they felt it was asking them to do. Hence they were stuck. To have told them what to do would in some sense have taken away the challenge, and in this lay the teacher's dilemma which I mentioned earlier. Edwards and Mercer (1987) say that, 'The teacher's dilemma is to inculcate knowledge while apparently eliciting it' (p 126). Perhaps another version of this is that the teacher has to elicit knowledge, without saying what knowledge he wishes to elicit. We saw the teacher's compromise in coping with the dilemma. This relates also to the *didactic tension* (Mason, 1988b) which I will elaborate on page 185, and discuss further in Chapter 7.

In Data item 6.13, 'Is it accurate?', the boys seemed to be working well and had developed a good strategy, but potentially it contained a serious flaw which the teacher through his own experience could anticipate. The practical work done was likely to be inaccurate to some degree. The ideal result, that the square of the third side should be exactly equal to the sum of the squares of the other two, was unlikely to be obtained in practice. However, with careful measurement, this result might be seen as an approximate pattern. I think the teacher was worried in two respects, that with very inaccurate working a pattern may not become evident at all; but also that justification of a rule from an approximate pattern is mathematically suspect. He criticised himself on another occasion for trying to get students 'to see a rule by drawing'. Yet did the scenario admit of any other possibility? Could he expect that students would move from their activity to some more formal proof of any pattern which might emerge? These questions, fundamental to notions of Mathematical Challenge, are very difficult for a teacher to contend with. Yet there was evidence of the teacher learning from such situations. The words 'see a rule by drawing' were a distillation from previous experience, an example of *abstraction by naming*. If the teacher's experience with these boys provided insight into expectations of mathematical rigour related to this

practical activity, such insight could be fed into the design of future activities, maybe providing scenarios which might more realistically challenge students to a high degree of rigour. I feel that there is a good example here of the teacher's developing his *teaching knowledge*, and potentially his *teaching wisdom*. He knows something about the dilemma in "seeing a rule by drawing" – this is his teaching knowledge. He can now anticipate circumstances in which this dilemma is likely to occur, and try to circumvent them – this is his teaching wisdom. These terms will be elaborated in Chapter 8.

I see a balance between SS and MC in these situations as follows. The girls could not make a start, so although the teacher did not wish to tell them what to do, he nevertheless offered them instances from his own experience to create a sense of what might be possible. The boys were getting on well and independently, so the teacher could 'toss out' his challenge. In Data item 6.14, 'Phil 1', the need was seen differently. Phil was not getting on well independently, and was not likely to take up a challenge unless it was impressed on him, so in consequence the teacher was more directive. I was struck by the outcome of this direction, and happen to have transcript from subsequent discussion between Phil and the teacher.

Data was not always orderly and well behaved. I collected data as it was possible in the classrooms which I observed. However, position of recorders and my own attention were not always directed at what I would later like to have data on. Thus, I have no record of what the two girls made of the 'Square Sums' task after their initial interaction with the teacher in Situation 3, so I am unable to make further remarks on any consequence of the teacher's intervention. However, I did have a recording of a subsequent conversation between Phil and the teacher which allows further consideration of MC where Phil was concerned.

Phil 2

After Situation 8, a little later, the teacher returned to Phil who animatedly referred to his subsequent work on 'Square Sums':

- (1) Phil I've got 26, and I'm working on – if I want to get 27 I've, I have to try and get the closest number – to do the sum – I have to use something like 1.5, cause, if I try to get the two, then that'll make four, if I try to use two squared plus five squared, er, that'll make 29, so I have to – cut em in half, obviously, cut em in half. (Mike "Right") I'm going to try and keep the five and use 1.5 squared.

Mike That's a nice idea. So you're going to try to home in to 27. Is it 27 you're working on?

Phil Yes

Mike Right

- (5) Phil If I can't do that, I'll take 4.5, I won't take five and a half, I'll take four and a half, and use two here.

Mike OK. / So you're going to have *something* squared –

Phil Hm, what's the word for – 1.5, er, *decimal*? Yeah, I'm gonna use decimal –

Mike Right. So, something squared, plus five squared equals 27.

Phil If it does. If it doesn't, erm, that's what I think, if it doesn't I'll try er 4.5 squared by erm 1.5. (Inaudible) then I'll go back to the two then I'll go

- (10) Mike Well, let's try working to the five squared, for the minute, and let's say that equals 27, now your problem is to find something squared, plus five squared equals 27. What can you tell me about this number, this *something* squared? Can you tell me anything about it so far?

Phil Erm, well, I know this five's important. That gets you into twenties.

Mike Right, How far into the twenties?

Phil Half way.

Mike So what does five squared equal?

- | | | |
|------|------|--|
| (15) | Phil | Twenty five. |
| | Mike | Right. Now, something squared, plus 25 equals twenty seven. |
| | Phil | I need to get two out of here. |
| (18) | Mike | Right. So something squared, – you’ve got to find a number, which // – now that seems to me to be a short cut. Can I leave that with you – to look at? |

Data item 6.15: Extract (4) from Pythagoras lesson 1 (30.1.87)

Some aspects of this situation may need clarification.

At Statement 1, Phil says he has got 26. I suggest this is by $1^2 + 5^2 = 26$. Also $2^2 + 5^2 = 29$. So, to get 27, he would need something like $1.5^2 + 5^2$ or $2^2 + 4.5^2$. At statement 9 he says that if these don't work, i.e. they don't give 27, then he will try $1.5^2 + 4.5^2$.

The teacher, after listening to Phil's initial statements, seems to think that Phil's approach is too random, so at Statement 10, he suggests staying with 5^2 , and considering aspects of this first. He tries to get Phil to analyse what it *is* that he needs, rather than varying two things haphazardly. Ideally, Phil should realise that what he needs to add to 5^2 in order to get 27 is 2, and so the number squared which gives 2 is $\sqrt{2}$. However, the situation is *not* ideal. The teacher could explain this to Phil, but is Phil ready to appreciate its sophistication? Phil seems to still be in a position of believing that with persistence he will find the number which he wants, which he expects to be a simple terminating decimal. Hence he believes, implicitly, that there is such a number. It is likely that the realm of irrational numbers is beyond his current thinking. In the few seconds which the teacher has in which to respond to Phil's statements, he has a very complex teaching situation to assess. Should he leave Phil to stab randomly, until perhaps he himself perceives the need for some other approach? Or should he try to move Phil towards what, according to his experience as a maths teacher, might prove to be more profitable? Mike said later that he was also aware of Phil's difficulty in coping with equations of the form $x + a = b$ from an intuitive perspective.

It is likely that the teacher drew on this recent experience with Phil, in which Phil responded well to direct challenge. The thinking which Phil exhibits in Data item 6.14 is witness to this. Despite the haphazard nature of his stabbing to get 27, he is nevertheless on the track of 27, and his

thinking is quite impressive in this respect. The teacher's consequent pushing results in Phil's articulation of the idea that he needs to 'get 2 out of there' (Statement 17), and so the teacher prompts at this point, "Right. So something squared ..."

I can express this in terms of Phil's ZPD. The teacher has to make judgements about the degree of challenge which it is appropriate to offer Phil at this point in order to enable him to move on. Too much challenge and the precarious position might be lost, and Phil might have to recreate the thinking which he has already achieved. However, too little challenge may result in Phil not making progress at a rate of which he is capable with the teacher's help. Theoretical knowledge of the ZPD is no panacea for a teacher; there has to be recognition of where a child stands and where he might reasonably reach. A great deal of knowledge of the child is bound up in this decision, and so Mathematical Challenge cannot be divorced from Sensitivity to Students.

MIKE'S OWN THINKING – AND RESPONSES FROM STUDENTS

In this section I shall change focus from classroom situations involving Mike's teaching, to Mike's theoretical perspective on his teaching. Related to this will be some of the remarks made by students about their own experiences of Mike's teaching. My purpose here is triangulation, supporting and extending what I have said earlier using data from Mike himself, and the students.

The *billiards* lesson seemed to me to have a similar structure to others of Mike's lessons where he began 'up front', getting students involved in the thinking, then setting specific tasks on which they would work in groups, interrupting this periodically for periods of thinking or sharing. The first *Pythagoras* lesson seemed different in structure. I remarked on this and Mike himself commented:

It was partly deliberate, in an attempt to have a different style in this lesson from what they'd experienced in previous lessons. They've had the style where I introduce a topic that is *closed* – in inverted commas – and they've had the style where I introduce a topic where it's not so closed – like the billiards – and they get out of it what they want, but it's still being led from the front, and they follow through a problem, and in their groups they're all working on the same problem, but they may find different things to investigate within the problems.

What I want to do is to add another dimension here and that is to try to add two different problems – they may be different in *their* perception – but to try to *deliberately* arrange it so the groups would be working on two different problems and perhaps there might be some cross fertilization of ideas during the two. (Mike, 30.1.87)

He implied that students could benefit from experiencing *different* approaches to lessons. He went on to say that offering the two parallel scenarios in written form was a new idea that he had been trying out:

One of my aims when I was thinking about this last night ... was to try something I hadn't necessarily tried before, the two different problems, the writing them out, the giving them out, and to see if I could find anything out and just to try it. I feel that the trial was a success, I feel that I got something out of that. I think it has given me questions. It has given me ideas. It is a lesson that I can come away from and not want to forget. So on that level I think it is a success.

One of the areas I'm thinking about, or I'm concerned with is that my aim in wanting two different problems to go on simultaneously in one group – that didn't happen in some groups and it did in others. Should I be disappointed if it didn't happen in some groups? Should I have forced it a lot more? Should I have said my objective is to have that going – let's make that a priority, let's keep pushing it? ...

So it has filled me with questions about that method, about what I should have done, what I should do next time, and that is what I call a success, a lesson I can go away from thinking about learning something from it. (Mike, 30.1.87)

Mike's focus in these quotes seems to be on the Management of Learning rather than on learning objectives per se. Yet implicit in his words seemed to be that his style or approach should foster learning.

I assumed that criteria for judging the lesson a success in terms of his own learning (about teaching?) must implicitly attach importance to students' learning, or to the meanings which students made from the activities in which they took part. For example, referring to billiards, he said "and *they get out of it* what they want ... they're all working on the same problem, but they may find different things to investigate within the problems".

I asked him about his learning objectives in a task such as *billiards*, which he said was 'less closed'. How did they compare to those, say in the *Pythagoras* lessons, where an end result was clearly important? He was emphatic about this:

Having a result is important in mathematics ... isn't mathematics about finding results? I mean, you don't just do things for the fun of doing them do you? You always want to get somewhere, otherwise after a while what's the point of it all? I mean, whether the result comes from you or from someone else, results are important, and I'd like them to develop towards perhaps looking for their own results, develop a feel, an internal monitor of where they are getting, whether they can see a result coming, whether they can see a conjecture, or they can see something or feel something, whether they can ask the right questions. I think that is part of the process. (Mike, 13.2.87)

These seemed to be more explicitly learning objectives, and Mike's words support my interpretations regarding his emphasis on developing particular ways of working.

In the Pythagoras lessons, where there had been a clear result to aim for, one group of four students, working on 'Triangle Lengths' had expressed overt frustration when, despite considerable effort, no pattern seemed to emerge. Episodes from their work had been recorded on videotape, and later Mike and I and three of the students looked at the tape together. The following three data items include parts of our resulting conversation:

Trust		
<i>I asked Susan, James and Simon what they thought was the intention behind the tasks which Mike set the class</i>		
(1)	Sus	There's a lot of lessons that Mike does that I don't see the point of. No offence!
	BJ	Do you think therefore that it would be better if he did something different in those lessons? Do you have a feeling that you are wasting your time?
	All	No! No, I/We enjoy it!
	Jam	You know you are supposed to be – you know – there's got to be a point to doing it. There has to be a point, otherwise you wouldn't be doing it in the first place.
(5)	BJ	Does that mean you trust Mike? ("Yes! Yes!") So he's not just making you do something for nothing?
(6)	All	For the sake of it. No we know that. We knew there was something behind it. We just didn't know what.

Data item 6.16: Stimulated recall (1) with students (11.3.87)

While it should be recognised that these remarks were made in front of Mike and me, they nevertheless convinced me of these students' feelings. They fitted with what I observed in the classroom in terms of students' willingness to participate in activities and evidence of their being interested in them despite frustrations.

This group had been particularly frustrated because, despite their efforts to find a pattern, no relationship emerged from their data. Mike asked how they had felt when they found it was not working, and one of them replied, "Frustrated. It does get up your nose when you think you've found something, then I mean you get another number which completely proves it wrong." Mike asked them what would have helped, would they have liked to have been told the answer?

Is there an answer?

- (1) Jam No I think, more than the answer we just wanted to be pushed along the right track.
- Sus I wanted to know if there was an answer, because I thought there wasn't anything to find, and that was why I was getting fed up.
- Mike I felt you were asking me to tell you what it was.
- Sus No, No, We just wanted to know if there was an answer.
- (5) Jam We don't mind working and finding the answer out for ourselves, as long as we know there is one.

Data item 6.17: Stimulated recall (2) with students (11.3.87)

Their stress on the importance of there being an answer was interesting, and reminds me of Mike's words on the importance of *results*. It seemed to be a part of their common epistemology that you had to be going *somewhere*. The students saw this in terms of reaching an answer. For Mike it was rather more general than just some prescribed answer, but was no less crucial. I had heard one of them explicitly refer to Mike's 'having the answer in his head', so I asked them about this.

Does the teacher know the answer?

- (1) BJ I think you told me that you thought Mike had a relationship in his head that he wasn't telling you.
- All Yes! Yes! *He* knew what it was. But *we* didn't.

	BJ	Do you think that's always the case?
	All	Yes! Yes!
(5)	Sim	Unless we come up with something that he never even thought of.
	BJ	Does that happen often?
(7)	All	No. Well, don't know really. He might not tell us. Yes, He'd probably just cover it up, and pretend that he knew all along! (Laughs)

Data item 6.18: Stimulated recall (3) with students (11.3.87)

I was struck by the similarities and differences in epistemology concerning 'telling' and 'knowing' between Mike and the students. The students seemed to believe in the existence and rightness of knowledge, although they were prepared to trust that Mike's methods had a valid purpose. There was a similarity here to views expressed in the questionnaire data from Clare's students. Mike seemed to want to foster their own constructions, but 'results' were an important requirement.

Perhaps for this reason, Mike had expressed dissatisfaction with what he referred to as 'the cognitive outcome' of the Billiards lesson. He had invited students to invent their own questions to explore and they had done literally that, being *very* inventive. In his words they had "made it too complicated – 37 degrees, going round a hexagon, in 3-D!". They had spent a lot of time making little progress because they had introduced too many variables into the problem and were going round in circles, not knowing how to handle it. The 'fuzziness' of the students' thinking had potential to be useful because it could help them to come to realise that with too many variables they could not expect to get very far. Yet, in terms of progress on the problem, not much had been achieved – there was little 'cognitive outcome', or no results. On another occasion, in a lesson in which he had asked students to make a poster to express their understanding of Pythagoras' theorem, he was dissatisfied with what occurred, saying of the activity, "I think perhaps it was not cognitively dense". Notions of cognitive outcome, or cognitive density, seemed to refer to the quality of mathematical thinking or perhaps the instance of mathematical results. There was potentially a tension between Mike's desire for 'results', and his deliberate creating of a situation (in *billiards*) in which students could see the need to simplify in order to get any results.

The tension between how a teacher wants students to work, and how she gets them to work in this way, was made manifest for me in many of these situations. It was bound up in the strategies which the teacher devised and the particular outcomes she had in mind, and also in students' responses and the teacher's evaluation of these responses. I felt that Mike was continually grappling with this tension as he identified objectives and evaluated the results of particular approaches. I shall quote one final situation in which the tension seemed to be manifested.

In one of Mike's KMP lessons, very close to the end of the lesson, Phil, who was quoted earlier, asked Mike for help. The substance of his query was that his KMP card asked him to find the area of a triangle; he had done this in two different ways and had got two different answers which he had checked and believed to be correct. His knowledge of areas of triangles was sufficient for him to realise that two different values could not apply to the same area. So which answer was in fact correct, and why was the other one wrong? In the short time that was left before the lesson ended Mike tried to get Phil himself to confront the apparent contradiction.

Speaking of the interaction later, as the result of a stimulated-recall session, Mike said:

That's why I think he put his hand upBecause I think he accepted that mathematics has got to be consistent – whichever way you do things you've got to get the right answer. He's got two methods for finding area of the triangle, multiply the two together; square one, square one and add them; and they were giving him different answers, and he was quite convinced it was a method. "I'm not confused. They're two methods. Just tell me why I'm getting this one wrong". My frustration there was that it was right at the end of the lesson.

...

There was also a part of that which was me trying to find out just what he did know. I just didn't know what to do in that situation. I didn't know how far to go back.

...

Also I was trying to create dissonance, without possibly realising it. It's the old, 'what's three fours?' 'nine', 'what's three threes?', 'Oh it was twelve' I was trying to do that bit by showing him that we got two different answers, the implication being it was wrong. But he wasn't buying that. That was the end of the lesson, and we had to pack away. (Mike, May 1987)

One of Phil's methods for finding the area of the triangle had been to square two sides and add them together. Mike put Phil's problem down to the work which the class had just been doing on Pythagoras, and felt that

his confusion had been with mis-use of the Pythagorean algorithm, mixing it up with that for finding area. He said:

I think that there is a big danger. Even the way *we* try to teach, in the end they will learn algorithms, and that's that. And however much practical work you give them to lead up to it, they regard that as the culmination of the day's work, or the week's work, or whatever it is. ... I still don't know how we get over that without abolishing algorithms or something, work things back from first principles. (Mike, May 1987)

"The way *we* try to teach", referred to the fact that Mike's department as a whole had a policy of using practical work, and activities which involved discussion, in order to get at concepts, rather than straight expository⁹ teaching. Yet Mike was highlighting the tension for any teacher that ultimately it was possible for students to disassociate algorithms from the processes to which they relate, and fall into traps of their misuse.

Again, a teacher wants particular outcomes from the teaching situation. Although recognising that pupils will make their own constructions from whatever is offered, the teacher has some very particular goals in mind. The goal here was that students should use algorithms in a meaningful association with the processes to which they related, not simply as short cuts to an answer. What can the teacher actually *do* to achieve this particular goal? This is a manifestation of the *teacher's dilemma* and is related to the *didactic tension*, which Mason (1988b) expresses as:

The more explicit I am about the behaviour I wish my students to display, the more likely it is that they will display that behaviour without recourse to the understanding which the behaviour is meant to indicate; that is the more they will take the form for the substance.

Significant in the above quotes from Mike is his own questioning of this practice, and this seems to indicate his, perhaps implicit, recognition of the tensions involved. I shall present further manifestations of these tensions in the next chapter.

CONCLUSION TO THE CHAPTER

This chapter has introduced the basis of a constructivist pedagogy for mathematics teaching through the practice of two experienced reflective mathematics teachers. The significance of their *reflective* activity, along with that of Ben in Chapter 7, will be the subject of Chapter 8. In this

⁹ See for example, Brissenden (1980) p 48

conclusion I shall explicate my perception that this teaching fits a constructivist perspective, and can moreover be regarded as investigative in style. I claim that, in being so, it does no disservice to the requirements of the mathematics syllabus – in current terms to the *delivery* of the mathematics curriculum. I saw high level cognitive demand, with emphasis on the development of high level thinking skills. I saw good classroom order, with particular attention paid to the needs of individual students.

I begin from a constructivist perspective, that is with the belief that individuals construct their own knowledge, and come to know through adaptation of their own experience. I saw teaching which not only *fitted* this perspective, but which explicitly recognised its implications, for example, both teachers' emphasis on *imagery*, and on *thinking*, and their differentiated approach for different students. I have pointed out ways in which I have seen various lessons as *investigative* in style. The investigative approaches have emphasised *thinking*, *asking questions* and *seeking patterns*; for example, Clare's 'hands down think', Mike's red books and his cued strategy for *asking questions*; looking for patterns in decimals representations of sevenths, and looking for what sorts of numbers can be represented as sums of squares. Encouraging questioning and exploration can be seen to promote involvement in mathematical ideas and influence mathematical constructions. The teacher, needing access to these constructions must listen to and talk with students about their thinking. Such activity has been evidenced in this chapter.

However, there has been no assumption by these teachers that simply providing opportunities for investigation is *sufficient* to ensure that high level mathematical thinking takes place. I have shown both teachers to have a well articulated agenda for how they want students to work. Clare talked of 'training' students in strategies for learning. I spoke of Mike's 'control' over the learning situation. Both teachers overtly asked pupils to think and to ask questions. Crucial to the ethos of their classrooms was their very overt Management of Learning, but also their levels of sensitivity to the students with whom they worked. The use of the word *training* is, I feel unfortunate in this context, for reasons which I expressed in Chapter 2 (see p 26). I would rather see what Clare was doing as *fostering* these strategies. I believe that the traditional connotations of the word *training* are ill-suited to the degrees of sensitivity which were present in the lessons I observed. On page 166 I asked in what ways Mike's 'control' might have constrained or inhibited the mathematical thinking of his students. I recognise that it prevented them from having *total* freedom

of thought, explicitly or implicitly pushing them in directions influenced by the teacher's emphasis. Yet this emphasis also encouraged their own perception and ideas. Ultimately, there has to be a balance between making something 'worthwhile' happen, and leaving pupils free to develop their own ideas, and this can be a source of tension for the teacher.

The meaning of 'worthwhile' is of course highly subjective. However, as curriculum delivery might be seen as having to be high on the agenda of any teacher, *worthwhile* might be interpreted in one way as *enabling* this delivery. As I conducted no testing of students, and no interviewing directed at their mathematical understanding I cannot speak of their *learning* as a result of the teaching I observed. However, I observed examples of their mathematical thinking related to the mathematical challenges offered by the teachers – Rebecca's development of an argument in the lines problem; Phil's process for tackling sums of squares; the girls' thinking on ratio and scale; Martin's questions about recurring decimals. These pupils were all involved in mathematical ideas. They were all seeking mathematical generalisation of some form. I claim that they were engaged in high level mathematical thinking in Desforges and Cockburn's (1987) terms. Drawing on my own experience as a mathematics teacher, I therefore suggest that they were engaging in effective mathematical learning in terms of building concepts and making links between their various schema.

I have described the teaching which I saw in terms of the teaching triad, which I claim is an effective device to enable characterisation of an investigative approach to teaching mathematics. Each of the elements of the triad has been mentioned in my linking of the teaching I have observed to a constructivist perspective above. The further testing of the triad was one of the main objectives of the Phase 3 research, and I shall report on this in Chapter 7, and ultimately link the triad to a constructivist philosophy.

I have mentioned on numerous occasions, in this chapter, connections which I have seen between *observed teaching acts* and the theoretical notion of *scaffolding across the student's ZPD*. Associated with this has often been references to *teacher's dilemma* and *didactic tension*. These are indicative of questions which the teaching act raises, creating issues for the teacher. I have not elaborated these strands in this chapter, as they will be seen to continue through Chapter 7, and will be discussed after an account of the Phase 3 analysis has been presented.

Finally, I recognise that in speaking here of a constructivist pedagogy I go beyond my limited observation of two teachers' lessons. In the spirit of Delamont and Hamilton's (1984) remarks quoted in Chapter 4 (see p 64), I believe that 'abstracted summaries and general concepts can be formulated which may upon further investigation be found to be germane to a wider variety of settings'. As a result of this research I shall suggest aspects of mathematics teaching which *fit* with a constructivist view of learning. It will be up to further research to explore the consequences of this, to practitioners to judge its viability for their classroom, and to theorists to examine the links which I have postulated.

INTERLUDE B

FROM PHASE 2 TO PHASE 3

The relationship between a constructivist perspective and observations of teaching becomes more overt.

Teacher and researcher awareness

Towards the end of the first term of the Phase 2 observation, I wrote the following diary entry about my current research work:

Sense-making

I believe that every teacher must ask the question, "What sense are the pupils making of mathematics in *my* lessons?". I am interested in how teachers go about, a) helping pupils to make sense of mathematics; b) finding out what sense is being made.

I am pursuing this by *observing* particular teachers in their classrooms, *noticing* aspects of their practice, *discussing* their work with them, trying to find out:

What most *concerns* them about the way they work?

What are *classroom issues* for them?

What *tensions* do they confront?

What *action* do they take to develop their teaching?

How do they present material?

How do they intervene, talk/listen with pupils?

How do they find out what pupils are thinking/understanding?

Are there common issues/concerns, beliefs about teaching, actions taken? (Fuzzy)

Is it possible to develop a language to describe classroom teaching development?

In many situations I am a teacher myself. I have concerns. There are issues/tensions which I confront; eg How does the way I present material influence/constrain what pupils do with it? When is it appropriate to make certain kinds of intervention - directive, non-committal, provocative, silent?

I have beliefs. How do my beliefs affect my observations of other teachers? How does my presence/observation affect other teachers' beliefs/actions?

Blocks to progress:

Very wide scope of study - need to narrow down to more specific questions. I'm interested in too many things and so not focussing closely enough on anything.

Theoretical basis is very fuzzy - investigative approach - constructivist approach - need to identify some theoretical starting point which might help in finding particular questions to focus on.

Data item B.1: Diary extract (6.12.86)

The literature on ethnographic research supports my retrospective view that the 'fuzziness' I experienced here is quite natural, that often this kind of research can feel overwhelming in its earlier stages. The nature of seeking characteristics is that until patterns emerge there is no clear focus. However, I recognise that despite worries about the fuzziness of my thinking and the scope of the study being too wide, I had started to be more specific about what I wanted to look at in terms of characterising an investigative approach. I had identified specific questions which I was addressing. I was beginning overtly to tackle issues relating my own theoretical basis to the observations which I was making.

The *development* of teaching was a clear focus at this time. The videotapes which I had recorded in lessons of the Phase 2 teachers proved a valuable stimulant for reflections from both teachers, separately and together. We spent many hours playing excerpts from the tape and talking through their reflections. I realise now that my view of 'the reflective teacher', which is discussed more extensively in Chapter 8, was largely formed by the discussions with Clare and Mike and their analysis for me of their aims and objectives for teaching. In thinking about their operation, I wrote, "What experiences have C and M had that has enabled them to work at this level? Is it a quality that they possess?" I felt that they were both extremely *aware* of what they were doing, what they wanted to do, and issues involved in this. I speculated on levels of pedagogic awareness which included, 'unconsciously unaware',

'consciously unaware', 'aware', 'consciously aware'¹. These were not well defined, but yet I felt able to place teachers according to my own intuitive notions. I felt that the Amberley teachers had moved from being unconsciously unaware of the way they taught and why, to being *consciously* unaware, and that during our work together they were moving to a greater awareness. I felt that the Beacham teachers were aware of what they were doing and why, and that during our work together they moved to a more conscious awareness. The word 'conscious' implies a level of knowing which involves the capacity for informed choice as a result of reflection. I found that Schön's (1983) levels of reflection in action related closely to this thinking. A level of conscious awareness indicated that the teachers were actively reflecting on their own awareness, which then increased their ability to make classroom decisions knowledgeably. I feel that this made their *teaching knowledge* more overt and thus increased *teaching wisdom*. These were terms which arose from the above thinking - they will be defined and discussed later. I recognise throughout the research my own development in the research process. I cannot readily fit it to the above levels of awareness because there are too many facets to consider. The very doing of research demands some level of reflective awareness from the start. However, its level of consciousness must be related to the clear making of choices. At this stage of Phase 2, I believe that I was moving from an intuitive to a more conscious position. Perhaps my inability to be more specific is due to my being less 'distant' from the research than I am able to be from the teaching. Who does the 'distancing' for the researcher?

This thinking inspired me at that time to write down in my diary

Hypotheses

1. Awareness is a prerequisite for effective change

A teacher cannot start to work on improving teaching if they are not self-aware. eg. Clare notices gender issues within the classroom because she is aware of gender problems.

Does self-awareness imply that you will ask questions about how you operate?

Are there levels of self awareness, eg unconsciously unaware, consciously unaware etc.?

¹ The notions of levels of awareness and consciousness are addressed in varying degrees and contexts in (e.g.) Kelly (1955), Frere (1972), Boud, Keogh and Walker (1985) and Claxton (1990)

2. “What sense are pupils making?” Every teacher asks this question at some level.
- If a teacher asks this question explicitly, does it say something about their self-awareness?
- For teachers for whom the question is only implicit, where are they?
- Does a genuine desire to answer this question imply links with a constructivist philosophy?
- Are efforts to involve pupils in thinking for themselves, or in taking responsibility for their own learning rooted in constructivism?

Data item B.2: Diary extract (27.2.87)

Recognition of an issue - The teacher's dilemma

There were many teaching issues arising from the work with Clare and Mike which I have discussed extensively elsewhere. However, one major issue arising from the Phase 2 work, involved Clare's ‘prodding and guiding’, Mike's ‘cognitive density’, and statements from both of them to the effect “it's only a KMP lesson”. It was about achieving significant mathematical development in pupils and the teacher's role in this. It was about how much to direct and how much to encourage pupils to go in their own directions at their own pace. It was about the need to cover certain mathematical ideas and the way in which these were approached. It might be seen to encompass the whole of teaching, but yet it could be encapsulated rawly in the phrase which came from Amberley, “When to tell” - which is one form of the teacher's dilemma (Edwards and Mercer, 1987) I find it hard now to recall just how I saw the issue at this stage, and how much my thinking on it developed in Phase 3. Certainly the issue became clearer in Phase 3, partly because I was able to discuss it openly with Ben . This helped me to refine my own awareness of it, and I shall include further discussion in Chapter 7. In Jaworski (1991) I wrote of Clare's ‘prodding and guiding’ dilemma which seemed to have the essence of the issue for Phase 2, and I quote Clare's own words below:

Clare - prodding and guiding

The way I work with these things is that if I know too much about where it's going, given that I do prod and guide, I may well prod and guide people into directions which may not be the most fruitful ones, may not be the most interesting ones for them. ...

Vicky and Ann were working in a way which I thought was not very fruitful ... I haven't prodded them very much, I haven't guided them very much, and the fact that Ann said a few things earlier on in this lesson helped actually, because I was able to say 'what was your idea?', 'what did you think you should do?' ... after all, I'm supposed to be a teacher and sometimes I do know that that some ways are more fruitful than others, but only ... oh dear, it's terribly difficult isn't it.

Sometimes I know and sometimes I don't know, and the ways that I know, I know because they apply in lots of different situations. I think this is it. I know it's fruitful to do clear diagrams and not fruitful to do tatty diagrams. And I know that it's fruitful to use apparatus and not fruitful to totally rely on the abstract. And there's other things that I know, I think. What I don't know is where this investigation can lead. I know some places it does lead, but I don't know where it can lead totally ... I don't think I know that about any investigation we do.

Data B.3: Extract from discussion with Clare (13.1.87)

Bound up in this seem to be fundamentals of constructivism. Pupils *will* construct for themselves. My interpretation is as follows: she wants to encourage their own directions of thinking - yet she herself has certain knowledge and experience, and she is a teacher with responsibilities to help further her pupils' knowledge. I think she recognises that their knowledge cannot be the same as hers, and that she should not attempt to make it so. Yet she wants to influence that knowledge. In expressing cognitive density, Mike indicated that there were activities in the classroom which seemed more intensely mathematical than others. I inferred from this that there was a sense in which these activities were more valuable than others. This raises questions about the nature and value of the other activities. Felicity in Phase 1, in the 'cutting and colouring' activity (Jaworski, 1986, example 7), had indicated something of this too. Although she had set the task, she felt that it was not mathematically dense (using Mike's terminology) and was eager to move on to more overtly mathematical thinking even though she resisted this to some extent. Both the Amberley teachers had concurred with the notion that pupils prefer to be told what they should know. In using the

individualised schemes, both mathematics departments subscribed implicitly to the need to follow a syllabus and get through a scheme of work. Their classwork or project work provided an opportunity for aspects of teaching and learning in which the scheme possibly was inadequate. Neither Clare nor Mike hesitated in 'telling' when this seemed appropriate, and this took different forms in different circumstances.

I feel that what I was valuably doing in this phase was characterising aspects of teaching by recording such circumstances as my data, analysing the nature of the interactions, and raising and refining issues in consequence. I have noticed that there are various stages in coming to terms with an issue. There is the initial, very fuzzy, yet potentially exciting stage in which the issue first begins to emerge. There follows a very frustrating and worrying phase when there seem to be irresolvable contradictions, and one is seeking for answers of some sort even if rationally one knows that this contradicts the whole nature of an issue. The next stage involves recognising that looking for answers is not a sensible exercise but that nevertheless there are ways of tackling the issue and its nature becomes much clearer. In understanding it, it becomes less threatening. The final stage is being in a position to tackle the issue, and to grow in knowledge and experience (or wisdom) as a result. This thinking is very recent, so I was not in a position to discuss these stages with any of the teachers.

The researcher's dilemma

As a result of analysing students' responses to questionnaires and trying to link this to my analysis of the Clare data, I wrote,

It's difficult. You could say that I asked the wrong questions, that I should have been more explicit about what I wanted. But then, beware the topaz effect - *the more explicit you are about what you want, the more likely you are to get that because it's perceived that you want it, not because it's actually the case.* [my paraphrasing of Brouseau 1984] (Diary, 3.3.88)

The topaz effect, which later became (I felt) more aptly expressed as *didactic tension* (e.g. Mason 1988b) coloured much of my thinking at this time, although I had not yet related it explicitly to what I was seeing in the classroom. I began to notice manifestations of it during the Phase 3 field work, and in my writing of the Phase 2 analysis.

I had hoped, perhaps unrealistically for some spontaneous utterances from pupils which would support my analysis of Clare's teaching. Of course, I could not expect pupils to express their thinking in my language, e.g. Management of Learning, Sensitivity to Students and Mathematical Challenge, and so needed to scrutinise their remarks for any 'fit'. There were a few potentially related comments in the questionnaire data, and interviews with pupils had produced others, (see Chapter 6). However, I felt that my interpretation needed further validation, and I felt that it would have been useful to 'probe the situation further'. For example, in response to the question, 'What is the most useful help that your maths teacher can/does give you?', one pupil had written (see p 154):

I suppose that being told to work it out for myself is the most useful help given, but I think a little more guidance would be helpful at times.

This seemed to fit with Clare's issue of 'prodding and guiding'. Perhaps the pupil concerned had some inkling of Clare's philosophy. I wrote that I should like to:

notice a moment in the classroom where such an incident occurs and then try to follow it through, discussing it particularly with both Clare and the student. (Diary, 3.3.88)

However, so far, situations in the classroom had been brought back to mind in subsequent discussions with Clare, after reading field notes or working on a tape or transcript. By this time the moment had passed and it was too late to resurrect it with the pupil. Noticing such moments when they arose required a level of awareness of issues acute enough for resonance to be triggered in the event rather than in subsequent reflection on it. This again related to Schön's (1983) 'reflection in action', and I was seeing a need for this here in terms of my research methodology, which developed further in Phase 3. The notion of *noticing in the moment* (a terminology introduced by John Mason – cf Mason, 1988; Davis et al, 1989) also gained significance as a device to foster teacher professional development. These ideas are developed further in Chapter 8

Significance

Bound up in the considerations above were questions about my attribution of significance to events and issues, both as I saw them *in the moment*, and as they appeared in subsequent analysis. It was in grappling with issues of significance that I finally began to make sense of the contradictions in

striving for objectivity in relation to the constructivist nature of my research. I include, in Chapter 4, a piece of writing which I did to express my thinking on significance at that time, as this was of major importance to subsequent work and thinking, and closely related to questions of methodology.

Significance became the subject of a seminar which I gave, after my analysis of Phase 2 data and before the Phase 3 field work, in order to articulate my own thoughts and to seek responses from colleagues. The exercise was valuable because it disciplined my own thinking as well as seeking other views. I reinterpreted what I saw myself doing as seeking the *essence* of a teacher's *teaching*. The following data item comes from my notes for this seminar.

'Essence' of teaching

In watching a teacher and a class over a period you start to notice things which happen regularly - build up an awareness of the teacher's style, make conjectures about why certain things happen, raise questions.

Of course I was able to talk *with* the teacher about what happened in the classroom, about her intentions, listen to her reflecting on what occurred.

An intuitive picture of that teacher emerges - you gain a vivid *sense* of that mathematics teacher.

The problem then is to try to communicate the *essences* of this teaching.

Data item B.4: Notes for a seminar (22.3.88)

I saw it being in this seeking for essence, that significance became crucial. I tried to invite seminar participants to work with me on notions of significance, by showing them brief classroom excerpts on video-tape, and inviting their own interpretations. I believe that I was asking implicitly for help with what significance might mean, and although I did not gain any fresh insights, there was some reassurance in others taking up the issue and exploring similar questions to those which I had raised.

One comment was, "What I noticed was what was significant. I can't be aware of what doesn't strike me.". Another person spoke of two sorts of 'striking' - the first where you simply notice something; the second where you notice something which is 'another example of ...'. I recognised that I

had reflected on notions similar to both of these remarks, and had a strong sense of resonance with what was uttered.

I recognise now that this communication with others, and consequent reassurance, contributed the beginnings of a modified methodology, based on an emergent epistemology, in which I tried not to be objective, but to support interpretations wherever possible while building on the strength of these interpretations. This strength lay in making explicit for myself the basis of my attribution of significance, and working on its communication.

Thus, rather than trying to say what *was* the case in any classroom, I should say what *I saw*, on what such interpretation was based and how what I saw related to a constructivist framework. The communication would depend on the construal of others of what I should offer. Thus my perceptions of different levels of construal extended to the research community, and the furthering of knowledge at this level became just another version of communicative sharing of individual constructions, as expressed in Chapter 2. These ideas were embryonic as I moved into Phase 3, and developed during the collecting and analysis of the Phase 3 data.

Implications for Phase 3

The strands of thinking which have contributed to my presentation in Chapter 7 of analysis from Phase 3, are complex. Firstly, and perhaps most simply, there is a progression in my work, across the three phases, with regard to the developing experience of my chosen teachers in terms of working investigatively. In Phase 1 the teachers were just beginning to set up investigational work, and had previously worked mainly in an expository style. In Phase 2, the chosen teachers were already experienced in working in a way which I regarded as investigative, although I do not think either of them described it to me in this way. In Phase 3, I chose to work with a teacher, Ben, because of his declared aim to put into practice an investigative style of teaching. Thus, in Phase 3, I expected to be able to talk with the teacher about what he saw as being investigative, because this language was explicit between us.

Secondly, there is my attempt to characterise, that is to describe and classify, what I observed in the various classrooms. Each teacher had very particular ways of thinking and operating, and it was my aim to distil from what I saw of their operation, characteristics typical of an investigative

style. The teaching triad, arising from Clare, being strengthened by its espousal in practice by Ben, and subsequently verified by its use in describing Mike, became central as a device to describe and present aspects of the practice of teaching investigatively. When I began Phase 3, the *teaching triad* was still very tentative, indeed it had not been unified under this name, consisting still of three separate categories with links between them. A major contribution of the Phase 3 work was to validate this triad.

Thirdly my study of investigative teaching became embedded in a constructivist theory of knowledge and learning. Although I continued to regard what I saw in classrooms in terms of *investigative teaching*, I was nevertheless developing a sense of how the classroom situations which seemed to be of significance related to this constructivist theory. I came to see this in terms of what sense pupils were making of the mathematics which they were offered, and how teachers could gain access to their construal. Thus, at some point I wished formally to link my observations to constructivism. During Phase 2, my own perception of constructivism was only starting to develop and it was difficult simultaneously to make overt links with my classroom observations. However, during Phase 3, I began more explicitly to see practical situations in constructivist terms, and during subsequent analysis of Phase 3 the links became more overt. I have thus decided to present Phase 3 in a way which draws links between classroom episodes, their description in terms of the teaching triad, and a constructivist theoretical base.

I hope by doing this to complete a story in which intuitive notions of an investigative approach to teaching and learning mathematics are embedded in theory in a constructivist philosophy of knowledge and learning; in which classroom observation leads to a means of characterising teaching approaches which I have regarded as investigative in style and recognition of those which are not; and in which this means of characterising allows aspects of the practice of teaching to be linked ultimately to a constructivist philosophy of knowledge and learning.

The following diagram aims to show this progression. The links between theory and practice may be seen to form the basis of a *constructivist pedagogy*. It is a question at this stage whether such a term is meaningful.

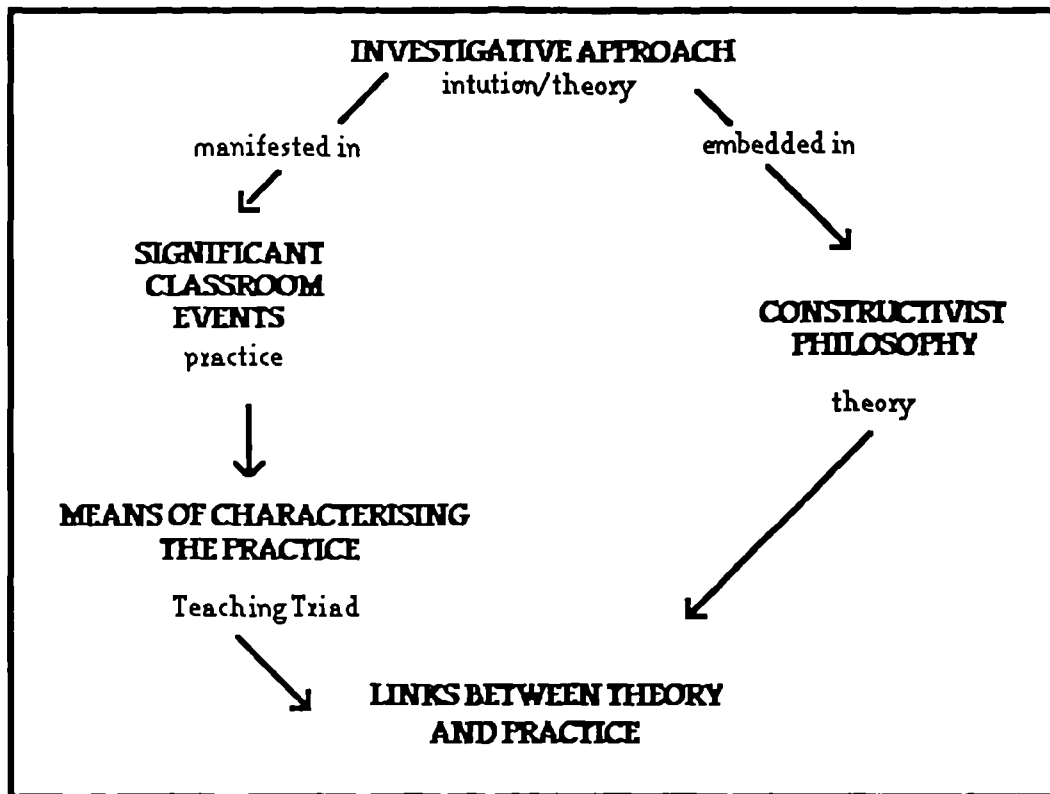


Figure B.1: The forging of links between theory and practice

CHAPTER 7

THE PHASE THREE RESEARCH

Testing and validation of emergent theory and its rationalisation with a constructivist theoretical base

Introduction

Phase 3 constitutes the second part of my main study in attempting to characterise an investigative approach to teaching mathematics within a philosophy of radical constructivism. It builds on constructs and issues which developed in Phase 2.

I chose to observe Ben because he was an experienced teacher with a declared aim to implement in his classroom an investigative approach to teaching and learning mathematics. He was Head of Mathematics at Compton, a small secondary modern school in the Midlands. Previously he had been an ESG advisory teacher¹ in the Midlands, and we had jointly run a course for teachers on “*An investigative approach to teaching and learning mathematics*”. I therefore believed that we had significant common vocabulary. I also knew him to be *reflective*². Thus I believed that I could expect discussions about an investigative approach related to the classroom practice in which he was engaged.

It was his second year as Head of Mathematics in this school. Before Ben had arrived, mathematics teaching had been mainly expository³ in style. Ben had introduced an investigative approach in his own classes, and one other teacher, Simon, claimed also to be trying to implement such an approach. Year groups in the school were set for mathematics, but there was nevertheless a considerable range of ability in any class.

Ben had developed a scheme of work for mathematics lessons at all levels which was not based on any published text, but such texts were used by teachers for various purposes at different levels. I observed Ben teaching a fourth year class, which he had himself chosen for observation because he

¹ A member of a team of advisory teachers on temporary contracts in a UK nation-wide initiative funded by an Education Support Grant.

² Taking Locke's definition – ‘the ability of the mind to observe its own operations’ (von Glasersfeld, 1987a)

³ See Brissenden (1980) – e.g. p 44.

had already been teaching them for a year and felt they were getting used to his way of working. There was no split between 'published scheme' and 'classwork' lessons as there had been at both Amberley and Beacham. All lessons were set up by the teacher himself, and in comparison with the other two schools they were all classwork lessons.

This meant that all pupils worked from the same starting point, but flexibility within the structure of lessons meant that they could diverge in emphasis once a particular activity had begun. Occasionally, lessons were labelled 'coursework' lessons, and in these pupils developed some area of work into an extended piece of coursework for GCSE purposes.

I gained no impressions of whether this pattern extended to other teachers' lessons within the school. Simon invited me to observe some of his lessons, and the class which I saw most was a fifth year class. They were the year least likely to be affected by changes to the department, having been in the school considerably longer than Ben himself, and this could have been one of the factors influencing what I observed in their lessons as I shall discuss later.

Methodology

DATA COLLECTION

The manner of data collection in Ben's lessons was very similar to that in the later part of Phase 2. I was a participant observer in the classrooms. I used audio recorders in lessons from the start, usually one carried by the teacher and often another placed with a particular group of pupils. I tried not to be seen as a teacher figure, but one of my chief aims, methodologically, got in the way of this. There were often occasions when, in analysing a particular Phase 2 interaction, I had felt that the student's view of it would have been invaluable. I was therefore determined that, in Phase 3, I should talk to students about what occurred in lessons much more frequently than I had previously.

In trying to achieve this, it was inevitable that I became closer to the pupils in these lessons than in the Beacham lessons, and so, in some respects, they came to see me as a teacher figure and would ask me questions about their work if the teacher did not seem immediately available. I could have resisted these questions, but usually I did not – partly because I wanted to maintain a good relationship with pupils, and partly because I wanted to learn from the interactions.

As I was also less concerned about objectivity, realising its impossibility and ultimately its decreased relevance to my study, I realised that there was much to gain from the interactions with pupils, so long as I did not pretend at any stage to be more than an interested and mathematically aware observer. I could thus choose what to respond to, and would often indicate that certain of the questions they asked me were outside my scope and refer them to the teacher.

The pupils in Ben's class were very welcoming, and most seemed to appreciate my interest. Many were more than willing to talk to me, either of their own initiative or in response to queries from me. However, I believe they were quick to learn what interested me, and Ben and I questioned sometimes whether particular responses were what the pupils thought I wanted to hear.

There were occasions when the teacher, some pupils and I talked together during or after a lesson spontaneously as a result of certain classroom interactions. Certain pupils sought to be the focus of attention, welcoming an audio-recorder at their group table.

Data collection from, and subsequent analysis of, Simon's lessons occurred under very different circumstances and I shall discuss this later.

DATA ANALYSIS

I undertook analysis of the Phase 3 data from Ben in much the same way as I had initially that from Clare after Phase 2. Thus I scanned transcripts trying to categorise what I found significant. However, by this time the teaching triad was established and much of what I was seeing became expressed in terms of the teaching triad.

Triangulation here was a more integral part of analysis than in Phase 2, as remarks from pupils formed a more regular part of data collection. However, ironically, I came to realise that despite having many pupil remarks, there were still many occasions when I did *not* have them where they might have been of use. I had noted in analysis of Mike's lessons that *data was not always orderly or well behaved*. It became clear that the lack of data which I should have liked regarding pupils' responses to certain episodes was really no different from the lack of data in other respects.

I could not hope to record everything which occurred, which included participants' views. In fact, there would inevitably be more gaps than continuity. I had to work from what I had, and could not expect more.

In attempting now to present a report of this analysis, I realise that it was much more manageable than at the Phase 2 stage. I had become more confident in recognising relationships between significant events and categorising as a result of this, which was undoubtedly influenced by categorisation in Phase 2.

The teaching triad was both an invaluable help in characterising the teaching which I had observed, and an unavoidable influence. The issues inherent in the teaching were clearer and more easily identified in relation to those which had arisen in Phase 2, and my awareness of the relationship between a constructivist theory and what I was seeing as significant was more overt.

It is important to realise that, in Phase 3, I was building on the Phase 2 experience. Although my principal aim at the time was not *theory-building* – I wished to characterise Ben's teaching just as I had wished to characterise that of Clare and Mike, without pre-conceptions – it was impossible to avoid the influence of Phase 2. I also wished to seek validation of constructs arising from Phase 2, in particular the teaching triad. Thus, there was some methodological tension in both characterising without pre-conception and validating constructs. I referred to this briefly in Chapter 4 (see pp 68 and 82).

In the Phase 3 analysis, there was very little redundancy in the data, so I had an overwhelming number of events presenting manifestations of aspects of an investigative approach. There were always nuances of difference between these manifestations, about which I could make relevant observations, and it was thus even harder to select those to present here than it had been in the previous phases.

The Study of Ben's Teaching

LESSONS FROM WHICH DATA WERE COLLECTED

I began to observe Ben teaching his fourth year class in September 1988, and continued observations until March 1989. I observed seventeen lessons in total, the chronology of which is given in the following table.

Autumn Term 1988	Spring Term 1989
1 Histograms	10 Coursework I
2 Statistics	11 Coursework II
3 Kathy Shapes	12 Sine and Cosine
4 Kathy Triangles	13 Continuing trigonometry
5 Graphs of Kathy Shapes	14 Height of the school
6 Moving Squares	15 $\frac{x}{y} = \frac{x+y}{y+2x}$
7 Vectors	16 Formulae
8 Surface Area	17 Reporting Back
9 Incorrect exam answers	

Figure 7.1: Lessons observed with Ben

THE TEACHING TRIAD

An important focus of my reporting of Phase 2 was the inception and validation of the teaching triad as a device for characterising the teaching which I observed. I make the assumption now that the device is an established means of talking about teaching situations, a part of the *vocabulary* of teaching. Before going further, I shall justify its use here and clarify its particular emphasis, by referring to Ben's own declared view of the triad.

During observation of the first five lessons, the teaching triad was not mentioned. However, after lesson 6, the *moving squares* lesson, I decided to offer Ben my language of the triad, saying little more than that I had found the three 'headings' useful in describing teaching which I had seen in other classes, and wondered if those headings might mean anything to him in terms of his own teaching.

His immediate response was, "I feel that management of learning *is* my job as a teacher. I think that those (referring to SS and MC) are a part of management of learning. As a teacher, that's my role in the classroom – as opposed to managing knowledge."

Subsequently, in our discussion before lesson 7, the *vectors* lesson, he produced a piece of paper on which he had jotted some notes under each of the three headings. I have reproduced this as faithfully as possible in Data item 7.1.

Management of Learning – as opposed to management of knowledge?

I like to be a manager of learning –

my role : – organiser of activity or questions
 chairperson
 devil's advocate
 challenger
 listener
 learner
 making pupils aware of other pupils

I am not a judge

Sensitivity – feelings – threat (need for success)

(everyone should be able to *start* the activity)
 success breeds success

choosing the activity level of difficulty chosen by the pupil – not today!

to the needs of 30 pupils – what a challenge.

Sensitivity to pupils by pupils my role

Mathematical Challenge – everywhere from the teacher good

from the pupils – and it takes off!

But how do we get there?

Data item 7.1: Notes written by Ben (November 1988)

I believe that Ben's account accords very closely with much that I have written in Chapter 6 of the triad as it emerged from Phase 2. In fact, although the material of Chapter 6 fits chronologically with my initial view of the triad before I presented it to Ben, it was hard, in re-analysing Mike's teaching, to avoid my perception of the triad which evolved during my work with Ben. I came to view the triad from Ben's perspective with SS and MC being closely related to each other under the umbrella of ML. The following diagram represents this view.

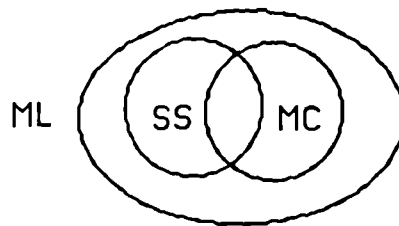


Figure 7.2: Ben's view of the teaching triad

In this chapter it will be from this image that I speak of Ben's teaching.

DIDACTIC VERSUS INVESTIGATIVE TEACHING

I chose to work with Ben because it was his declared aim to use an investigative approach to his teaching of mathematics. However, on a number of occasions, he referred to his teaching as being 'more didactic than usual'. It was in exploring what he meant by the term 'didactic' that I gained insight to important issues for Ben himself where an investigative approach was concerned, and came to be clearer, myself, about links between constructivism and classroom manifestations of an investigative approach.

I shall begin with a conversation which took place before the *vectors* lesson. Ben was talking about his plans for the lesson, and seemed to be apologising because he felt it would be less investigative in spirit than he would like, or I would expect. My degree of influence here is curiously in question. I tried to make clear that my purpose was not to judge what I saw, but sincerely to find out as much as possible about what motivated it and what effects it had. Yet, because I was overtly exploring the characteristics of investigative teaching, Ben may have felt some need to justify what he did in terms of what he expected that I should regard as 'investigative'. There seemed to be some way in which he did not regard what he was about to do in the vectors lesson as *investigative*. He referred to it as *didactic*, and other than his normal style. The same day, I had observed Ben cover for another teacher in a fifth year probability lesson. A question had been raised concerning certain formulae relating to probability. This is referred to in the conversation between Ben and myself from which the following situation is taken.

“Very didactic”

- (1) Ben Very didactic, I’ve got to say, compared to my normal style. But we’ll see what comes out. There’s still a way of working though, isn’t there?
- BJ That’s something that I would like to follow up because you say it almost apologetically.
- Ben Yeah, cos I /
Yeah, I do. Erm
We’re back to this management of learning, aren’t we?
- BJ Are we?
- (5) Ben Can I read what I put here? (Referring to his written words on ML in Data item 7.1 above) I put here, “I like to be a manager of learning as opposed to a manager of knowledge”, and I suppose that’s what I mean by didactic – giving the knowledge out.
- BJ Mm. What does, ‘giving the knowledge’ mean, or imply?
- Ben Sharing my knowledge with people. I’m not sure you can share knowledge. Mathematical knowledge is something you have to fit into your own mathematical model. I’ve told you about what I feel mathematics is?
- BJ Go on.
- (9) Ben I feel in my head I have a system of mathematics. I don’t know what it looks like but it’s there, and whenever I learn a new bit of mathematics I have to find somewhere that that fits in. It might not just fit in one place, it might actually connect up a lot of places as well. When I share things it’s very difficult because I can’t actually share my mathematical model or whatever you want to call it, because that’s special to me. It’s special to me because of my experiences. So, I suppose I’m not a giver of knowledge because I like to let people fit their knowledge into their model because only then does it make sense to them. Maybe that’s why if you actually say, ‘Well probability is easy. It’s just this over this.’, it doesn’t make sense because it’s got nowhere to fit. That’s what I feel didactic teaching is a lot about, isn’t it? Giving this knowledge, sharing your knowledge with people, which is not possible?

Data item 7.2: Extract (1) from transcript of discussion with Ben (23.11.88)

Ben seemed to be saying that if we offer probability to pupils as simply a formula, “this over this” it is likely to have little meaning for pupils because they have no means of ‘fitting’ it into their experience. Ben’s statement uses ‘fit’ in the von Glasersfeld sense (see Chapter 2), and statement 9 seems as clear an articulation of a constructivist philosophy as I would be likely to find, ‘off the cuff’ in a discussion of what a lesson was going to be about. However, Ben and I had never discussed constructivism. Could it be that Ben was a successful practitioner in working consistently in an investigative style *because* he had a philosophy so akin to constructivism? If so, what did he mean by the term ‘didactic’?

Two aspects of the above conversation stands out.:

1. the result of my probing at statements 4 and 6. This caused Ben to define what he meant by *didactic* teaching – ‘giving knowledge out’ – in Chapter 2 terms, ‘a *transmission* view of teaching’. His words seemed to deny both a transmission view, and also some absolute view of knowledge, indicating a relationship between the building of knowledge and a person’s past experience.
2. the third sentence in statement 1, “There’s still a way of working though, isn’t there?” What I understood by this was that despite using a didactic approach, he believed there might nevertheless be a way of working which would fit with constructivist views.

I pushed harder towards what I saw as being a fundamental tension – *the didactic/constructivist tension*, of didactic approach versus constructivist philosophy. The conversation continues from that in data item 7.2.

“A conjecture which I agree with”

- | | | |
|-----|-----|--|
| (1) | BJ | I’m going to push you by choosing an example. <i>Pythagoras</i> keeps popping up, and <i>Pythagoras</i> is something that you want all the kids in your group to know about. Now, in a sense there’s some knowledge there that’s referred to by the term ‘ <i>Pythagoras</i> ’. And, I could pin you down even further to say what it is, you know, what <i>is</i> this thing called <i>Pythagoras</i> that you want them to know about? |
| | Ben | My kids have made a conjecture about <i>Pythagoras</i> which I agree with. So, it’s not my knowledge. It’s their knowledge. |

- BJ How did they come to that?
- Ben Because I set up a set of activities leading in that direction.
- (5) BJ Right, now what if they'd never got to what you class as being Pythagoras? Is it important enough to pursue it in some other way if they never actually get there?
- Ben Yeah.
- BJ What other ways are there of doing that?
- He laughed and then continued.*
- Ben / You're talking in the abstract which then becomes difficult, aren't you now? Because we're not talking about particular classes or particular groups of pupils etc. Because I've always found in a group of pupils if I've given them an activity to lead somewhere there are some pupils who got there. It sounds horrible that. *Came up with a conjecture which is going to be useful for the future if I got there, yes?* And then you can start sharing it because pupils can then relate it to their experiences.
- BJ So, it's alright for them to share with each other, but not alright for you to share with them?
- 10 Ben If I share with them I've got to be careful because I've got to share what I know *within those experiences*.
- BJ OK. So, if we come back to didactic teaching then, if you feel they're at a stage that you can fit – whatever it is that you want them to know about – into their experience, isn't it then alright? You know, take the probability example this morning. If you felt ...
- (12) Ben That is nearly a definition, isn't it? That is, I suppose that's one area I'm still sorting out in my own mind.
Because things like \overrightarrow{AB} and *vector* is a definition. What work do you do up to that definition?

Data item 7.3: Extract (2) from transcript of discussion with Ben (23.11.88)

The tension seemed to be between having some particular knowledge which he wanted pupils to gain, and the belief that he could not give them the knowledge. The above conversation seemed to summarise his pedagogical approach – the presentation of activities through which the pupils could construct knowledge, and his monitoring of this construction,

“My kids have made a conjecture about Pythagoras which I agree with. So, it’s not my knowledge. It’s their knowledge.” Implicit in this is his need to know about their construction, to gain access to their construal, “a conjecture about Pythagoras which I agree with”. Pupils have to be able to express their thoughts in a coherent way for the teacher to make this assessment, so he has to manage the learning situation to encourage such expression. In Data item 7.2, he distinguished between being a *manager of learning* and a *manager of knowledge* (statement 5). In Data item 7.3, statement 12, he referred to a *definition*. The probability example involved a definition, as did the notion of vector and its representation as \overrightarrow{AB} . His, “I’m still sorting out in my own mind” seemed to refer to the status of a definition in terms of knowledge conveyance or construction, and indeed the nature of knowledge itself. There seemed to be some sense in which you could only *give* a definition. If this is the case, what preparation needs to be done so that the pupil is able to fit that definition meaningfully into their own experience? Here again is the *teacher’s dilemma* (Edwards & Mercer, 1987) and it is closely allied to the *didactic/constructivist tension*. Pupils construct their own meanings; the teacher offers something from which they can construct. There is some concept which the teacher needs to elicit or to inculcate. However, inculcation is likely to result in lack of meaning, and eliciting of what the teacher wants may never occur. (For further discussion of these ideas, see Jaworski, 1989.)

I was keen to explore what *didactic* teaching meant for Ben, as opposed to *investigative*; and also what preparation was necessary in order for didactic teaching to be successful in terms of pupils’ conceptual construal. In the next two sections I shall look at the *moving squares* lesson as an example of an investigative approach, and then at the *vectors* lesson as an example of a didactic approach as identified by Ben. I hope to show some overt differences, but other subtle and arguably important similarities, with the aim of clarifying the didactic/constructivist tension as it applied in Ben’s teaching. The teaching triad will be used to characterise features of the lessons

THE MOVING SQUARES LESSON

1: CREATING AN ENVIRONMENT FOR THINKING AND INVOLVEMENT

Management, of Learning, Sensitivity to Students and Mathematical Challenge all played important roles in the creation of an environment at the start of this lesson. I shall indicate Ben’s management role in setting up the activity, encouraging pupils to start thinking, managing the whole

class sharing of initial ideas, and launching the class into further work and thinking. Part of this management role involves levels of sensitivity in enabling all pupils to make a start, and particular pupils to offer their thinking as part of the whole class discussion;. It also encompasses mathematical challenge in offering questions which stimulate thinking and promote mathematical activity.

The activity which Ben had planned for this lesson involved consideration of a square grid on which an object at one corner had to be moved so that it ended up in the empty space at the opposite corner. The intervening spaces were filled with objects which could be moved only into an empty adjacent square.

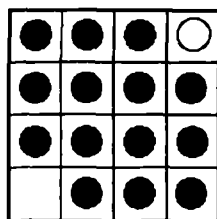


Figure 7.3:
Moving Squares

Part of Ben's planning involved creating opportunity for development of pupils' ability to make decisions in setting their own parameters for problem solving. He set this in the context of investigational work in a style with which the group was familiar. The following is an extract from Ben's words in our discussion before this lesson.

Planning for the Moving Squares lesson

Ben I'm going to get them to draw the square, give the counters out, tell them the rule, and we'll see how many moves it takes to get from one to the other, and we'll keep a record on the board / and hopefully at some point I will say / "what's the minimum number of moves?", because obviously you can do any number you want, above the minimum, and at that point / I don't know what I'll say – I think it'll depend on what occurs, 'cause I can say to that group "Now *investigate!*", ... or I might need to be more specific by saying "what happens to *other sizes?*" There's one or two not used to it, and I think I'll go round to them and say, you know, "what happens to other size squares?", limit it for them a bit. // But, I don't like limiting it – generally. I like to let people be free. ...

I expect a rule problem to come up, 'cause I'm going to use the words "move to adjacent squares" and that will then lead to 'what about diagonals?' and I think that's something the group's got to decide.

Data item 7.4: Extract from discussion with Ben (9.11.88)

Ben had declared (see Data item 7.1) that an aspect of SS was that “everyone should be able to start the activity”. The first few lines above indicate a task in which everyone hopefully could engage – moving counters across a square grid. Then he said, “I don’t know what I’ll say – I think it’ll depend on what occurs”, from which he indicated various possibilities from the very open, ‘investigate’ to questions which would limit the situation for pupils who needed more support. He had expectations of what *might* occur. For example, he anticipated that pupils would ask about diagonal moves and hoped to use such a question to provide experience for pupils in making their own decisions about how to proceed. When I asked him whether he *wanted* them to use diagonal moves or not he replied, “I don’t mind – either way can lead to interesting conjectures”.

I found his planning here typical of many of his lessons. He tried to set up an activity clearly and unambiguously so that everyone could get involved. The activity had to have plenty of scope for varying inclinations and levels of thinking so that pupils could progress in different directions and to differing depths depending on personal characteristics. His planning was open in allowing for the ways pupils responded and the particular needs which he discerned. However, there were certain criteria which he determined to fulfil, about which there was little scope for compromise – in this case the decision-making. In this lesson, ‘decision making’ was part of the lesson *content*. There was little overt *mathematical* content. A challenge was to respond to the task and to pursue whatever mathematics arose. This embodied high cognitive demand as analysis will show.

2: THE LESSON OPENING

In the classroom at the start of the lesson he engaged the class’s attention, and said:

The object of the game – I call it a game is to move *that* different coloured counter (See fig. 7.3 – top right) to here (bottom left). Obviously it’s very easy, as someone said, you can just pick it up and *put* it there, and if we did that there wouldn’t seem anything to do. So the rule I’m going to put on this – just *one* rule – and that is you can only move a counter to an *adjacent empty square*. (Ben, 9.11.88)

After the words, ‘adjacent empty square’, there was a buzz of activity in the room. Pupils started making squares of counters, and then moving the counters. There was a cacophony of voices with comments, questions and suggestions from individuals. He had been successful in engaging attention and getting pupils involved. The result of the involvement was

potential classroom chaos – everyone speaking at once, asking questions, demanding the teacher’s attention. He could not respond to everyone. He made eye contact, listened to various remarks without any comment, then picked on a question he wanted to pursue. Someone had asked, “Can you move diagonally?”, and he responded to this addressing his comments to the whole class.

“Can we move diagonally?”

- (1) Ben Can we move diagonally? I think we’ve got to decide on that.
 Ps No No
 Ben Who says no then?
 P If you do it diagonally you can do it in 19.
 (5) Ben Who says yes you can move them diagonally?
 P Me
 Ps No, no. No you can’t.
 (8) Ben Can people put their cases ...

Data item 7.5: Extract (1) from transcript of *Moving Squares* lesson (9.11.88)

There were many replies to the question at statement 8 – pupils talking to each other or raising voices to make the teacher hear. Ben responded to particular replies but it was impossible to hear or respond to every pupil. The vociferous response made demands on his management. He wanted pupils to think and express their thoughts, which they were doing. However, he also wanted them to listen to each other which demanded order and quiet. He compromised by allowing moments of hubbub where energy was expressed and released, then demanding order:

“You’re the teacher, aren’t you?”

- (1) Ben Could we just stop for a moment please, can everyone just stop moving a sec. I know it’s addictive [‘it’ refers to moving the counters on the square] can you just stop. I think we need to decide the rules – otherwise you’re giving me all these numbers won’t mean anything will it. Now I had a couple of people saying why they think it should *not* be a diagonal – anyone like to say why it *should* be a diagonal? /// How are we gonna decide?
 P Well let’s stick to the rules.

P If you're allowed to do it diagonally, its gonna be less, you'll have less moves.

There were various comments from pupils to which the teacher responded in a fairly non-committal way, without implying any judgements. The whole class seemed to be engaged in the thinking – some arguing the point together, others directing their comments to the teacher. Then,

(5) Tony Why don't you say one and tell us to do it?

Ben Sorry?

Ton We're going to be here all day – just say diagonal or not diagonal.

Ben That's passing responsibility onto me and not ..

Ton Does it really *matter*? You're the *teacher* aren't you.

(10) Ben Well I had a vote and only about 6 people took part last time

Ps I'll vote it. I'll vote it.

P ...look up adjacent and see if it means you're allowed to go diagonal and ... take it from there.

Ben Sorry, I didn't hear you.

P Get a dictionary, look up adjacent, and if it says you can – it's diagonal or just to the sides, then you know.

(15) Ben Got a dictionary?

P No

P Shall I go and get one?

P No Catherine'll have one. (many voices)

Ben Can we just stop – I don't know what's up with us this afternoon. We're not giving other people a chance to talk. Sorry Nicole ..

(20) Nic If you move it diagonally – it's not ...(hard to hear)

Ben You think if we move diagonally its going to be too easy?

Ton Does it matter whether we move diagonally or not?

	P	Yes
	Ben	Does it matter?
(25)	Ps	Yes. Yes.
	P	It's going to be less moves if you're allowed to do it diagonal. (Many voices)
(27)	P	It's less complicated.

Data item 7.6: Extract (2) from transcript of *Moving Squares* lesson (9.11.88)

I particularly noticed the remarks made between statements 5 and 9 above. The pupil's words and tone of voice indicated to me a frustration which seemed to make demands on the teacher. It was a serious conscientious boy, Tony, who made the remark at statement 5, and I felt it was not intended to be flippant or disruptive. Ben indicated, at statement 8, that he was not going to accept responsibility for the decision, and despite the pressure from Tony, he continued to receive other pupils' comments. Some pupils appeared to disagree with Tony. Many argued for or against diagonal moves and gave the impression of believing that the decision *did* matter for them. I saw interesting levels of focus here. Some pupils were bound up in the diagonals decision. Tony, and maybe others, wanted the decision out of the way, possibly not caring much which way it went. The teacher wanted to focus on the importance of pupils making decisions themselves. The teacher was aware of the different levels of focus, as became clear in our discussions later, but it is likely that most of the pupils were not.

It is possible to see Tony's response as having been made in order to define the objectives more tightly, i.e. to get the teacher to be more explicit about what he required and thus reduce cognitive demand, as Doyle (1986) and others have pointed out. My personal knowledge of Tony, arising from my observations, suggests that it was not the case here. Tony always seemed more than willing to rise to cognitive demand. Significant here, I felt, was that he did not rate *this* decision as being very demanding.

When we talked after the lesson, Ben identified what might be a conflict between some of his own intentions. He had said, as I quoted earlier, that he did not like *limiting* the pupils' exploration, that he liked them to be *free*. However, he recognised that in homing in on the question about

diagonal moves, he had in fact been focusing their attention in a way which might have limited their freedom.

Freedom v Control

Ben There is a conflict there – I don't think its a yes or no conflict – it's a sort of grey conflict – at certain times certain ideas have priority, certain concepts ..

When they brought up the diagonal moves – and I picked on that – and I actually got the whole class together – I'm controlling the direction at that moment – I'm controlling that direction because I think that people should have freedom – which is a complete contradiction. What I'm actually saying is you've got freedom and its not my control – I'm trying to let go by saying you as a group can make that decision –

Interesting that isn't it, because I actually gained control then gave it back again.

Data item 7.7: Extract from transcript of discussion after *Moving Squares* lesson (9.11.88)

The issue of control is strongly manifested here. What is the teacher controlling and why? Of what is he relinquishing control? Ben had a number of objectives for the lesson, including the content of the lesson in terms of mathematical activity, including aspects of working mathematically which he wanted pupils to develop; including requirements of GCSE coursework which he wanted to fulfil. In respect of this he made many decisions in the lesson, one of which has been explored above. It is difficult to identify the effect that these decisions had on the pupils and on their learning, and indeed even more difficult to predict what different effects would have been manifest as a result of different decisions. However, the teaching intentions are more recognisable. In what I have discussed above, ML is extremely overt. The teacher set out to create an opportunity for pupils to engage in mathematical thinking. He encouraged initial exploration followed by discussion of the task, aimed at clarifying what was possible. Part of his agenda was the making of decisions by pupils, and he was determined not to make the diagonals decision for them.

I was interested in Tony's reaction, and I asked Ben how he had felt about the boy's response, "you're the teacher, aren't you". Ben replied:

... one of my philosophies is that pupils should have choice. Either I haven't put that over, or he feels he would prefer to pass the responsibility onto me. So it doesn't worry me. Also I feel that frustration is a part of life. I mean we have to learn to live with it. (Ben, 9.11.88)

I saw an element of sensitivity here, the recognition that this pupil could cope with the frustration and possibly also learn from it. I asked Ben's permission to talk with Tony, to gain access to some of his perceptions by asking him about the incident.

I asked Tony to tell me, "anything at all about what you were thinking then".

The highest authority in the classroom

- (1) Tony I was just thinking the place of the teacher ought to be above a pupil, you know and instruct, not totally in everything, he ought to give you some freedom, not give such a choice, it just got as though we were going through, I don't know how to put it really ..
- BJ Just try.
- Tony I just felt we were going on for quite a long time wasting time, and then so I just thought that Mr. West was the highest authority in the classroom so I thought that he might as well tell us.
- BJ Why do you think he didn't?
- (5) Tony Because he likes to give us more freedom and // to / I know why, I think it was just so that we could be more independent, so that we could learn for ourselves
- BJ And how did you feel about it at the time?
- Tony I didn't really mind.
- BJ You didn't?
- Tony No. I just wanted to, it was all set and I could start from there. I didn't mind either way. But I think it needed set rules, and they have to be set by someone.
- (10) BJ Could that someone have been you?
- (11) Tony Yeah, but it would have been easier if the whole class was doing the same thing so that you could compare notes at the end. But, if just a pupil stood up and said the rules have got to be that, ... the rest of the class wouldn't have accepted it. But if Mr. West said you've got to do that, they would have. That's about it.

Data item 7.8: Extract from discussion with Tony (9.11.88)

I was particularly struck by Tony's words at statement 5, referring to freedom, independence and 'learning for ourselves'. He seemed to share my perception of the teacher's philosophy, despite acknowledging his preference that the teacher should have made the decision. The teacher's response above might seem insensitive to Tony's immediate needs, but might also be seen as catering for his longer term needs. It indicated to me that ML involves potentially painful moments and decisions for the teacher, which require a strong motivational philosophy. This might be related to higher level cognitive demand.

The teacher's management of learning here seemed to include overt sensitivity to the pupils concerned. Those who were bound up in the diagonals decision perhaps needed the teacher's consideration more than others, like Tony, who could see through it. If the decision mattered for them, then perhaps they had to be encouraged to take it themselves. Others had to be encouraged to take part in discussion. At statement 19 Ben said, "Can we just stop – I don't know what's up with us this afternoon. We're not giving other people a chance to talk. Sorry Nicole ..." Nicole was one of the higher attaining pupils in the class, but was very softly spoken and fairly diffident. She would not push herself forward over the more vociferous characters. Ben said on one occasion that he had to be aware enough to make space for people like Nicole, and for girls particularly, to contribute. It would be too easy for them to be swamped and therefore to stop trying to offer ideas. *Listening* was something which he emphasised continually.

To summarise, I saw this lesson-opening creating a basis for pupils' construal in

- 1 allowing everyone to make a start on the activity; feel comfortable with what the activity was about; feel freedom to explore in whatever directions seemed interesting;
- 2 building awareness of making decisions; negotiating with others who see things differently; being sensitive to the needs of others to contribute and to express what they think.

3: WORKING ON THE TASK

An important aspect of Ben's management of learning was the way in which his classroom was set up physically, and the way in which pupils worked together supporting each other. The creation of groups and the ways in which groups were encouraged to work seemed an important

factor contributory to the ethos of his classroom. The development of this ethos occurred over a period of time and was not therefore particular to any one lesson.

However, each lesson contributed to the building of the ethos. The main substance of any of Ben's lessons involved pupils grouped around tables, working both independently and cooperatively. Sometimes this working involved a group of from two to six pupils working very closely, discussing ideas together. Sometimes Ben overtly encouraged particular forms of group working. For example he encouraged a group to work collaboratively on one occasion:

What happens if you look at a bigger one then, or a smaller one? And there might be some pattern between those two numbers, yes? If you all do the same it's a waste of time isn't it? So can you get yourselves organised? (Ben, 28.9.88)

I shall refer to one group of six pupils in the *moving squares* lesson, as an example of the type of work which ensued as a result of a lesson opening such as the one above.

In this lesson I sat with these pupils, quietly observing their activity, an audio-recorder on the table at which they worked. They paid me no obvious attention except when I addressed remarks to them towards the end of the lesson

They began individually by making squares, counting moves and writing down results. Most worked systematically on a number of special cases, going back from the initial example of 4 by 4, to 2 by 2 and 3 by 3, and on through 5 by 5, and 6 by 6. There was no audible agreement here as to who should do what. They seemed to decide for themselves what to do, do it, and then compare results. Of course, the activity of the others around them may have influenced the way some pupils tackled the task.

The group sharing was at the informal level of discussing results and looking over to see what results another person had got. During this activity, aspects of their work were shared.

For example in the situations below, one pupil, Lesley, had just articulated some of her thinking and another, Jenny, tried to understand what she had said.

Two by two equals five

- (1) Jen Lesley, what are you on about?
- Les Look, four times four is sixteen, right? But, the answer to the counters is twenty-one. So, to make it up to twenty-one, sixteen plus five is twenty-one.
- Jen Right. So two by two equals five, [According to the counters', she seems to imply]
- Les Yes
- (5) Jen Minus 1, yes. [She seems to imply, 'two times two is four. So you have to take 1 away from 5 to get 4']
- Les But, it's one to make it up. But when you get to these, seven times seven is forty-nine, whereas that's forty-five, so you need – you have to –
- Jen So there's not a pattern there.
- (8) Les No

Data item 7.9: Extract (3) from transcript of *Moving Squares* lesson (9.11.88)

Because I was sitting with the group, conversations such as this made sense to me, as they clearly did to the participants. My interpretations in brackets are based on this local sense-making. However, reading the transcript later, the words uttered are minimalist in so far as they are no more than enough for each contributor to understand the other. Although this conversation might appear strange to the reader, it did seem to make sense. This is an example of the incidence of *prolepsis* (see Chapter 2 page 20) Jenny and Lesley gave every indication of knowing what each meant, and, partly to their thinking to some extent, I felt I could make a reasonable interpretation of their words. This *reasonable interpretation* is my way of *constructing a set of assumptions in order to make sense of the utterance*.

Lesley had tabulated her results as follows – I have put in the headings to clarify the situation for the reader.

Square	Minimum moves
2 by 2	5
3 by 3	13
4 by 4	21
5 by 5	29
6 by 6	37

She seemed to be inspecting the difference between the minimum number of moves, and the area of the particular square, and not finding a pattern. In thinking further, she modified her table as follows, again *I* have added the headings.

Square	Minimum moves	Difference
2 by 2	5	8
3 by 3	13	8
4 by 4	21	8
5 by 5	29	8
6 by 6	37	

At this point Lesley called Ben over and said, "I've put all the numbers in order, and there's 8 between each of them". She showed him that she had tabulated results systematically and she conjectured that the 7 by 7 case should be 45. He replied, "Now the question I would ask – why is there 8 more when you increase the square by one? Because if we can sort out why there's 8 more always, won't we have solved the whole thing?"

This seemed to incorporate a high degree of MC, yet it seemed to me to be appropriate to the stage Lesley was at. Because I was sitting with the group listening to what they said, observing their activity and seeing what they wrote, I felt I gained a good sense of their construal. Ben had been moving around the room talking with different groups, so he had not seen as much of this group as I had. He was yet able to offer what appeared to be an entirely appropriate challenge. On various occasions when I questioned the basis of certain decisions he indicated that they depended greatly on his knowledge of the group.

For example, once, when contrasting planning with spontaneity he said, "So that's one interpretation isn't it, that Ben sticks slavishly to his planning and won't be pushed off. Or is it the other way of looking at it, that Ben knows his group fairly well and can fairly predict their reactions?⁴". On another occasion he said, "That's the real role of a teacher, isn't it – knowing your pupils and knowing when you can throw ideas at them?"

⁴ Here, Ben offers alternative construals, much as Clare did – see pp 153 and 159

I was aware that the classroom ethos owed little to chance, and I saw Ben working on particular aspects of classroom ethos overtly, such as the importance of listening to others as with Nicole in Data item 7.6. During my observation of the above group, a boy, Colin, said that he had a *prediction* for the 5 by 5 case, and Lesley replied, “Don’t you mean a *conjecture*? In maths it’s a *conjecture*!”

Mathematical language was something which Ben strove to develop and I knew that ‘conjecture’ was one of his words, so it was interesting to hear Lesley’s emphasis of this. In response to Colin’s ‘conjecture’ another pupil, Pat, said, “Don’t tell us yet”, but Colin responded immediately with, “29”!

At the point where Lesley was reminding Colin about the language of *conjecture*, Pat and Julie were having a conversation about Colin’s unfairness in ‘*spoiling their fun*’. Ben said, later, that they had picked up his language here too, as he often urged that they consider whether they might spoil another person’s fun by telling them an answer or a result. This emphasised to me that aspects of Ben’s philosophy, through his ML, had observable impact on his pupils, and I found this comparable with aspects of Mike’s ‘control’, and Clare’s ‘training’ (see Chapter 6).

A little later Colin said, “Are you sure that 6 by 6 is 37, because that was my conjecture?” Having been alerted to the mathematical term, he was now using it, which emphasises how an ethos is propagated through social interaction. The pupils in the group were now overtly working together, sharing results and checking each other’s results. The larger squares were more difficult to check so other reasoning came into play. Someone said, “7 times 7 is 49. That’s 4 more than 45”, again trying to relate the square of the side to the least number of moves. Then Colin said something which sparked off a most significant conversation where I was concerned.

Teacher knows the answer		
(1)	Col	Mr West ... got the answer.
	Jen	He hasn’t, has he?
	Col	Yeah
	Jen	No-o. Because all the patterns we do, he’s never actually told us the right answer.
(5)	Col	He does

- | | |
|----------|---|
| Jen | He doesn't . |
| Col | Course he does. |
| Jen | I've never heard him ... |
| Col | He doesn't <i>tell</i> you – but of course he's got all the answers |
| (10) Les | How do <i>you</i> know? |
| (11) Col | Cause he always has |

Data item 7.10: Extract (4) from transcript of Moving Squares lesson (9.11.88)

They went on to discuss some formula which had come up as a result of work in an earlier lesson, and asked who had been responsible for the introduction of this formula. Colin claimed that it had been Ben, or that at least Ben had known the formula. Others said that they had reached that formula themselves. It was interesting to contrast Colin's view with that of the others, and to compare it with responses from Mike's students when we discussed with them whether Mike always had the answers. (see Chapter 6, p.182) I believe that neither Mike nor Ben encouraged this epistemological standpoint, but for some pupils it was nevertheless deeply ingrained.

Lesley had progressed to thinking about moving counters on *rectangular* grids. She called Ben over and asked if a 2 by 1 was allowed, or whether 'the two numbers had to be the same'. He replied, "I didn't make any restrictions, did I?" He could have asked her to stay with square grids for the time being, but did not. I felt that the thinking about rectangular grids complicated the situation, when the pupils had not yet really generalised the square grid case, and, had I been the teacher, I might have tried to constrain the situation for them at that point. However, this was yet another example of Ben's trust in pupils' ability to come to terms with their own thinking. Perhaps he felt that Lesley could cope with the wider scenario and that she herself would rationalise the situation.

There was suddenly a hiatus in the activity, where not much seemed to be happening, as if the energy had drained and everyone was taking breathing space. I took the opportunity to ask them a question about how they perceived their current stage of work and thinking.

Pupils' views of *investigating*

Statements 1–9 came rapidly and together:

- (1) BJ Can I ask you all something? ... Could you say where you think you're *at*, at the moment with regard to the investigation?
- Col We haven't really started yet. We're really nowhere near starting.
- Pat We have started it ...
- Les As we go on we keep finding new things to ...
- (5) Jen We've started on the even sided shapes, things like 2 by 2 ...
- Les But not the odds. I'm just starting to do ...
- Pat Yes, we've done 3 by 3 , 5 by 5...
- Jen No! Like different sides, 1 by 2 , like that, rectangles ...
- Col What we've got to do now. We know what to do.

It was hard to hear what anyone was saying as they all spoke together, eager to say what they thought. I asked if it was possible to speak one at a time. Colin said,

- (10) Col For a hundred, to work that out, you have to find out 2 plus 8 plus 8 plus 8 – till you get to a hundred. What we've got to do is find a formula, so that you can just get to a hundred straight off.
- Jen Or without adding nine on.
- Col Without adding eight on, yes. So that's what we're aiming towards first of all. Then we can work it out on the other ones, like 2 by 1.

I asked if what they were doing was related to things which they had done before. They said it was, and I asked how.

- Jen Changing the sides and all that ... not the actual moving of the shapes but changing of the –
- Les – the lengths and the sides

(15) Jen Like on billiard tables⁵.

Les Yes

I asked if they were using particular strategies that they knew about.

Les In other investigations we changed, like on the billiard table one, we had to see how many bounces it was ...

Pat Yes but first of all you've got to find the solution to both the same lengths and then you can move on to ...

Jen Yes, we have to find a formula for ..

And later,

(20) Pat The different sides – what you've got to do is just find the next, because once you find out a four by four and a five by five like that, you find a formula for that then you can go onto like three by five or five by seven. But until you find these out, with even sides, well you're just going to get totally confused if you go onto the other ones.

Data item 7.11: Extract (5) from transcript of *Moving Squares* lesson (9.11.88)

Their articulation of what they were doing was, not surprisingly, rough and imprecise, yet it captured well aspects of a way of working which I had seen Ben foster. For example, Ben frequently referred to the importance of looking for patterns, making conjectures, and expressing generality, and their use of these processes was implicit in the pupils' reporting. They related what they were doing in this problem to processes that they had used in others. They talked about extending a problem, and indicated a need to finish one level satisfactorily before proceeding to the next.

They sought an expression of the generality which they perceived in terms of a *formula*. I asked how they went about finding the formula, whether they had particular strategies, or whether it was trial and error. Some said it was trial and error, but others said it was using "knowledge" – "what you learn". They tried to tell me what they meant by 'knowledge', but their expression of what they understood was difficult and came across as knowing how to make a table, to add and multiply and so on. Pat said that there were lots of questions, and when I pushed her on this she said, "Once you ask yourself one question it leads to another question." Despite

⁵ The 'Billiards' investigation, described in Chapter 6, is well-known and used.

their inexperience in expressing perceptions of their own learning, and my own inability to help them do it, I gained a strong sense of their awareness of what mathematical problem-solving was about. What I heard seemed to fit well with Ben's philosophy, and I felt once again that it provided evidence of his management having its desired effect on pupils.

As well as being impressed with pupils' approaches to thinking, the mathematical nature of this thinking seemed to me to be of a high quality. In Data item 7.11, statements 10, 11 and 12 seemed particularly revealing mathematically. At 10, Colin indicated the desirability of a *formula* to avoid having to work out all special cases leading to the one you wanted, for example, a hundred, or ten by ten square. Jenny (at 11) concurred with this, indicating that she understood his statement by her remark, "Or without adding nine on". I believe that prolepsis led to misunderstanding here where Colin was concerned. I conjecture that Jenny, in thinking of the ten by ten case, realised that it was possible to get to the *ten by ten* by adding on something to the *nine by nine*. However jointly, they seemed to have a clear view of the value of generalising and the expression of such generality. When I listened carefully to pupils' conversations, as I had here, I gained frequent evidence of their involvement in high level mathematical thinking of this sort. This contradicts suggestions made by Desforges and Cockburn (1987), to which I referred in Chapter 3, about the impossibility of teachers' encouraging such high level thinking within the prevailing school system.

It seemed to be this level of thinking which was the ultimate achievement of the classroom ethos which was created. In learning to work well together, and appropriate ways of tackling mathematical problems, pupils achieved a basis from which a high level of thinking could emerge. However, not all pupils responded positively to the shared environment. Whereas Jenny and Lesley, in Data item 7.9, had seemingly understood each other well, Colin seemed not to be tuned in to Jenny, at statement 12 in Data item 7.11 above. In fact there was evidence that Colin did not listen to others very well at all. Ben had some problems in fitting him into groups in the class as he was not very willing to play a group role. I saw Ben's coping with this as a combination of ML and SS, managing the situation through trying to do his best for the pupils concerned, Colin included. This often involved remonstrative remarks to Colin about ways of working cooperatively. However, it also involved, on occasion, allowing Colin to work alone or with Tony, who also found difficulty in working with others. Ben felt that Colin had much to gain from the different working situations.

The teacher's need to gain access to pupil construal (cf the *fruit metaphor*, p 42) raises questions related to the interactions described above. In Data item 7.11, there seemed to be inconsistent use of the terms 'even' and 'odd' (e.g. statements 5 to 8). Had I been the teacher, hearing this articulation by pupils would have alerted me to explore pupils' understanding of these terms. The pupils themselves did not push their different interpretations further, although Pat may have inferred something of Jenny and Lesley's usage, leading to her own use of 'even' at statement 20. I also pointed out earlier the possible difference in meaning between Jenny and Colin in statements 11 and 12. The *prolepsis* implicit in these utterances is a natural feature of conversation between individuals leading to development of shared meaning, as I pointed out in Chapter 2. Encouraging pupil-pupil conversations allows meaning to be explored, and provides the teacher with access to such meanings. Yet there is also evidence of misunderstanding or proliferation of possible misconception. Could pupils usefully be *alerted* to differences in meaning? Could *prolepsis* be made more explicit? These questions raise issues related to discourse analysis (see for example Sinclair and Coulthard, 1975; Edwards and Westgate, 1987), but they seem also to be pertinent to the teacher working from a constructivist philosophy in terms of making use of pupil-pupil discussion.

4: AN INVESTIGATIVE LESSON – WHY?

Finally, with regard to the *moving squares* lesson, whose initial selection I made as an example of one lesson which Ben had regarded as investigative, what was investigative about it?

In the first place, the task which was set might in current common parlance be regarded as 'an investigation'. Like 'billiards', discussed in the Mike analysis of Chapter 6, and referred to by Ben's pupils who had also worked on it, 'moving squares' would be recognised by many teachers as an *investigation* which they might offer pupils in a mathematics lesson. It had no particular, required, mathematical content, (such as *area*, or *equations*, or *fractions*). *Algebraic symbolism* might have been expected, but no *one* mathematical outcome was sought. Certain processes were important to the conducting of the investigation, for example *pattern spotting*, *conjecturing*, and *generalising*. Pupils were encouraged to justify conjectures. Different directions could be pursued, for example Lesley's group started to look at *rectangular grids*, but this was not common to other groups in the class.

It might be asked which of these characteristics were present in other so-called investigative lessons, or perhaps more crucially which were not? Other lessons which Ben regarded as being investigative were *Kathy shapes*⁶ and $\frac{x}{y} = \frac{x+y}{y+2x}$, (see figure 7.1) Both of these had very explicit mathematical content. In *Kathy shapes* pupils had to seek shapes whose area was numerically equal to perimeter. In undertaking the task they came up against properties of various shapes, methods of calculating area, and ultimately Pythagoras theorem. In $\frac{x}{y} = \frac{x+y}{y+2x}$, pupils had to substitute values into an algebraic formula and to work on sequences of fractions and decimals. The tasks of both of these lessons might have been regarded as investigations, but they were investigations involving the manipulation of mathematical objects, which *moving squares* and *billiards* were not.

Pupils were free to follow their own directions and develop their own thinking. Challenges from the teacher came mainly in response to the pupils' own positions, for example his response to Lesley, quoted earlier: "Now the question I would ask – why is there 8 more when you increase the square by one? Because if we can sort out why there's 8 more always, won't we have solved the whole thing?" This would not necessarily have made sense to another pupil who was proceeding in a different direction. However, there were expectations about ways of working and mathematical processes. Often when the teacher emphasised aspects of process, pupils were able readily to respond. On one occasion Ben asked a pupil what question he was just about to ask her, and without pause she replied, "Is there a pattern?". Like Mike in encouraging *the asking of questions* (Chapter 6 page 161), for Ben, encouraging *the seeking of patterns* was an example of *cued strategy*. In each of these cases the process was a part of classroom rubric and thus of pupils' experience. Both teachers actively encouraged the developing of experience and expectation to accommodate styles of thinking and learning which they wished to foster in pupils.

THE VECTORS LESSON

The *vectors* lesson differed in two important ways from the *moving squares* lesson. It was the second lesson of the series so Ben was not initiating the topic as he had been with moving squares. One influence which this had was that instead of some initiating activity, the initial stages of the lesson involved 'recap' of ideas from the previous lesson. The

⁶ A description of this lesson is provided in Appendix 5.

second major difference was less circumstantial, and more a consequence of Ben's own perceptions of the topic on which the lesson was to be based, i.e. *vectors*. The topic of the *moving squares* lesson had been described by Ben himself as an *investigation*. However, *vectors* seemed to lie firmly within *mathematical content*, and Ben saw the need for a difference in approach because of this. Briefly, at this stage, the difference might be seen in terms of the ground rules for the two topics. In *moving squares* there was only one ground rule, and pupils were explicitly encouraged to make decisions about setting their own conditions in deciding what to explore. Where *vectors* was concerned there were many more ground rules which needed to be established and understood. Perhaps for the teacher there was a significant difference between conventions which he could set himself, and established (mathematical) conventions to which he needed to induct pupils.

1: THE LESSON OPENING

The lesson began with the teacher's words, inviting pupils to 'recap' on previous work:

... last Monday we started with a thing called vectors ... could we just sort of recap on what we were doing and see how far we can get? Could we think please, instead of doing a lot of talking. / Come on Tony! (To Tony who was not attending) / (To the class again) I introduced you to *that*. Could anyone explain what that object is? (Ben, 23.11.88)

He had written on the board \overrightarrow{AB} . Nicole responded but it was very hard to hear what she said as many other voices were interjecting. Ben remonstrated,

Now, hang on! We're forgetting the first principle, that is to listen to other people. Nicole said something and people were talking. We need to listen please. Nicole, would you like to repeat it a bit louder please? (Ben, 23.11.88)

Nicole replied that it was "the journey from point A to point B". Ben repeated, "the journey from point A to point B". There were still voices interjecting, which he was trying to contain. He acknowledged another pupil's attempt to enter the discussion – "Pat, you were saying something ...". Pat's response made reference to a grid, and Ben asked, "Do you want to draw it?" She came out to the board and proceeded to draw lines of a grid. Conversations continued momentarily, but when she started to speak the class mainly listened to her. She had drawn some vertical and horizontal lines, and put on two points A and B, as shown

below. Then she traced out a number of paths between A and B, for example, across one, up one, across one, up one; and up two and across two; as she said,.

“Right, you’ve got a grid, right? It’s a square, well it’s a grid, yes? And you’ve got to find, you’ve got that point there, and you’re gonna get from that point there, A, to B, yeah? // Or you can go all the way up and across, and that’s it!” (Pat, 23.11.88)

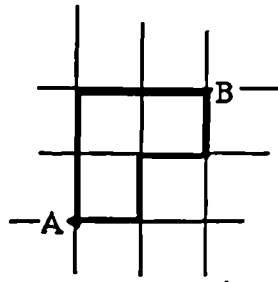


Figure 7.4: Pat’s diagram

At this point there were many loud interjections as others in the class commented or asked questions. It was hard to distinguish remarks, but many of the class were actively and loudly involved in expressing ideas. Ben came in to control contributions. As I saw more of his operation I realised that his tolerance of certain periods of noisy energy release was actually important to the flow of the lesson. When he asked for quiet he usually got it fairly quickly.

Drawing a vector

- (1) Ben Hey, one at a time, come on!
- Ps You could go up like that. You’re going the wrong way. ... arrow shows direction.
- Ben The arrow on the top shows direction, yes. What else do you want? No one has told me the name of this object.
- P Vector
- (5) Ben Vector // On here, where’s vector AB? Can someone actually draw in vector AB?

A pupil offered to draw, and was invited to do so.

- Ben Come on then. (Pupil draws from A to B – see Figure 7.5a below) OK, so that's your vector AB.. How would I actually describe that ... the one that Pat has actually drawn me as vector AB?
- P Two two. [He writes this as in Figure 7.5b below]
- Ben What does the first two tell me?
- P Two along
- (10) Ben Two along. And the other, the second two?
- P Two up.
- (12) Ben So that's the vector AB – it's two along and two up (he traces out a path on Pat's diagram on the board).
- Actually, we very often do it *that* way (he traces out another path – see Figure 7.5c below) two along, two up. That's two ways to do it.

Data item 7.12: Extract (1) from transcript of *vectors* lesson (23.11.88)

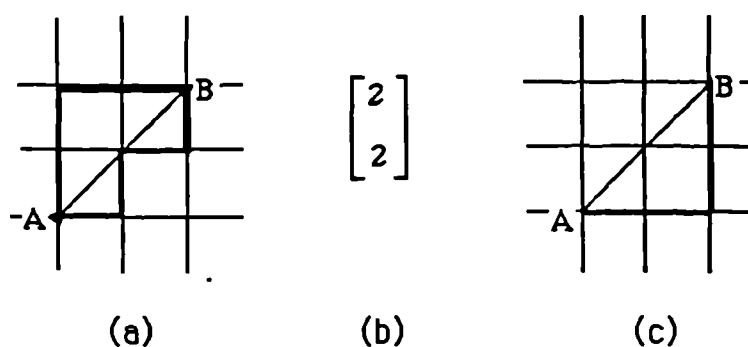


Figure 7.5: Drawing a vector

Here the second difference between the two lessons is exemplified. There were certain 'facts' about vectors which needed common currency. They had been introduced previously, and the above discussion, focused by Ben, encouraged pupils to recall what they already knew and understood. In providing pupils with opportunity to express what they think and encouraging them to do this, it is likely that misconceptions, that is conceptions that do not fit established mathematical convention, might emerge which can then be addressed explicitly. The above approach gave Ben the chance to re-emphasise rules which he considered to be important. This emphasising of rules on his part, is what I believe he meant by *didactic* style. In the investigation of *moving squares*, there was no need

for him to engage in this didactic mode, because he genuinely wanted pupils to set their own rules, and refused to do this for them. He seemed more comfortable with this, as if it accorded more strongly with his beliefs. Yet he saw *necessity* for the didactic style where vectors was concerned. However, I see, in both cases, elements of the teacher's belief and motivation being unequivocally addressed. In *moving squares* there was no compromise over *diagonal decisions*. In *vectors*, no compromise could be made where *conventions of vectors* were concerned – certain aspects of vector representation and definition needed to be established as *common knowledge* (Edwards and Mercer, 1987), for example, the meaning of $\vec{3AB}$, of \vec{BA} and of the difference between \vec{AB} and AB .

Rather than emphasising a polarity between the two types of lesson, I now want to focus on similarities. In what had occurred so far in this lesson, pupils took an active part. The teacher controlled the direction of the lesson, but he did this no less in the *moving squares* lesson. In both cases he had a particular agenda and well considered objectives.

He continued by asking what they thought $2\vec{AB}$ might mean. Responses included, "From A to B and from B to A", "AB, AB", "AB to AB", "Two times AB". To one of them Ben said, "Show us." and Colin came to draw on the board,

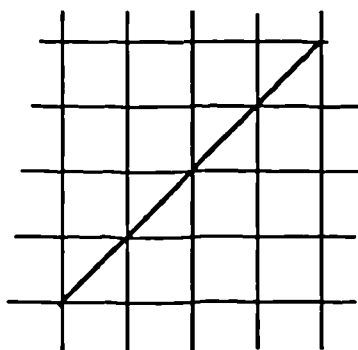


Figure 7.6: Colin's drawing

Nicole, who had said, "Two times AB", explained with help from other girls around her, that if you multiplied (2,2) by 2 you got (4,4), and something else (I did not hear what it was) gave you (6,6). Ben asked a boy who did not appear to be attending, "Luke do you agree – how do you get (6,6)?" After a couple of false starts, Luke expressed it as, "It's AB plus 2AB." I was quite impressed by this, as it was a different way of expressing $3\vec{AB}$ to all that had been offered so far, and I expected Ben to

take this up and emphasise it. However, he went back to the girls' representation without further comment to Luke. When I asked Ben about this later, he said that his question to Luke had been because he was not sure of Luke's attention and he was satisfying himself that Luke was in fact thinking. Luke's response more than adequately showed that he was, but the novelty of his offering was not something which Ben wanted to emphasise at that stage. The thinking of the class seemed very delicately balanced between those like Luke who had a clear conceptual understanding of $3\vec{AB}$, and those who were still struggling to interpret it at all.

There were many similarities with the *moving squares* lesson. Luke did not appear to need the continued discussion about $2\vec{AB}$ and $3\vec{AB}$, just as Tony did not need to argue about whether there should be diagonal moves or not. Ben focused his attention on the pupils who were struggling to interpret what was involved. In this case it was a group of girls who bombarded him with questions about aspects of vectors which they did not understand. In *moving squares* it was pupils who disagreed about the value of diagonal moves. In both cases Ben encouraged the pupils to ask their questions and express their ideas.

There seemed here to be a manifestation of Ben's earlier remark, "There's a need for success, otherwise Maths is very threatening. Everyone should be able to start the activity". I felt that he was here trying to ensure that everyone had at least reached some conceptual level with vectors at which they could start work on the task he would set.

When three vectors of (2,2) were drawn in succession to produce a vector of (6,6) someone asked in a puzzled tone, "Where's B on there, though?", and Ben replied, "That's a good question, I don't know the answer to that!" People were clearly struggling with the various notations and their compatibility. If \vec{AB} represented a journey from A to B, what journey was represented by $3\vec{AB}$ and in particular where was B?! The atmosphere of the lesson, in which pupils were encouraged to air their thoughts and worries, allowed such conceptual difficulties to emerge rather than be suppressed, and allowed the teacher to indicate that he did not have ready answers to everything that they might ask. After further discussion where pupils offered suggestions Ben said:

There's the vector AB. There's another vector AB, and there's another vector AB. (He points to the diagram, figure 7.7 below) And so I've got three vectors. Three lots of AB. ...

I think at some point we have to go away from this idea that a vector is a journey from A to B, to a point where a vector is – this line, this quantity. And to think of that as the vector AB, as that line, not necessarily a journey. (Ben, 23.11.88)

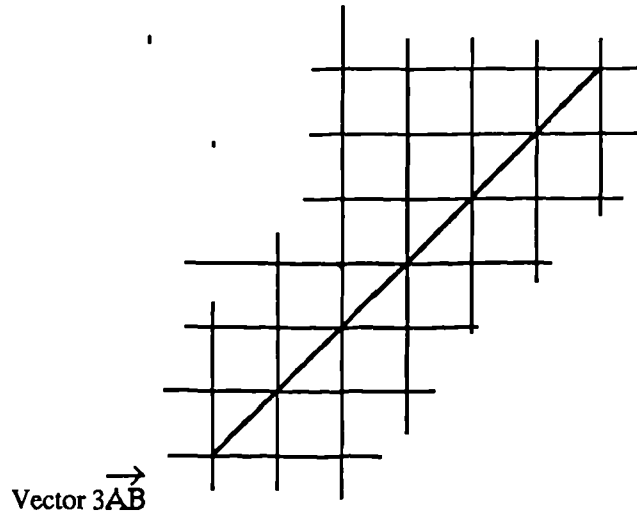


Figure 7.7: Ben's diagram

Much of this opening was spent in establishing meaning. As people contributed ideas, meanings developed. At various points Ben took opportunity to express *his* meaning, as in the words just above. This might be regarded as teacher *exposition*, and perhaps a part of what Ben regarded as the *didactic* nature of the lesson. However, pupils were quite ready to question Ben's exposition. At one point a girl said, "I just don't know what we're doing!", and Ben replied, "Can I come and help you in a minute, when other people are busy?" There did not seem to be any sense of simply accepting the teacher's meaning. I felt a dynamic urge in the class to sort out the meanings for themselves. Ben seemed to feel obliged at one point to qualify the rules which he felt were being imposed. He said,

"We're just talking about the ways a mathematician writes things down, yes? We're not learning anything really new. Those two are different. (\overrightarrow{AB} and \overrightarrow{BA}) Those two are the same. (AB and BA). You tend to write the first one down because they're in alphabetical order, and we rarely write down BA, yes? We're just talking about what mathematicians write." (Ben 23.11.88)

An important sameness in the two lesson openings, as far as I was concerned, was the way in which the majority of the class were actively involved in the thinking, and while Ben was quite prepared to focus and offer his perspective, there was a feeling of freedom for each person to

contribute to ask a question or to express an opinion about the mathematics. This freedom necessarily carried penalties. It was possible for someone to be inattentive, to opt out or disengage, or to focus on something other than the mathematical context of the lesson and this be unnoticed in the general *melée*. Ben usually picked up on occurrences of this sort however, as with Luke. It was often extremely noisy, and Ben's chairing role involved overt remonstrance with regard to pupils taking turns and listening to each other. When the discussion was orderly, he was frequently the mediator of remarks, since it was hard to have a genuine discussion between individuals in a group of 32 without the conversation getting out of hand. It could be said that in this he controlled the direction of discussion, but it was hard to see how it might have been otherwise.

The lesson opening, which had been quite lengthy, concluded with considerations of the length of a vector and how one might find this length. One pupil said, "draw it and measure it.", to which Ben replied, "measure it, great!". But he went on to be more precise about what he actually wanted, "How could you *calculate* the length of the line? I agree with your first answer, yes, but I've changed the question now"! This was twofold in its significance. Ben had certain requirements of the lesson about which he was not prepared to compromise. A pupil came up with a legitimate interpretation of Ben's remarks, but not what Ben wanted. Thus he 'changed the question', acknowledging that the pupil's response was respected while making clear that he wanted a particular approach.

Finally, Ben set the task for the rest of the lesson,

"What I would like you to do please / is copy what you need from the board and then / wait a minute – before you start – just listen to the rest! Then, I'm not going to put any questions on the board. I would like you to make your own questions up and write your own answers out and then share your questions with a neighbour. Could you be inventive please. Don't put up a whole series of boring questions – could you sort of try and choose them. // Does anyone here not know exactly what they've got to do?" (Ben, 23.11.88)

2: WORKING ON THE TASK

Implicit in Ben's instruction seemed to be that the task involved vectors and their lengths. It felt investigative in nature – make up your own questions and write your own answers. The task required pupils to appreciate the generality of lengths of vectors, which might not have been necessary if Ben had simply provided a list of vectors himself of which pupils were to find the length. Pupils' responses were interesting. There

were some who had not understood, and Ben had to repeat his instructions for them. There were others who would not or could not invent their own questions, and pressured Ben to do it for them, perhaps trying to reduce the cognitive demand.

I shall provide three examples in this section which show different aspects of the class working on the set task, and the opportunities for pupil construal which can arise from an activity of this sort.

The first occurred in response to Ben's final question above, "Does anyone here not know exactly what they've got to do?" Various people said that they did not. He went on, "OK. Could you make a question of your own up and then pass your question on to your neighbour. A little later a pupil asked, "Is that alright Mr West? Is that the sort of question you want?", and he replied, "Yes, you work out the answer, then pass your question on and see if you get the same answer."

Later, another pupil said, "I don't know what question to ask", and he replied, "Well, a simple question is, find the length of the vector AB there, isn't it? Yes? To go past that, you need to be a bit more inventive, yes? But do that as a start. It's a good starter isn't it?"

It might be asked, why he did not just set them some questions himself, rather than leaving pupils in such an uncertain state. For some of them, he had to come very close to stating their questions anyway. One reason is to do with challenging them to think about what they were doing, rather than just mechanically responding to given questions with a prepared technique. Another, rather more subtle, is to do with the constraint which the teacher's own questions could impose. There were particular concepts which he wanted pupils to grasp. His questions could have been tailored to bring pupils up against these concepts. But could he guarantee that the pupils would actually see in them what he saw himself in setting them up?

My second example, of what happened in the case of one pupil, Mandy, to some extent vindicated his strategy. The next situation contains the conversation between Mandy and Ben, later in the lesson.

Making questions more interesting

(1) Man All my questions come to the same answer. I've got all the same coordinates.

Ben What do you mean, the same coordinates?

Man Look, I've done these right, and then that one's the same and that one is.

Ben Can you see why. Is there a reason why?

(5) Man Because they're the same triangle.

Ben ... the same triangle?

Man Because I just put them out anywhere, right? And then I sort of put coordinates with them, and I didn't look at the size of the triangles they come from, but they're all the same. So shall I just carry on because some of them are different?

Ben Some are different and some are the same. Maybe you've got another question. How can you predict which will be the same?

They went on to inspect two that were the same. He asked what was special about them, and she replied that they were both three by one. They looked at diagrams of the vectors. One of them was (3,-1).

Man Does it make any difference to that *Pythagoras*? It doesn't does it? Because I'm not doing vectors, I'm just doing their lengths. It doesn't make any difference does it? It doesn't make any difference because it won't change the length of the line will it?

(10) Ben Good, that's good thinking, yes? You had to think about that though, hang on a sec [to another pupil], you had to think about that didn't you?

They talked further about the vectors being different, but the lengths being the same. Ben again asked why, and Mandy said something about them all being 3 by 1. Ben said,

Ben So can you ask which – how can we sort of make that into a question? I don't want to do it for you. You've noticed something, yes? When you notice things you can very often make it into a question, can't you?

She tentatively tried, "What others are the same?", and he asked her if she felt she understood what he meant, to which she said, "yes".

(12) Ben So you've got a new question haven't you? Which is a bit more interesting I think than saying "Find the length of those lines", Yes? That's what's nice about questions, twisting it round to make it interesting.

I did not observe this interchange. It was recorded on the recorder which the teacher carried while I observed other pupils, so I cannot give details about the particular vectors on which Mandy was working. However, the force of the teacher's intervention seems to be independent of these particulars. From her own starting point, Mandy had noticed aspects which were the same and others which were different. By urging her to reframe her questions the teacher seemed to indicate that there were patterns which she could observe which might lead to general principles. He did not guide the substance of her work, but did push her quite strongly in terms of her approach to it. This seemed a very significant case of MC. Mandy's particular examples were very meaningful to her, thus she could be pushed to generalise from them in a way which might not have been possible if the teacher had provided the examples. It is interesting to compare this with Skemp's (1971) view of providing examples to enable concept development. Here the pupil provided the examples and the teacher worked with her on how she might use them. The teacher seems to be tackling the learning paradox (Bereiter, 1985) from a 'higher level' than Skemp suggests.

If Ben had said what he wanted from pupils in setting their own questions, he might have described something like the interchange above. Although many of the pupils did not go beyond finding lengths of vectors which they invented, others like Mandy, did find further relationships on which to remark. One boy for instance noticed that two of his vectors were parallel, and as a result of this started to look for others which might be parallel, and thus approached generalities for parallelism.

For quite a few pupils the use of negative numbers in vectors caused some problems. My third example concerns Luke and Danny, with whom I was sitting, whom I saw begin to tackle the task. Luke explained to Danny what he thought they had to do. He wrote down the vector \overrightarrow{AB} , as below, placed points A and B on a grid, drew the triangle around them, drew squares on two sides of the triangle as shown below.

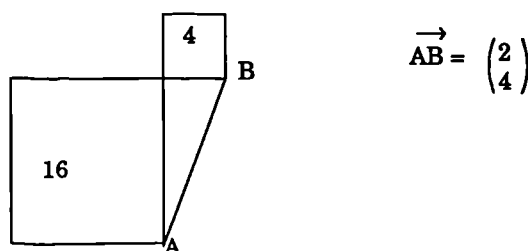


Figure 7.8: Luke's explanation to Danny

He wrote the square numbers in the squares, and then worked out mentally aloud: “16 plus 4, that’s 20; square root // about 4.5”. Danny seemed to follow what he had done, and the pair set about independently inventing vectors and finding lengths. In each case, Luke drew a diagram similar to the one above, writing the square numbers into the squares. He then performed the calculation mentally and wrote down the result.

This might have seemed unremarkable, except that in a fairly recent previous lesson, Luke had been struggling with the application of Pythagoras’ theorem, and Ben had remarked at the end of the lesson that Luke had not really grasped the Pythagorean concept. Apparently no significant work had been done by the class on Pythagoras in the meantime, but here was Luke appearing to be quite fluent in its use. Denvir and Brown (1986) point to a similar occurrence in their work with young children, which showed significantly enhanced performance in a delayed post-test to that in the immediate post-test. They suggest “It does seem likely that the improved performance was on skills in which the teaching had provided more ‘relational understanding’ (Skemp, 1976) which not only made it possible to remember the new skills which they had acquired, but also to build on and extend their new knowledge”. I suggest that Luke had similarly developed a good relational understanding of Pythagoras’ Theorem. This was reinforced by subsequent work in the group.

In the course of drawing various vectors Danny drew one with a negative slope. When they wrote down the vector as below, they were worried by the negative sign in it.

$$\begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

This led to discussion with a group at another table. Some pupils said the negative sign made no difference, others that it did. However, Luke returned to his method which I described above, drawing his squares and writing the square numbers in the squares. His argument was that if you multiply -3 by -3, you get 9. So 9 plus 4 is 13, and you just have to get the square root. Most of the others then seemed to be convinced by this. The discussion reinforced my impression that Luke now had a good understanding of the concept of Pythagoras. It was in watching his activity and listening to his words, in an activity which the teacher had created, which provided me with this insight into his construal. That /

made this observation and not the teacher is beside the point, which is to support earlier remarks that a teacher can learn much about pupils' construal just by watching and listening once an opportunity for pupil-pupil discussion has been created.

3: A DIDACTIC LESSON – WHY?

As I have indicated, I saw much that was investigative in this lesson. So why did the teacher classify it as didactic?

It had very particular mathematical content. Pupils were expected to focus on certain aspects of vectors. There were aspects of vectors which were not negotiable. Pupils had to understand what a vector was, and its many representations. This had two important aspects: (1) Considerable time was given to exploring meanings. The teacher offered his own meanings to the class in an expository style. Pupils were encouraged to say what they understood and to question when it was not clear. Different perspectives were encouraged. Helpful images were shared. (2) The teacher pointed out that what he was asking them to accept was simply "the ways a mathematician writes things down". This seemed to suggest to them that he was not dictating a truth, merely a convention, but whether the pupils gained any such sense I cannot know.

There were other lessons which Ben classified as *didactic*, the trigonometry lessons in particular. In these again there were certain non-negotiable aspects, what a sine or a cosine was, for example. However, in these lessons too, once terms and conventions had been introduced and meanings negotiated, investigative tasks were set. For example in the *sine and cosine* lesson, pupils were asked to use their calculators to key in numbers, obtain their sines and jot down both number and sine. Then Ben asked, "Has anyone found a number whose sine is zero, or 1, or more than 1? As a result of answers to such questions pupils moved again into pattern spotting and attempts at generalisation.

I perceived that the term *didactic* was used when Ben felt that information had to be conveyed which he could not approach through exploration or questioning. In statement 12 of Data item 7.3, he referred to an aspect of probability as "That is nearly a definition, isn't it". Vectors too involved definitions, as did sines and cosines. Perhaps for him, *didactic* was associated with exposition, and giving definitions, and he saw that giving definitions, although inevitable, seemed to be, in my terms, a reversion to the transmission process rather than an encouraging of active construction.

Tensions and Issues

In this section I shall focus on three tensions, or dilemmas, to which I have referred in this chapter and elsewhere earlier, namely the *teacher's dilemma*, the *didactic/constructivist tension*, and the *didactic tension*, and show how I see these linked to each other and raising issues for the mathematics teacher who chooses an investigative style of teaching. I shall begin by setting each of these into a context, either that in which they arose as part of this study, or relating to some external source which is of relevance to the study.

THE TEACHER'S DILEMMA

Edwards and Mercer (1987) refer to the teacher's dilemma as:

to have to inculcate knowledge while apparently eliciting it.

or as:

the problem of reconciling experiential, pupil-centred learning with the requirement that pupils rediscover what they are supposed to.
(p 126)

They quote Driver (1983) who, in writing of science teaching, made the following remarks:

Secondary school pupils are quick to recognise the rules of the game when they ask 'Is this what was supposed to happen?' or 'Have I got the right answer?'. The intellectual dishonesty of the approach derives from expecting two outcomes from pupils' laboratory activities which are possibly incompatible. On the one hand pupils are expected to explore a phenomenon for themselves, collect data and make inferences based on it; on the other hand this process is expected to lead to the currently accepted law or principle.

Edwards and Mercer claim that these expectations lead to pupils trying to guess from teachers' 'clues, cues, questions and presuppositions' what it is that the teacher actually wants them to know rather than truly making inferences from their own experience, and that this leads to a *ritual* form of knowledge in which pupils can provide 'right answers', but not principled explanations.

Investigating in the mathematics classroom may be seen to parallel experimenting in the science laboratory. If investigation is expected to lead to particular mathematical laws or principles, the charges laid by Driver may be as true of mathematics teaching as they are of science teaching. Edwards and Mercer report on a classroom activity in which

pupils were invited to explore the variables relating to pendulum swing, and in particular the relationship between length of string and period of swing. Although the pupils explored other variables such as the material of the string, the angle of swing, and the mass of the bob, ultimately, perhaps through the teacher cues and emphasis, these variables were put aside in favour of those which are considered to fit the pendulum 'rule'.

In Chapter 6, I reported on Mike's offering of the two activities *square sums*' and *triangle lengths* as an introduction to work on Pythagoras' theorem. There was no point during my observation of the Pythagoras lessons at which I felt that Mike 'cued' the Pythagorean relationship from any students. This is not to say that no such cues were given. Interestingly, what I did observe was that one of the students offered the relationship to his peers⁷ and these pupils reconciled it with the data which they had collected in their investigation. Their sang froid with respect to this revelation was instructive for me in leading me to question the value which we might place on pupils 'discovering for themselves'. If ultimately their knowledge of the Pythagorean result is 'principled' rather than 'ritual', in Edwards and Mercer's terms, does it actually matter *how* that knowledge was gained? I suggest that the pupils who fitted the Pythagorean result to their data could have developed a principled understanding through their bringing two sources of information together and reconciling them. That they had a principled understanding was demonstrated in future activities which I saw the class undertake, but its source is difficult to identify. What was clear to me was that they took delight in working on the revelation and convincing themselves of its truth, and this seems to be of motivational importance whereas continued investigation revealing no obvious relationship might have resulted in apathy or boredom. A question which this prompts is could a teacher capitalise on this experience by similarly offering 'the rule', or is the peer-offering a vital ingredient of the success in this episode?

Two justifications for conducting mathematical investigations in the classroom were mentioned in Chapter 1 – offering children experience of exploring mathematical ideas for themselves, and providing experience of mathematical thinking and problem solving which they can then draw on in reaching understanding of conventional concepts. The *moving squares* activity, described above, could be justified in both of these ways. The pupils I observed worked on mathematical ideas which had arisen in their exploration, and most of them reached results at some level. Their activity

⁷ This incident is described in Appendix 5.

of looking for patterns, making conjectures and seeking general formulae provided experience which is valid for exploring traditional mathematical concepts. Indeed, it is likely that this activity contributed to subsequent working on the *vectors* activity. However, no conventional results were sought from the *moving squares* activity. Results which were obtained were required by the teacher to be justified, but there was no cuing to suggest that the teacher required particular results. However, the 'rules of the game' here required that certain *processes* were of value, and the discussion of 'conjecturing' and obtaining formulae indicated that pupils were entering into this game. In the vectors task of making up questions to ask, pupils asked Ben, "Is that alright Mr West? Is that the sort of question you want?", and he replied, "Yes, you work out the answer, then pass your question on and see if you get the same answer." Could this be regarded as the 'game' of the lesson – to come up with the sort of questions which Mr West wants?

This raises questions of which games *is* it valid to play in the classroom, and which not. Driver talks of intellectual dishonesty. Is it dishonest to indulge in any classroom enterprise which could be labelled as game playing by the teacher? Ben himself expressed worries about ways in which his activity might be seen as game-playing during one of the *coursework* lessons which I discuss in Appendix 5. In conversation after the lesson, when he expressed his worries, I asked if he could give me an example of something which he felt might be regarded as 'game-playing'. His response was:

Well, when they've got a pattern, if you're not careful you can finish up saying, "Are you sure?" if you know they haven't got them all, and if they have got them all you say, "Great!" It's nearly a game isn't it that? Instead of saying you're right and wrong you're now just choosing different words to mean the same thing. ...

We've now got a new *code* ... and that's a game isn't it? It's very difficult, when people are insecure, to say, "Are you sure?" when you know they've got everything. (Ben, 11.1.89)

I am aware that there are questions here which could form the basis of further research, indeed which I could explore further myself from my current data. This is, however, beyond the scope of my present study. The issue raised concerns the relationship between the knowledge which the teacher wants the child to acquire, and the teaching practices which are employed to enable the construction of this knowledge. This links very closely to my second tension/dilemma.

THE DIDACTIC/CONSTRUCTIVIST TENSION

Ben declared his intention to employ an investigative approach to his mathematics teaching. Discussions with him led me to believe that his philosophical base for this might be regarded as constructivist. Yet, on a number of occasions he indicated, apologetically, that a lesson would be *didactic* in style – an apparent contradiction. Two features which emerged from my analysis of the so-called didactic lessons were as follows:

1. There were definitions or conventions which needed to be in common currency. Expecting pupils to discover these for themselves was unrealistic, and could have resulted in the intellectual dishonesty to which I have referred above.
2. In order to establish such definitions or conventions, the teacher became involved in an expository style of telling or explaining more frequently than might be the case in an investigation lesson.

What seems ultimately to be crucial is what sense the pupil makes of the mathematics in the lesson. If this mathematics involves knowing the rule for pendulums, then it is hoped that pupils would have a *principled* knowledge of this rule. Edwards and Mercer claimed that the pupils they observed did not, that the knowledge demonstrated was of a *ritual* form. Methods of teaching which fit a transmission view of teaching and learning may also be accused of encouraging only a ritual form of knowledge. Such methods can be seen to provide too little opportunity for pupils to make the ideas their own and to develop principled understanding. The pendulums teacher appeared to create such opportunities for her pupils, yet due to her cues, or emphasis, or whatever, a principled understanding was not achieved. There seems potentially a paradox here, which I might express as follows:

Transmission teaching – telling and explaining – results in ritual knowledge. For principled knowledge, learners need to explore ideas themselves and reach their own conclusions. These conclusions may not be the ones which convention or the teacher wants. Thus the teacher needs to interact with the learner's exploration in order to guide the learner to the appropriate conclusions. This is intellectually dishonest. Thus it would be more honest to tell the pupils what you want them to know. This is transmission teaching.

So can principled knowledge ever be achieved?

The answer to this question is clearly 'yes'. Despite the limitations of this study with regard to making observations about pupils' learning, I can point to particular indications of principled understanding. A mistake in the above logic lies in the polarised statements it contains – for example, not *all* telling and explaining results in *only* ritualised knowledge. The example of William, telling his peers about Pythagoras' theorem (see Appendix 5), in Mike's *triangle lengths* task, was a case in point. This is not to suggest that the *telling*, in and of itself, was responsible for their subsequent principled understanding, but that the telling was timely as part of their overall activity and thinking.

In Ben's *vectors* lesson, when faced with the question, "where is B in the vector $3\overrightarrow{AB}$?", Ben initially seemed at a loss to answer, and eventually responded with explanation. He was however honest about the difficulty of answering the particular question. The classroom ethos which encouraged pupils to express their ideas for others, as in Pat's and Colin's contributions, and in which asking questions was the norm, allowed this question to be raised and tackled. Seeing Ben struggling for an answer himself contributed to this ethos, as did his ultimate explanation. Other analysis of the transcript data may reveal teacher's cues or emphasis which I have not noticed. Interviews with pupils directed at exploring their knowledge of vectors may have revealed more ritual than principled knowledge resulting from the vectors lesson. However, certain pupils provided indications of thinking in a most principled way, Mandy and Luke, for example, and the boy who worked on parallel vectors.

I should like to suggest that Ben's apology for what he regarded as didactic teaching was misplaced, as was the implicit suggestion that if it was to be regarded as *didactic*, it was therefore not *investigative*. The dangers of labelling seem to be apparent here, as labels carry with them their own connotations. Teaching in a way consistent with a constructivist philosophical base does not necessarily exclude telling or explaining, just because these teaching modes are more commonly associated with a transmission form of teaching. Constructivism speaks fundamentally of construction of knowledge and modification of experience. Such construction and modification is enabled by a mode of enquiry in which expressing, sharing, and questioning of ideas is fundamental. The teacher is a powerful figure in this, and has potential for being regarded as a source of truth, so her ideas, statements or cues ought to be offered only in an overt knowledge of their potential influence. However, this does not

preclude their being offered, and it is more honest to do this openly than covertly.

The pupils in Ben's *moving squares* lesson debated Ben's prior knowledge of the formula which they had found in a particular investigation. Some felt that Ben knew the formula and expected it, whereas others felt it was their own formula, that they were not just jumping to the teacher's tune. This speaks also to the ethos of this classroom, although there were still elements of the epistemology observed in Mike's students with regard to the status of knowledge and the teacher's knowledge in particular. I spoke at different times with all three teachers, Clare, Mike and Ben, about the degree of explicitness they brought to their teaching – that is in how far they shared their objectives, intentions, and philosophy with their pupils. This leads to my third tension/dilemma.

THE DIDACTIC TENSION

The didactic tension seems to me to be fundamental to considerations of investigative teaching which I saw cropping up over and over again in working with Ben, and to which this pedagogic issue of *how explicit to be* relates strongly. The *didactic tension*, not to be confused with didactic/constructivist tension, is a term coined by John Mason in response to a phenomenon which Guy Brousseau (1984) termed the 'didactic contract'. Mason (1988b) refers to it in this way:

The didactic contract is between teacher and pupil although it may never be made explicit. The teacher's task is to foster learning, but it is the pupil who must do the learning. The pupil's task is to learn, or at least to get through the system. They wish to be told what they need to know, and often they wish to invest a minimum of energy in order to succeed. Guy Brousseau, who coined the expression, 'didactic contract' points out that it contains a paradoxical dilemma. Acceding to the pupil's perspective reduces the potential for the pupil to learn, yet the teacher's task is to establish conditions to help the pupil learn

Put another way, the more the teacher is explicit about what behaviour is wanted, the less opportunity the pupils have to come to it for themselves and make the underlying knowledge or understanding their own. (p 168)

Elsewhere (Mason, 1988c), he refers to the same issue in terms of a *didactic tension*:

The didactic tension can be summarised as:

The *more* explicit I am about the behaviour I wish my pupils to display, the more likely it is that they will display the behaviour without recourse to the understanding which the behaviour is meant to indicate; that is the more they will take the *form* for the substance.

The *less* explicit I am about my aims and expectations about the behaviour I wish my pupils to display, the less likely they are to notice what is (or might be) going on, the less likely they are to see the point, to encounter what was intended, or to realise what it was all about.
(p 33)

An example of didactic tension, arose as a result of a conversation which I had with Ben after one lesson where I had observed a group of pupils throughout the lesson, and had noticed what occurred when Ben himself came and talked with the group. I pointed out that I had found their discussion very desultory, except when Ben himself had been present. Then, discussion had become more focused, due to the teacher's questions, and pupils had made much more coherent statements about their ideas and thinking than when he had not been present. I raised the question of how a teacher can get a group to engage in productive discussion when not personally present with the group. This led to a discussion of intentions, and whether it would be valuable for the teacher to make his intentions about group discussion explicit for the group.

The tension for the teacher here lies in considering when it is appropriate to be explicit with pupils about what you want, and when it is inappropriate. He wanted discussion between pupils, when he could not be with them, to be as fruitful as discussion which might occur when he was present. One way to affect their discussion would be to talk with pupils about what *sort* of discussion he wanted. In order to make this clear for them he would have to give examples, and hope that they could generalise from his examples. However, a possibility might be that the resulting discussion would be very stereotyped and not as fruitful in learning terms as he would hope. Mike similarly grappled with the didactic tension in deciding how to handle the girls' query in Data item 6.12 (see p 168).

I observed differing degrees of explicitness in Ben's emphasis on mathematical processes. For example, *looking for patterns* was extremely explicit, and in the main seemed to be successful in terms of pupils behaving as he would have hoped. However, the process of *specialising*, that of trying out special cases when looking for patterns, seemed never actually to be made explicit. Yet pupils exhibited plenty of evidence of

using this process, for example as in the *moving squares* lesson. Ben encouraged the process implicitly by encouraging consideration of special cases when he talked with pupils. However, I realise that when I use the terms 'explicit' and 'implicit' they carry meanings related to 'saying' or 'not saying'. Was not Ben's *implicit* encouragement of the process of specialising just as influential as his more *explicit*, ie declared to the class audibly, emphasis on pattern spotting? The classroom ethos on which I have commented was not achieved by Ben *telling* the class what ethos he desired, but by a mixture of his statements and actions in working with the pupils.

One manifestation of the didactic tension, to do with use of processes, occurred when Ben referred to the project work which pupils had handed in for assessment. He said to them "Lots of people are into the routine of, 'let's do ten examples, put them into a table, look for a pattern' – people are on automatic – giving me a lot of information and not much thinking" He had explained to me before the lesson, that there were some cases where ten examples were more than was necessary and others where ten were too few. What he actually wanted was pupils themselves to discriminate and decide what was necessary, and then to account for what they had done by explaining their thinking. He had never said that ten examples were either necessary or sufficient, but somehow pupils had got the idea that this was what was required, and had presented the data without any rationale for it. A question which occurred to us was, 'what created this impression, and what could have made it otherwise?'

Another manifestation of the didactic tension occurred in the vectors lesson, when one pupil required reassurance, "Is that alright, Mr West? Is that the sort of question you want?", and another had said, "I don't know what question to ask." Ben said, after the vectors lesson:

Jessica was floundering. She wanted questions given to her and she wasn't getting them. She asked me three times for a question and I said the same thing three times. And then she says, 'What's Becky doing?' I said, 'Ask her'. 'What's Nicky doing?', 'Ask him!' She was trying everything to get a question out of me. (Ben, 23.11.88)

In this case Ben had refused to be drawn into defining what questions he wanted. I said to him during our conversation:

I think there are questions about management of learning in here, in the way this was done.

I mean, in deciding that you would go along with the idea of letting them make up their own questions you were allowing for the possibility that the questions they made up wouldn't include all the different cases that you might have included if you'd set them an exercise.

And yet if you set them the exercise they don't get the chance to think it through and investigate for themselves. (BJ, 23.11.88)

I was seeing that Management of Learning involved grappling with the didactic tension. At the end of this lesson there were probably many questions on vectors which had not been tackled, but which Ben, possibly because of his syllabus, *wanted* to tackle. However, the setting of an exercise which comprehensively introduced such questions may not have been successful in getting pupils genuinely to grapple with ideas of vectors. Ultimately I saw a compromise. Pupils tackled their own questions, many of them, like Mandy, coming up with ambitious questions, others going not much beyond finding lengths of certain vectors. As a result of this, Ben felt that some ideas had not been comprehensively tackled. Subsequent lessons were then devoted to these.

A further manifestation of didactic tension occurred at the end of the *incorrect-exam-answers* lesson, in which Ben had given pupils a set of exam answers (for a test which they had themselves taken a few weeks earlier) each containing a number of errors which they had to try to spot. He explained to pupils *why* he used the incorrect-answers technique, rather than the more traditional one of explaining the correct answers, and then asked them to comment on their perceptions of the technique.

Making incorrect exam answers explicit

Ben I think you should go over your tests because I think it's good for you to learn where you go wrong. But the problem is how to actually achieve that. How do we actually get you to look at what you've done, actually go over it, and sort out where you went wrong? Because where you went wrong you'll learn from, yeah? Now the normal – a lot of teachers do it, and I don't like doing it this way, I could stand at the board and give you perfect solutions, yes? And you could check your solutions against mine. That's one way, – Dawn, I need you listening and thinking please! – That to me is pretty boring, cause when people are going through things that you have answered right, you're bored, and when people go through things you've done wrong, you want to stop and ask questions, because sometimes the teacher doesn't give you what you want. So that's one way of doing it. ...

Could you put your hands up and could people listen to what others are saying ... because it's quite crucial because we've got to go through this process two or three more times, and we need to make doing tests a useful exercise, not just an exercise of finding out how good or bad you are.

Data item 7.14: Extract from transcript of *Incorrect Exam Answers* lesson (14.12.88)

This seemed to me to be a clear example of a teacher trying to be explicit about his intentions, and seeking feedback from the class to gain insight into their perceptions of what he had asked them to do. Various pupils offered their perceptions of the technique, and in particular Colin offered one comment:

People might see where *you've* gone wrong, but they might think, "But where have I gone wrong in my test? How's this helping me?" (Colin, 14.12.88)

then, later, a further comment which seemed significantly different.

I think it's much easier to see other people's mistakes than your own. I mean you might say, "A lot of this is easy, I can see where he's gone wrong", because you've got a better idea now, but if you try and see it on your own you can't see where you've made mistakes. It's maybe helping you slightly, but not entirely. (Colin, 14.12.88)

Ben felt that, in the second comment, Colin, not necessarily intentionally, was trying to give him what he perceived that he wanted to hear; that somehow Colin had picked up cues to this effect. There was some imputation of intellectual dishonesty in Ben's remarks.

In my study of Ben, I had many conversations with pupils, sometimes during, sometimes after lessons, often with Ben present. I have quoted two instances from the *moving squares* lesson. Some of the pupils seemed very ready to talk with me about their experiences in mathematics lessons, when they observed that this was something which I welcomed. On one occasion I spoke to the two boys Tony and Colin after a lesson, and in the course of conversation, the following remarks were made by one of the boys:

To tell you the truth, I mean Mr West, he's taught us, cos he's a different kind of teacher completely. Before you had sums and you've been set it, you've come across him and, at first to tell you the truth I didn't like him as a teacher. I thought, 'No. Pathetic! You know, this isn't maths. What's this got to do with maths.?' And as I've come along, I've realised that it's got a lot to do with maths. (Tony, 30.11.88)

The boys elaborated on what they *now* saw maths as being. I regarded the discussion as particularly revealing of these boys' perceptions of Ben's style of teaching, as well as confirming what I observed myself. However, Ben was more sceptical, and wondered how much the boys perceived of what it was that we wanted to hear, that perhaps he and I were giving, albeit subconsciously, cues to what we wanted to pupils to believe and to say to us.

In terms of the didactic tension, the business of cues and 'playing games' adds another dimension to questions of being explicit. When the teacher is explicit, it is as a result of a decision overtly to offer advice or instructions, knowing that pupils will interpret these in diverse ways. The teacher can then knowingly look out for these interpretations and, if they fall short of his expectations, offer remedial action. However, pupils interpret all remarks made by the teacher, and it is when the teacher is not being knowingly explicit that this interpretation can offer most surprises. For example, Ben was surprised when most of the class presented ten special cases in a table as their interpretation of looking for patterns. What cues had he given that led to this level of consistency in their response?

Constructivism and the Teaching Triad

The three tensions discussed above seem fundamental to considerations of implementing a constructivist philosophy in the teaching of mathematics. In preparation for the Open University course ME234⁸, 'Using Mathematical Thinking', I interviewed Paul Cobb whom I quoted extensively in Chapter 2. The following question from me and response from him are from audio material which forms part of that course:

- BJ If I just ask pupils to construct for themselves, how can I be sure that they will construct what I want them to construct?
- PC A lot of people tend to assume that constructivism means that basically anything goes, and we have this beautiful unfolding into how children learn or whatever. This is of course lunacy. In other words, the idea that we give children some blocks or some materials, and we leave them alone, and we come back in fifteen years' time and expect them to have invented calculus, just makes absolutely no sense whatsoever. The teacher is still very much an authority in the classroom. The teacher still teaches.

⁸ Open University, 1988

My question was deliberately naive, but nevertheless it captures in essence the paradox of trying to turn a constructivist philosophy into practice. This includes the learning paradox which I discussed in Chapter 2, but for the practice of teaching it is more than this. It is encapsulated in Cobb's final sentence above, "The teacher still teaches". I have been exploring of what such teaching might consist.

I shall summarise here what I feel I have done, in my study of Ben's teaching and that of the earlier teachers. Having embedded investigative teaching in a constructivist theory of knowledge and learning, I have analysed data, particularly from Ben's teaching, to seek manifestations of a constructivist philosophy in practice, and issues arising from this. I have looked particularly at one teacher's creation of opportunity for mathematical thinking and learning in pupils and have asked questions about how the teacher has encouraged pupils to make their own sense of the mathematics which they have encountered. I have been assisted in my identification of such manifestations by my earlier synthesis of the teaching triad from the work of the Phase 2 teachers. I have found that the teaching triad provides a way of viewing teaching which is particularly illuminating where noting manifestations of constructivism is concerned.

The teaching triad identifies three aspects of teaching which seem fundamental to the teaching act, in enabling pupils' mathematical sense-making. The teacher in teaching has to create and organise, not just the situation in which learning will take place, but also the underlying ethos of this situation which is bound up in social context. Within this ethos, teacher and pupils interact with mathematics. Where the teacher is concerned, the link between pupil and mathematics is fundamental to the exercise of teaching. Mathematical challenge has to be offered in order to stimulate thought, enquiry and activity in the pupil. Inappropriate challenges are likely to result in ineffectual learning and so the challenge must be appropriate. This requires a high degree of sensitivity of the teacher to the pupil's needs, which demands a considerable knowledge of the pupil.

Thus in looking at Ben's teaching in terms of his management of learning, his sensitivity to pupils and the allied degree of challenge offered, I have been able to perceive aspects of this teaching which seem to contribute directly to pupils' sense-making, their fitting of the mathematics into their current experience, and their modification of current experience in the light of new learning.

Two consequences of my resulting awarenesses from this work and thinking are as follows:

- 1 I can observe a particular teacher's teaching and suggest ways in which it fits or fails to fit with a constructivist philosophy of knowledge and learning.
- 2 I can offer the teaching triad to others in order to develop awareness of teaching, not necessarily to try to invoke teaching which fits with constructivism, but to foster awareness of teaching processes which can enhance the learning experience of pupils.

I illustrate the first with reference to another teacher, whom I studied briefly, who claimed to want to implement an investigative approach to teaching, but who gave no observed indication of doing so.

The Study of Simon's Teaching

Simon was a teacher in the mathematics department at Compton, with responsibility for Information Technology (IT) work within the school. He had already been teaching at Compton for some years when Ben joined the school as Head of Mathematics. Ben's style of teaching had been new to the department, as were schemes of work which Ben devised for pupils entering the first-year of the school. Classes already within the school at this time continued to be taught by the established teachers in their established styles. Simon, however, declared an interest in Ben's style of teaching, and claimed to be himself trying to work more investigatively in his own classes.

LESSONS FROM WHICH DATA WERE COLLECTED

I observed six of Simon's lessons, as follows:

Lesson	Class
1 Information gathering	3rd years
2 Travel graphs	5th years
3 Conversion graphs	5th years
4 Consolidation of graphs	5th years
5 Probability 1	5th years
6 Probability 2	5th years

Figure 7.9: Lessons observed with Simon

In the beginning, I was able to go to the school for one day a week only. After the first week it was no longer convenient for me to observe the third year class on this day, and the fifth year class was the only other available. I began to observe the fifth year class, but stopped after five weeks because Simon was experiencing some difficulties in his relationship with the class, and my presence was not a help.

I must emphasise that I saw very few lessons and that these could have been unrepresentative of Simon's approach to teaching. However, the one third year lesson which I observed had many of the same characteristics as the subsequent fifth year lessons. I shall use one of the fifth year lessons, *Consolidation of graphs*, to highlight some of these characteristics. I have chosen this lesson because of my particular reflections which resulted from transcribing audiotape from the lesson.

CONSOLIDATION OF GRAPHS

As with Ben, I talked to Simon before a lesson about his intentions for it, and afterwards about his reflections on it. The class had been working on graphs for a number of lessons, two of which I had observed in previous weeks.

1: BEFORE THE LESSON

Simon indicated that he wanted the focus of this lesson to be 'tangents to graphs' and that one thing which he wanted 'to try to bring out' was the use of a tangent to a graph to indicate the speed at which an object would be travelling after it had been dropped. He indicated that the lesson would be mainly abstract, and that he felt the pupils were ready for this.

I asked whether the class had done any work on graphs of motion of this sort, or whether this would be a new idea. He said that some work had been done and described it as follows:

One of the first things they did was, er, just a straightforward exercise with not much hanging on it. It was an object dropped from a tall building. ...

I gave them all the essential information. For example, I gave them a table of values; I gave them an explanation of what was happening to the object; and I asked them to generate their own axes, plot the points, draw the graph. (Simon, 5.10.88)

I asked if any experiment had been done, but he said not, qualifying this as follows:

Certain kinds of experience of that sort would be quite useful to do, I'm sure, ... In practice, some experimental form of graph generation would be possible, but the kind of graphs that I'm thinking about would be / difficult to do – this kind of situation is difficult to do ... (he referred to an experiment which he had done at college, dropping a ping-pong ball and the difficulty of timing it.). (Simon, 5.10.88)

He continued:

At this stage we are moving on a little bit towards abstraction. This lesson is going to be, as I say, about tidying up loose ends, as indicated on the syllabus. ...

Topic that I want to try and get in to this lesson – skills that I want to try and get across, are skills to do with tangents to curves, skills to do with calculation of gradients, skills to do with interpretation of graphical data, and skills to do with symmetry – the symmetry of a parabolic – I mean of an object dropping / and I want to try and encourage kids to think in terms of the symmetry of certain kinds of function. Obviously it's been worded and worked in a way that makes it easy to digest. (Simon, 5.10.88)

We went on to talk about what pupils had experienced already, and what would be new to them, of what they would perceive about the purposes behind finding gradients as opposed just to being able to find them. I asked if pupils had already started to make the link between the gradient of the tangent and the speed at that instant, and he replied:

No, no. It's going to be something that I'm going to ask questions about, and see if the group between them can work out what the meaning of that tangent is in that particular context.

At this stage I'm not going to try to push it too far – if they can get from today's lesson a selection of processes, physical, manual skills that they can perform, and deduce certain – realise that certain kinds of information, not inherent in the original, can be deduced by doing extra things to it, then I shall be satisfied with that. My expectations for today's lesson are not terribly high. (I said "really?" at this point)

Well, no, no, that makes it sound bad, no. / I see the lesson as a consolidation of skills that they've already learned, essentially, – skills such as graph drawing skills, certain forms of calculation skills, some interpretive skills. Today's lesson is basically about tying up loose ends, within an area of the syllabus. Nothing spectacular. (Simon, 5.10.88)

I observe that Simon talked almost entirely about the mathematics of the lesson, and what mathematical objectives he had for the pupils. He mentioned the syllabus twice. He used language which suggested a transmission view of teaching, for example:

I gave them all the essential information ...

... skills that I want to try and get across

if they can get from today's lesson a selection of processes ...

He said nothing about *how* he would offer this material, or what approaches he might employ for the benefit of different groups of pupils. Perhaps he hinted at this in statements like,

I want to try and encourage kids to think in terms of ...

... something that I'm going to ask questions about, and see if the group between them can work out what the meaning of ...

However, I had the very strong impression that he saw the lesson from the point of view of the mathematics which it was about, and the skills which pupils needed, and that the motivation for this was the mathematics syllabus. Perhaps with a fifth year lesson this is not surprising.

2: INTRODUCTION TO THE LESSON

He began by asking the class to copy a table of values which he had put on the board, leaving space around it to do some calculation. Then he said,

Introduction to Consolidation of graphs

Simon What I'm going to ask you to do in a moment, is to do some calculations to find the missing values, but the first thing I'm going to do, I'm just going to go *through* one of the calculations *for* you, just to remind you how it's done.

He had written up beside the table the function, $y = \frac{x^2}{2}$, and he proceeded to work through an example starting with $x=2$:

The function is y equals x squared over two. In other words, given any value on the x axis, we square it, divide the result by two, which will give us the value to go with it to be used on the y axis. So, for example, – I'll just do one calculation for you, just to remind you, as I say, of how it's done – let's take the value x equals two. When x equals two, to get the y value –

What's two squared Steff?

(Response from the girl addressed, "Four")

Divided by two?

(Response from the girl addressed, "two")

So the y value when x equals two, y is also two. So that gives you a coordinate pair. In other words the coordinates of this particular point are (2,2). OK, by the calculation, when x equals two, y comes out as two also.

What I'd like you to do please is to, in the first instance, perform the remaining calculations to get the results in the remaining boxes – on the graph sheets.

Incidentally, just as a point of interest, what's nought squared?

(The response to this was a chorus of, "Nought!")

Data item 7.15: Extract from the *Consolidation of Graphs* lesson (5.10.88)

3: ANALYSING THIS INTRODUCTION

In the process of transcribing the above material from the audio tape, I stopped to make a record of my own thinking. The lesson as it appeared to me, listening to it on the tape, seemed curiously different to the lesson as it had appeared when I was in the classroom. As I played the tape I could hear only the teacher's voice, and it was as if I saw the lesson from his perspective rather than from my own perspective, which had been from the back of the classroom.

I wrote the following paragraphs:

Reflections on transcribing audiotape of 5.10.88

I stopped to make this note because I'm finding that something interesting is happening as I work on this tape.

I came out of Simon's lesson feeling very unhappy – mostly with my own reactions to the experience. I felt very negatively critical of what I had seen. I now need to substantiate that feeling because I find that in working through the tape I am becoming much more sympathetic to Simon and *perhaps* closer in understanding his perceptions of the event. As I slowly transcribed his exposition [given in the section above] I felt myself enter into some of his possible feelings as he talked to the class. His words became mine as I wrote them down. I entered into the function – the 2x2-ness of the calculation, the responses from the pupils, the nought-squared equals *nought*, from many voices. *Surely, this is so obvious that all the class have heard, and appreciated, and understood it!* I can see that from Simon's point of view it may have felt like this. He was so close to it. He had spelled it out. Pupils had responded favourably.

However, *in the event itself*, as I sat at the back of the room listening to this exposition, making my own notes, recording some of his words, copying from the board, it had no such impact. I was aware that it happened. I 'heard' it. Pupils around me no doubt 'heard' it in much the same way, ie it washed over them to some extent, the degree of this depending on what else they were doing at the same time – eg copying from the board, sharpening a pencil, talking to a neighbour.

Maybe the point that I am making is quite trivial. It is certainly no radical observation that exposition 'washes over' pupils without necessarily having any 'impact' at all. However, my own realisation is to do with the teacher's perception of what occurred – in being so close to it that it is hard to see its distance for others in the room. The teacher is *doing*. The teacher is *thinking*. The teacher is *involved*. So are most of the pupils for that matter – in *something*. But it is all too easy for the teacher to assume that what *they* construe is what the teacher construes during the particular period of time.

As I later walked around the classroom, I saw some pupils gazing at the table which they had copied, not knowing what to do next. One boy prompted another that he had to fill the table in using the formula, but the boy didn't know how to start using the formula. I tried to help out, but response from the boys was unwilling and vaguely insolent – quite unlike what I experienced in Ben's class. On at least two other occasions I talked with pupils who did not know what to do or how to do it. There were Sonia and Jane, who were trying earnestly to *do*, without being sure *what*, with a reply at one stage of "no, it's too fast – we were still drawing the curves", when I asked if they had followed a particular instruction. One boy told me that he was "useless at maths". He didn't understand the formula, but when I prompted him through it he could apply it quite well.

I thought about the lesson afterwards in terms of 'learning'. What had been learned by the pupils? How did this fit with (a) what Simon really wanted out of the lesson; and (b) what he actually perceived to have occurred?

Data item 7.16: My own reflections (11.10.88)

Looking back on my reflections I recognise firstly my negative feelings about the lesson and secondly my realisation of how the lesson might have felt to a teacher who had never been in my critical position. I am very sceptical, as a practitioner, of what exposition and practice alone can achieve in terms of learning. I recognise that, because I say something to someone, it does not mean that they have perceived in it the sense which I intended to convey. However, if a teacher has not been used to thinking in these terms, could it not look to him as if there was every reason to believe

that pupils had understood his words and were ready to undertake the task he had set? What would cause him to think otherwise?

4: THE REST OF THE LESSON

After the introduction, the class settled down to the task set, and Simon responded to pupils who put up their hands, answering their questions and queries. The form of these interactions was mostly one of Simon explaining to the pupil in a form similar to his initial explanation to the class. He himself put emphasis on explanation as indicated in the two remarks which follow.

He interrupted their work on two separate occasions to speak to the class as a whole. First of all:

When you've completed those calculations, I'd like you, please, on the graph sheet to draw axes with that kind of format [pointing to a drawing of axes on the board]. The x axis will need to go along the base; the y axis somewhere in the middle. / I will explain in due course. I don't want to explain it now. (Simon, 5.10.88)

Then, a little later:

If you've done the calculations and plotted the points correctly, it should come out as a *smooth curve*. It's not a straight line, it's a curved graph. For reasons which I will explain in a few minutes, I'd like you to make that curve as smooth as you can possibly get it. (Simon, 5.10.88)

It seemed that the purpose of this part of the lesson was that everyone should have in front of them a smooth graph of $y = \frac{x^2}{2}$, in preparation for the next stage of the lesson. I recognise that in doing this they were possibly consolidating skills previously encountered, as Simon had said to me before the lesson. I wondered however, why so many pupils were having such difficulty with what seemed a relatively simple task, assuming they had done this sort of thing before. This raised questions about what had been learned in previous lessons. I also wondered why Simon needed to spell out everything for them, rather than getting pupils to recall themselves what they had learned to do previously.

As a result of having observed previous lessons, presented in a similar format, my feelings were that pupils were not actively encouraged to think things through for themselves, and were used to the teacher's explaining and spelling things out in detail for them. It seemed to encourage their

dependence and make them less likely to think or make decisions themselves. Indeed it left them with very little to think about or achieve.

I spoke to Simon of the boy who said he was 'no good at maths', and he commented:

Yes, academically he would be towards the lower end of this particular group.

I replied, "When I pushed him though, he could do it", and Simon said:

Yes, that's right. He's one of a number of people that we've got in the school at the moment who're not really as dull as they'd like you to believe. (Simon, 5.10.88)

This seemed a very unsympathetic response, although there may have been background to it which I could not know. However, I felt unsurprised that the boy gave an impression of being dull. There seemed to be so little of interest or challenge to encourage him to be otherwise.

After about half an hour of the lesson, Simon again called the class to attention, using the words, "We're going to have to press on *fairly* quickly because we're getting a *little* bit behind schedule." This was followed by quite lengthy exposition about tangents and gradients. Some pupils had finished the original graph, but others were still calculating and drawing and not really attending to the explanation. At the end of it I spoke to the two girls Sonia and Jane who indicated that they did not know what to do next as they had been finishing their curve, and the explanation had been too fast for them. I enclose an extract from my field notes which focuses on Simon's explaining.

Explaining

I recognise: Needing to get thro' agenda of lesson;
Pupils cannot keep up with pace;
Some don't listen, ∴ don't know what to do.

Simon explains at board – some listen and *do*. He then walks around, explaining what he wants again. More people now *do*. He explains next stage – many still on last stage. Some people keeping up – many not. He walks around and explains again. Eventually all get somewhere, but where is that? However, he keeps on relentlessly. He has an agenda. I have been in his position – it is very hard to break out of.

Data item 7.17: Extract from field notes – Consolidation of Graphs (5.10.88)

The chief characteristics of this lesson seem to be:

- An agenda expressed mainly in terms of mathematical content, with overt emphasis on explanation.
- The apparent assumption that pupils will know something because it has been said. If not, then the only reason can be that they have not listened, so the remedy is to say it again.
- An apparent lack of warmth of relationship between teacher and pupils.
- Keeping to the agenda, regardless of whether pupils are ready or not.
- A lack of conversation between pupils, or between teacher and pupils about their mathematical thinking.

5: SIMON'S VIEW OF THE LESSON

During my transcribing of the audio-tape, I gained a glimpse of what I felt might be Simon's perspective, which was at odds with my own view of what occurred in the classroom. Simon's declared view of the lesson, after it had ended, fitted with my perception of his perspective. In our discussion after the lesson he commented:

Bearing in mind that the subject matter was fairly abstract, I didn't think it went too badly. Looking round I think most people – just about everyone's eyes that I saw – eventually cottoned on to what was going on. I'm not convinced that many of them understood why they were doing it, er, nevertheless the actual methodology I think got through to most people. (Simon, 5.10.88)

I asked, "Can you say anything about *how* it got through?", and he replied,

Yes, erm, they had, acquired the skills necessary to produce the result required.

This seemed to indicate an emphasis on being able to *do* what was required, without any requirement of conceptual understanding. Simon had declared his emphasis on skills earlier, and seemed to feel that this requirement of the lesson had been fulfilled. I was less sure that this was the case. Perhaps pupils could reproduce the activity of drawing a graph of $y = \frac{x^2}{2}$, but I was in doubt as to whether they could extend this to any

other graph. Having the skill seemed to me to require more than just being able to reproduce what had been done in the lesson.

I mentioned my conversation with Sonia and Jane, and Simon responded,

Jane is a little bit, she's always been a bit, shallow – in a sense that she likes to, impress. And I think that, where she feels she can't impress, she's not confident. She doesn't have the confidence to tackle a new situation, a new set of circumstances. She likes – no shallow is the *wrong* word, that's totally wrong – she likes, despite what she might say, she likes a particular routine. She needs to go over things several times, and sort of minutely dissect them at each stage to make sure that she knows what she's doing. She is not, of those in the group academically, certainly in terms of test results and exam results over three years that she's been here, she's lagged towards the bottom end of the group. I think she associates herself with Sonia because she knows Sonia's a bit better mathematician than she is, and uses her to keep herself from falling behind, which is not necessarily a bad thing. It only gets to be a problem when Sonia doesn't know what's going on either. (Simon, 5.10.88)

This lengthy reference to Jane seemed to indicate that Simon did indeed have considerable knowledge of Jane which he could draw on when making decisions about what levels of challenge to offer her. However, as with the boy earlier, there seemed little warmth in his account, or desire to help Jane make the most of her abilities. My impression in the classroom was that pupils were offered a fairly standard response with not much individual variation. In the following comments he speaks of a 'norm', of a 'lesson of that type' and a 'fairly rigid sequence of actions', which support this impression.

Commenting on the lesson as a whole Simon made the following remarks:

The whole point of the lesson was to get through a range of things – tidy up a syllabus topic.

Some pupils could have gone faster. Some are hurried, some held back. It's the norm, not an unusual situation in a lesson of that type where it's fairly formal and I've set a fairly rigid sequence of actions to go through.

Neither do I think that it's – given the constraints that we do have placed upon us – something that you can entirely get away from. I mean, in the lower school there's much more time, and you can approach things in a much more relaxed manner. But at this stage it does come down to pressures to get topics covered. (Simon, 5.10.88)

His words "given the constraints that we do have placed upon us" fits with some of Doyle's (1986) remarks which I quoted in Chapter 3.

Simon said that the next topic would be probability, which he liked to teach because he “could be more relaxed”. I looked for evidence of a “much more relaxed manner” in analysing data from the other lessons which I observed, and shall comment on this in the next section.

6: THE OTHER LESSONS

Features of the above lesson which were common to other lessons were:

- The mathematical agenda for the lesson, which left no flexibility for pupils’ individual thinking or interpretation of planned tasks.
- The teacher’s expository style, explaining what he wanted, often several times for the benefit of different pupils, but usually in substantially the same form.
- Moving through his agenda without real regard for pupils thinking or needs.

He remarked that lessons in the lower school could be more relaxed. However, in the one third year lesson which I observed, having set a task in which pupils had to produce a tally chart from a set of data, he said to the class before they had all finished, “Another moment on that. I should like to press on”. The words, “press on “ were used in the lesson above, and I noticed them on various other occasions, indicating a desire to complete a pre-planned agenda.

In this third year lesson, there were periods during which Simon interacted with the class rather than just talking at them. This might euphemistically be called whole class discussion, but it was rather a case of Initiation, Response, Feedback (IRF) (Sinclair and Coulthard, 1975), where he invited pupils to comment in response to his questions. One example of this from the lesson is,

- | | | |
|---|---------|--|
| I | Teacher | What is a bar chart for? |
| R | Pupil | To make information look easier. |
| F | Teacher | Yes, easier to see, follow and to <i>interpret</i> . |

(Simon’s 3rd-year lesson, 14.9.88)

A number of times during such interchanges, Simon asked pupils to explain. In one case, the following interchange took place involving an explanation from one pupil, an invitation to others to comment which was not taken up, and feedback on the initial explanation:

- I Teacher What does the chart tell us? / What general information can you get from the shape of the chart?
- R Pupil The average weight of the group is 45 to 49.
- I Teacher Anyone agree, disagree? Roy? Were you shaking your head?
- (There was no response to this from the pupils)
- F Teacher It *appears* to show an average in the region of 45 to 49 kg.

(Simon's 3rd-year lesson, 14.9.88)

On another occasion, after a response involving explanation from a pupil, he replied, "Yes, I see what you mean", without commenting or inviting others to say if they understood or agreed. Instead he asked,

Is anyone falling asleep? Anyone not know what we're talking about?

There were no responses at all to either of these questions. I had written in my field notes prior to these questions, 'Some pupils not contributing. What are they doing/thinking? Some doodling'. Pupil attitudes during the IRF sequences were much as I had observed during expository sessions. Some pupils seemed to listen and take part, but others gave little sign of attending to what was taking place. They seemed not to be involved, and gave no indication of being interested or motivated. The tasks in the lesson were routine and undemanding. Simon said afterwards that he was disappointed that there had not been more discussion, but in the IRF sequences his questions had not stimulated discussion, so it was unsurprising that pupils had not responded. Perhaps the lesson could be described as *more relaxed* than the fifth year lesson discussed above because the IRF sequences seemed to lack urgency, but it was no more relaxed in terms of teacher-pupil relationships.

In the fifth year probability lesson (Probability 1), I again looked for evidence of this lesson being 'more relaxed'. It differed from others which I saw in that pupils were asked to work in groups of four on a practical task, that of rolling two dice 144 times and recording the score for each roll. Each group had to record their scores on a class chart on the board prior to drawing a graph from the class totals. A great deal of time was spent in rolling and tallying, then in copying the class chart from the board. Some entries in this chart were incorrect and so it had to be altered, delaying the process further. Most of the activity seemed to be mechanical and devoid of thinking. Simon commented afterwards that it was a pity that the chart had taken up so much time, as he had hoped to end the lesson with a discussion of the results which pupils had obtained. I asked what he

felt had been learned, and he replied, "From a given set of outcomes, they are not all equally likely". I wondered how many pupils would have come to this conclusion or realisation. In terms of relaxation, there had not been the same drive to 'press on', and the group work had provided freedom for social chats and a less formal atmosphere. Little mathematical thinking seemed to have taken place, and again there seemed little motivation or interest. However, there had not been time for discussion at the end, so this thinking may have been deferred until the next lesson.

Of the remaining lessons, two stand out as encouraging more participation and interest from the pupils. The first of these was *Conversion graphs*, which was undertaken by pairs of pupils at a computer using a simple BASIC program. Pupils were encouraged to modify the program to change conversions from, for example, inches/cm to centigrade/fahrenheit. This had scope for exploration and discussion. It was not a class lesson in the sense that most other lessons were. Simon moved from group to group, inviting pairs to say what they saw to be happening with the program, helping with syntax and offering explanations. It was in this lesson that I saw most thinking taking place, and Simon himself at his most relaxed. I wondered what the computers, and his role with regard to IT in the school might have contributed to this. Certainly I felt that pupils were more interested and motivated than seemed usually the case.

The other lesson which seemed to interest and motivate pupils rather more was *Probability 2*. This revolved around a set of playing cards – the probability of drawing a card of a certain suit or type from a pack, and the related odds. After the introduction, pupils were set an exercise which was tackled with interest, and pupil discussions provided evidence of mathematical thinking. The context of the lesson seemed to inspire pupils as had the computers in *Conversion graphs*. Simon's interactions with pupils in *Probability 2* were less explanatory and more questioning. However, at the end of the exercise, he chose to read out answers to the class, having to remonstrate when pupils did not listen, rather than encourage pupils to offer and explain their own answers, with the possibility of further discussion.

7: THE ISSUE OF EXPERIENCE/ABSTRACTION

In our discussions Simon used two words frequently – *experiential* and *abstraction*. After the very first lesson he said that he preferred pupils to gain experience of an idea before making it formal by introducing particular terminology or notation. So, in the third year lesson, pupils in

the class provided as data their own weight, height and hand-span and, from the accumulated data, Simon asked them to draw a graph of height against span. He said that he wanted this to lead to ideas of *correlation*, but would not use this term until they had some experience to which it might be related.

After the first fifth year lesson, on *travel graphs*, Simon indicated that he was not very happy with the way it had gone. The focus had been on certain questions from a text book, and Simon had tried to get pupils to say what they understood by the questions. However, this had taken much the same form as the IRF sequences to which I referred above, with many pupils seeming not to take part. He referred to the “limitations of the textbook”, saying that interpretation was required. This led to discussion of the provision of ‘experiential work’. Simon said that it bothered him, at this stage with the fifth years with time and syllabus pressure, how much experiential work was necessary or possible.

One thing that does bother me at this stage, I don't think I've made my mind up about how to approach the time leading up to an exam for fifth years. With second, third, or even fourth years you can get away with a large amount of experiential work. ...

I tend to do quite a lot of investigational and problem-solving work with the younger pupils, but something I haven't quite worked out in my own mind is how much of that could be included by the time I get to the fifth year.

I haven't got a decision either way. It's something I'm finding difficult to have a good compromise. It's something I don't feel clear about, and I reckon in my position I should be clear.

As they get older my approach to teaching them is changing, in a positive sense to more abstraction of ideas. (Simon, 21.9.88)

After the lesson on conversion graphs, with the computers, he referred again to the experience/abstraction issue, saying that his use of computers was to provide the experience in this case. In the probability lessons the rolling of dice and the playing cards exercise were designed to provide experience. However, he repeatedly indicated that fifth years should be able to cope with more abstraction.

There seemed to be clues here to Simon's personal pedagogy. The metaphor ‘get away with’, at the end of his first paragraph, made the inclusion of experiential work seem almost surreptitious, contrary to expected or necessary practice perhaps. His recognition of being unclear about how much problem solving or investigational work to include in the fifth year was perhaps indicative of a lack of clarity of what purpose such

work might fulfil. The words, "I reckon in my position I should be clear", seemed an honest reflection of the concern which his lack of clarity caused him. I gained the impression that he was struggling with the realisation of difficulties caused by the abstract nature of much of the mathematics that pupils were traditionally offered, genuinely concerned about alternatives and how they might be managed, and having yet not developed a strong enough sense of what balance he wanted to enable him to develop successful management. Simon's predicament raises many questions about the development of teaching approaches which fit the needs of the learner, and ways in which teachers can be enabled to develop such approaches.

THE TEACHING TRIAD

I recognise that the introduction of an investigative approach with pupils so firmly established in traditional methods would be difficult for any teacher, so perhaps the observation that I saw little which could be regarded as investigative teaching in observing Simon's fifth year lessons is unsurprising. However, I feel it is difficult for a teacher who works in an investigative way with other classes to give *no* clues to this even in his work with a fifth year group. In trying to fit what I saw to the teaching triad I had little success.

In some of the lessons observed, Mathematical Challenge was almost completely lacking. Tasks were trivial and were often laboriously explained. Pupils, used to the teacher explaining, depended on these explanations and there was little evidence of pupils doing any creative or original thinking.

Sensitivity to Students was not much in evidence. I saw little indication of Simon taking account of pupils as individuals, rather providing a diet of instruction and explanation which was much the same for all. Where he spoke to me of individuals it was in quite negative terms, which led me to think that pupils were not respected, or encouraged to value their own thinking or ability. When pupils were praised, it was usually because they had completed a set task correctly.

I do not believe that the term 'Management of Learning' applied to Simon's teaching. Simon managed the pupils and the classroom. He set tasks and ensured that the tasks were undertaken. There was little emphasis on thinking, and I do not know how he evaluated pupils' progress. When I enquired what he felt had been learned, this was usually

expressed in terms of what *he* had set out to achieve, rather than in terms of what any pupil achieved. He rarely talked of pupils thinking or learning, rather of what they could *do*.

In constructivist terms, it is hard to say what sense the pupils made of the mathematics they encountered. I gained little insight into this from the time I spent in these lessons. The teacher's approach to teaching was extremely narrow, involving little beyond instructions and explanations. I suggest that he gave little thought to what construal might be made of these by pupils. There was little attempt to talk *with* pupils, or to encourage them in ways of working which might develop their own ability to think and learn. The teacher's reflection on his teaching involved mainly an assessment of whether he had succeeded in what he set out to do, usually in terms of completing an agenda.

CONCLUSION

I believe that Simon was bound up in a transmission view of teaching which did not allow considerations of the individual learners beyond their ability to respond to what the teacher offered. I was not able to explore his view of mathematics, but his planning and presentation of lessons seemed to indicate a belief in the existence of invariant concepts which it was his task to deliver, rather than of personal concepts which individuals could be encouraged to develop, share and negotiate. His issue of experience versus abstraction seemed to indicate a belief in backing up delivery of abstract concepts with related experience. This experience was then used as a basis of explanation about the concept rather than as an environment in which pupils could develop their own thinking.

I found little to study here in terms of an investigative approach, because I could see little of such an approach with this fifth year class. Consequently, when difficulties arose with the class, I allowed the work with Simon to come to an end. I cannot offer much insight into the difficulties because we discussed them only very superficially. Relationships between Simon and the pupils seemed to have deteriorated. He was unhappy with the work which they were doing, and there were behavioural problems. I do not think my presence was a help, but I cannot know how much it affected what occurred or the problems which arose.

My experience in observing Simon's lessons throws new light on earlier observations and leads to a number of questions:

- How far did the fact that it was a *fifth* year class affect what I saw? Evidence in the school suggested that these students had been used to a traditional approach to teaching mathematics. They were likely, at this stage, not to be receptive to new approaches, particularly if these approaches were not well designed.
- How does a teacher *begin* to implement new approaches? The Amberley teachers began with first year pupils who had as yet no experience and few expectations of secondary education. It is much more difficult, even for a teacher with experience of investigative work, to combat the expectations of pupils who have only experienced a traditional approach.
- How does a teacher *learn* about teaching? Clare, Mike and Ben were established in a pattern of development of thinking and practice, which was a part of their professional life. The Amberley teachers came new to investigative work, although they perceived much about its philosophy and purpose. They were in a learning situation, and openly used our work together to develop their knowledge and experience. Simon was an established teacher confronting new and threatening ideas without perhaps the realisation that he needed to rethink his personal philosophy and seek support in implementing new approaches.
- Which comes first, change of practice, or change of philosophy? The implementation of investigative work is not just a matter of *doing* things differently, it also involves a different way of thinking about them. How can teachers develop new philosophies within the lonely environment of their own classroom and their past experience?⁹

A study which I feel it would be valuable to undertake would be to offer a teacher such as Simon notions of the teaching triad with encouragement and support to undertake classroom experimentation and reflect on the outcome. This work would be in the area of teacher development. I have suggested that my current study has implications for teacher development, and discuss this extensively in Chapter 8.

⁹ See also Claxton (1989), pp 120–121.

Conclusion to the Phase 3 work

The aim of my analysis of the Phase 3 data was to draw links between characterisations of the practice of teaching mathematics and a radical constructivist philosophy of knowledge and learning (The reader might refer to the diagram at the end of Interlude B, p 199) The *teaching triad* was an important vehicle in making these links. The triad, emerging initially from characterisations of practice became ultimately a device through which practice might be regarded. Thus in terms of *Ben's* teaching, in Phase 3, I was able to recognise manifestations of the three elements of the teaching triad, and through this make links to my constructivist theoretical base.

I have not up to this point made explicit the links between the triad and constructivism. In the figure which follows I link the two principles of constructivism, quoted from von Glasersfeld in Chapter 2 (see p 13), and set these against the three strands of the triad. The centre column is designed to elaborate the links both between elements of the triad and the principles of constructivism. Thus elements of teaching, O and C may be seen to relate directly to the first principle, and element C and K to the second principle.

Radical Constructivism	Teaching	The Teaching Triad
1. Knowledge is actively constructed by the learner, not passively received from the environment. 2. Coming to know is an adaptive process which organises one's experiential world. It does not discover an independent, pre-existing world outside the mind of the knower.	Offering mathematical challenges appropriate to the learner (O)	Mathematical challenge
	Creating opportunities and an environment for mathematical thinking and exploration (C)	Management of learning
	Knowing the learner well in order to perceive both appropriateness of challenges and their fit with a learner's past experience (K)	Sensitivity to Students

Figure 7.10: Linking the teaching triad with radical constructivism

The relationship between constructivism and teaching might be seen as in figure 7.11 on the left below:

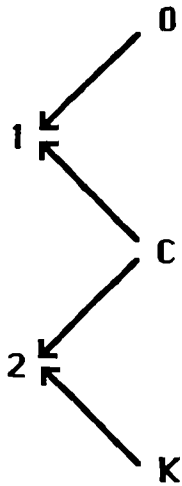


Figure 7.11:
Constructivism and
the triad

Figure 7.12 gives an image of the triad which emerged from Ben's teaching, with ML encompassing MC and SS; in which ML is seen to encompass C, O and K; MC to be closely associated with O, but linked to K; and SS associated with K although linked to O.

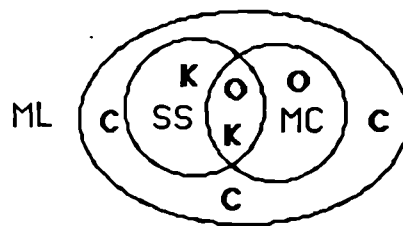


Figure 7.12: Elaborating the teaching triad

The relationship between the teaching triad and the practice of teaching may further be illustrated by manifestations of the triad in the vectors lesson, as in Figure 7.13 below *The Vectors Lesson and the Teaching Triad*. I offer this here as an exemplar. I wish to emphasise in offering it, that the exercise of its production is an individual and personal one. Its benefit lies in the thinking that is demanded by the production process. Another person's diagram may look very different. What I offer is *my* diagram, purely as an example of how such a diagram might appear.

It would be impossible for me to construct such a diagram using any of the lessons which I observed with Simon. I find this very instructive despite its negativity. It validates my view that the teaching triad is a device closely linked with a constructivist philosophy of learning, and associated ways of working in the classroom. What I set out to do in this study was to explore the *nature of an investigative approach to teaching mathematics*, which became a form of words indicating *manifestations of a constructivist philosophy for teaching mathematics*. I feel that the emergence of the teaching triad is a powerful step in identifying this nature.

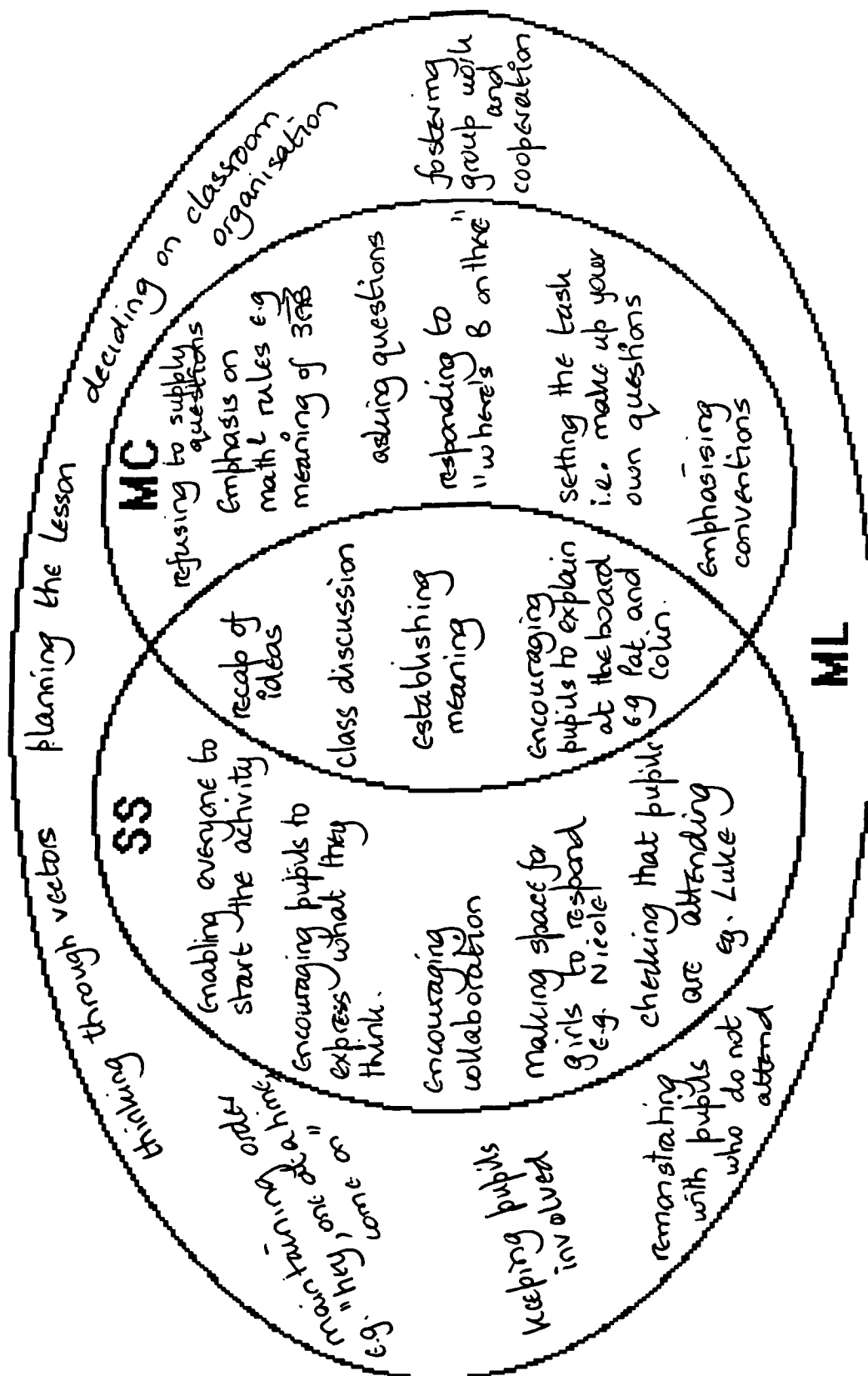


Figure 7.13: The vectors lesson and the teaching triad

PART 3

CONSEQUENCES & CONCLUSIONS

CHAPTER 8

REFLECTIVE PRACTICE

The focus of this study began very firmly in the domain of mathematics teaching, with emphasis on teaching acts designed to promote pupils' conceptual awareness, and the thinking which lay behind these acts. A teacher's thinking was an overt consideration of my study, but implicit in this was that teachers *do* think, and that this thinking influences their classroom activity. My belief that this study has implications for individual teacher development has sprung out of the relationships which I experienced with teachers in the study, and my observations of their personal development during this research.

As my study has evolved, and I have come to focus more and more on levels of interpretation, and on *whose* story I am actually telling, the position of what I have called *the teacher-researcher relationship* has grown in prominence in this research, as has the role of *the reflective practitioner*, both teacher and researcher.

My perceptions with regard to the development of teaching have arisen to a great extent through recognition of the role which reflection has played in my own development.

This has brought me in contact with a fresh area of literature, as I have started to consider reflective practice more widely, in particular that related to reflection in teaching. Associated with this are aspects of teacher-thinking, teacher-knowledge, and teacher-theory. Although much of this literature can be seen to be related to my study at some level, I shall restrict consideration here to that which impinges closely on conclusions which I shall draw from this study.

The purpose of this chapter is to speak of the teacher-researcher relationship which has developed between myself and teachers with whom I have worked, and to present my own conceptual model of this relationship. I then discuss how I feel this model underpins reflective practice on both sides of the relationship.

claiming that “the defect of the reflective approach is that it is severely constrained and limited by what it ignores.

Being critical, or engaging in critique, involves analysis, enquiry and critique into the transformative possibilities implicit in the social context of classrooms, and schooling itself.

I feel that Smyth’s critical pedagogy, whose intent “is that of ‘liberation’ (or emancipation)” which encourages freedom of choice, is relevant to my more limited focus, the teaching of mathematics as I discuss below.

However, Smyth’s remarks beg the question of what precisely *reflection* is seen to be, and how it relates to teachers’ thinking.

There are few studies of reflective practice which do not cite Dewey in establishing what reflection is. For example, Dewey (1933) writes:

Active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends constitutes reflective thought. (p 9)

He goes on to claim further that:

reflective thinking, in distinction to other operations to which we apply the name of thought, involves (1) a state of doubt, hesitation, perplexity, mental difficulty, in which thinking originates, and (2) an act of searching, hunting, inquiring, to find material that will resolve the doubt, settle and dispose of the perplexity. (p 12)

and

Demand for the solution of a perplexity is the steady and guiding factor in the entire process of reflection. (p 14)

This last statement supports what I have taken reflection to be in my study. Moreover, I feel that it points towards what I have come to believe is a necessary result of reflection, if possibly not a natural component of it, which is some *action* resultant on the reflection. Smyth, above, seems to suggest that reflection requires some critical process to lead to action. Kemmis (1985), writing of *the nature of reflection*, claims that “reflection is ‘meta-thinking’ (thinking about thinking) in which we consider the relationship between our thoughts and action in a particular context”. He adds:

We do not pause to reflect in a vacuum. We pause to reflect because some issue arises which demands that we stop and take stock or consider before we act. We do so because the situation we are in requires consideration: how we act in it is a matter of some significance. (p 141)

In my own experience, and supported by the literature (e.g. Cooney, 1984), it is often the case that teachers' knowledge or theory which guides their classroom action is implicit. Polanyi (1958) introduced the term *tacit knowing* to speak of this implicit knowledge, and Schön (1983) suggests that such tacit knowing lies *within* the action.

Our knowing is ordinarily tacit, implicit in our patterns of action and in our feel for the stuff with which we are dealing. It seems right to say that our knowing is *in* our action. (p 49)

Polanyi (1958) claims that "tacit knowledge cannot be critical" He emphasises the necessity of "the assertion of an articulate form" for what is being criticised. It seems clear that if knowledge is implicit then, to engage in some process of critical reflection, some means has to be found of explicating this knowledge. In my research, I felt that I played a role in this explication for the teachers.

The teacher-researcher relationship

In order to gain insight into the teaching acts which I observed in the mathematics classrooms which I studied, it was necessary to try to gain access to the teachers' thinking. In work with all the teachers in this study, considerable time was spent in conversation or discussion between the teacher and the researcher (myself). Typically, I talked with the teacher both before and after each lesson, and on some occasions reflection on one lesson led directly to discussion of the next. Over a period of time (with most of the teachers it was more than six months), I believe that we developed a level of trust and understanding which allowed deep and searching questions to be tackled. In the main, these questions arose from lessons which I had observed, as a result of reflection on the lesson.

In Fig 8.1 I have included a flowchart which highlights the main aspects of my conceptual model of the teacher-researcher relationship. This involves teacher and researcher each moving through a number of stages of reflective process, in which they separately perceive and reflect on perceptions, and jointly negotiate these perceptions at a number of levels. The left hand side of the flowchart indicates the teacher's stages, and the right the researcher's stages. Arrows which cross from one side to the

other indicate interactions between the two. The flowchart is an iconic representation of my perceptions of the relationship. In the text which follows I elaborate the various stages, and the flowchart can be used by the reader as an aid to visualising the process as a whole.

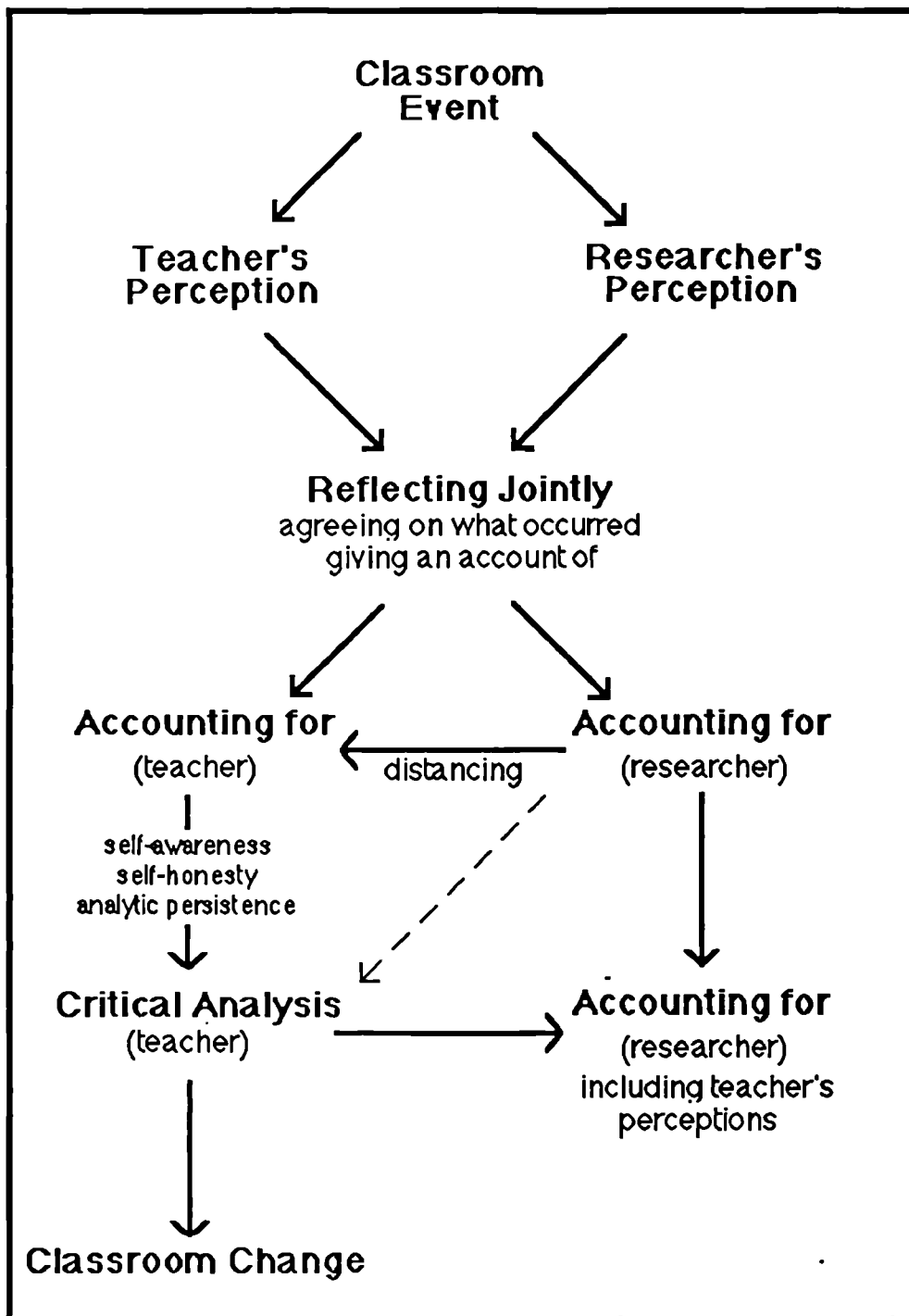


Fig 8.1: The teacher-researcher relationship

Initially the process involved both teacher and researcher participating in an event of which each has particular perceptions. They then discussed what occurred, either agreeing or negotiating, and jointly gave an account of the event. As a result of this the researcher was likely to account *for* what occurred, based on her own perceptions of the event. Part of the researcher's role was to encourage the teacher to reflect *on* what occurred and to account *for* it. This encouragement usually took the form of questioning, and its effect was that of *distancing* the teacher from the practice. For the teacher this was likely to lead to a *critical analysis* of the planning and motivation for what occurred, and this could lead ultimately to a developing of *teaching knowledge* and *teaching wisdom*, and to changes in the practice. Essential to the last two steps where the teacher is concerned are a high degree of self-awareness, self-honesty and analytical persistence. Interaction with the researcher can considerably influence these capacities. Interaction with the teacher, during these levels of analysis and change, can lead to further conclusions for the researcher. I shall look at these stages in more detail below, providing examples to illustrate what I mean by the various terms.

STAGE 1 – REFLECTING

Agreeing on what occurred could either be something which the teacher-researcher pair set out to do, or a point from which they start. When I have observed a lesson it seems natural to me that what I *saw* was what *occurred*. I recognise that there were things I did not see, and am prepared for the teacher to add to my account, however, I am surprised if the teacher's account is materially different from mine. The act of *giving an account of* is one in which teacher and researcher 'try out' their own images of the lessons on each other, and for each of them it serves to recall events and to remind them of significant moments. The differing perceptions of the event lead to negotiation which often results in valuable insights for both participants.

The roles of teacher and researcher are not equal. The teacher talks aloud for the benefit of the researcher from *within* her perceptions of the lesson. The researcher engages in conversation from *outside* her perceptions of the lesson, trying to operate simultaneously at two levels – engaging with the teacher about the lesson, and keeping an overview of the conversation with the teacher and what, as researcher, she is learning from it. Thus the researcher starts to draw conclusions, which at this stage are based on her perceptions from the classroom and what she is beginning to hear from the teacher.

The situation which follows contains one example of this stage in practice. I should like to draw attention to (a) the incidence of prolepsis (Stone, 1989) as researcher and teacher use only minimal statements to reach common understanding, implying shared knowledge and perspective; (b) the way in which the teacher's silence validates interpretations which the researcher makes; (c) the move by the teacher into *accounting for* at one point.

$\text{Luke's } 3\vec{AB} = \vec{AB} + 2\vec{AB}$

Talking to Ben after the vectors lesson, I brought up the incident concerning Luke who had offered a novel way of explaining the vector $3\vec{AB}$ (See Chapter 7 The vectors lesson, p 229)

- (1) BJ Oh, another thing I recall now, do you remember when you'd got three AB up there, six, six? (Ben said, "yeah") And you turned round and you asked Luke. And my understanding of that was, Luke's not paying attention. You're checking that he knows what's going on. And you asked him to explain that. And he clearly hadn't listened at all, but he comes up with an alternative correct representation.
- (2) Ben But that's Luke. That's the sort of person he is, isn't it? I think.
- (3) BJ I mean, I was quite surprised not to/ for you not *then* to make the link, but you decided to go on and –
- (4) Ben I felt there was so much around, that I had to sort of / it's these judgements again isn't it? You make judgements all the time.
- (5) BJ What I read into this was, 'Ok he wasn't listening, but what he's given me is OK so I'll let it go'.
- (6) Ben I've also made my note to him that, 'I know you're not listening'. Yes?

Data item 8.1: Extract from conversation with Ben (24.11.88)

Ben's reply in statement 2 indicated an acceptance of what I had offered in statement 1, thus validating my perspective, as well as providing a further remark about Luke. I was therefore encouraged to pursue my further interpretations of the event.

Prolepsis occurred at statements 3 and 4. At statement 3, I indicated my own surprise that Ben had taken a certain course of action, and before I had finished, Ben indicated that he understood what I meant, and started to *account for* his action. “This business of judgments” was part of our common understanding from discussions of his lessons. He needed to say no more than this for me to understand what he meant

At statement 5, I offered my further interpretation. Ben made silent assent to this, validating my interpretation by not contradicting it, but then added further information about his intention in the act.

In this conversation we gave a joint account of the event. There was no apparent disagreement, but the teacher added to my account by indicating more of his purpose than I had perceived. There was movement into the next stage of the process, which is *accounting for* what occurred. Ben did this with his talk of judgements. The process:

reflection —> accounting for —> critical analysis

might be perceived as being linear with respect to any item of discussion, but in fact we moved in and out of its various stages throughout a conversation. Although in the statements above, we said no more of judgements, our conversations on judgments had reached the stage of *critical analysis* in former conversations, and Ben’s recognition of it in this incident indicated to me that he had become very aware of making judgments, possibly as a result of our conversations. I shall say more of what I mean by critical analysis shortly.

I made the above analysis of the conversation after it was over. It is hard now to recall what particular conclusions I reached during it, but these might have involved observations about the teacher’s validating of my perceptions, and recognition that the teacher’s spontaneous reference to judgements indicated significance for him of previous conversations.

STAGE 2 – ACCOUNTING FOR

It was important for me, as researcher, to gain access to the inner motivation behind the teacher’s classroom acts. Thus having made observations about *what* occurred, it was important to encourage the teacher to *account for* it. At its simplest level this involved asking ‘why?’ Sometimes it was unnecessary for me to ask. In the above example, Ben anticipated my query from the words “I was quite surprised not to/for you not *then* to make the link, but you decided to go on”, and accounted for his decision not to make anything special of Luke’s reply. He indicated that

there were many possible judgements here, and that this had been one of them. In fact later, it emerged that he had been concerned about a group of girls who were having difficulty with basic understanding, and this was more important for him at that point than was Luke's new view of $3\overrightarrow{AB}$.

An important role for the researcher at this stage is that of *distancing*. By *distancing*, I mean, firstly, enabling the teacher to take steps back from the event to try to see it less subjectively, in order to examine it critically; and secondly pushing (with sensitivity) to enable the teacher to go beyond first reactions.

I believe that the act of *accounting for* is essential for any practitioner when reflecting with a view to becoming more aware of and improving practice. However, it is very difficult for a teacher, reflecting alone, to force through to the deeper levels of perception and awareness of motivation and belief. Reaching these levels can require high degrees of self-honesty and persistence and can be painful. The researcher, in pushing a teacher to these levels, has to be aware of the sensitivities involved.

Teachers with whom I have worked have spontaneously said:

- 1 that I ask 'hard' questions;
- 2 that they have found working with me and trying to answer my questions helpful for their own development.

I believe that the first observation recognises the *distancing*. In order to reach the deeper levels of motivation and belief, hard questions have to be tackled. If they were easy questions, then either they would require only superficial attention, or they would address areas with which the teacher was very familiar and did not need to search deeply for.

The second observation, I feel, has nothing to do with the particular researcher, but is recognition of the value professionally of reflecting and accounting. However, there is here also the question of trust. If this is lacking it is likely that little of value results, and this may have been the case in my work with Simon.

The four data items which follow contain some particular instances of questions which had *distancing* effects of different kinds. In the situation below, Ben was initially threatened by the question I asked, interpreting it as criticism, which I had not intended. However, the threat possibly

pushed him further than he might otherwise have gone, to consider other aspects of his role in a coursework lesson. This allowed him to focus on 'listening' and to analyse its value in relation to the activity of the pupils.

A threatening question triggers reflection

I asked Ben a question, "Do you always work like that?", related to his role in a coursework lesson. He commented as follows:

Ben I was sort of thinking about coursework – what's my role in it? You know, maybe I've read more into your comment, 'Do you always work like that?', maybe I read that as 'Don't you do anything else?'

I pointed out that I was not being judgmental, just 'asking questions on what I see'.

Ben Yeah! And that was my reaction to that question. It's the wrong reaction, but maybe it wasn't because you then start to say, 'Well, what is my role while they're doing coursework? What am I doing? What is the teacher's role?' And I decided from listening to that tape [audio tape of one of his lessons], one of my biggest roles was listening. I was actually encouraging, saying 'Yes, great, super, that's a good idea!', which is a very important role. It's nothing mathematics but it's still a very important role. I got a lot of questions like "Is that the way you do it?" And with a lot of them, yes it was.

Data item 8.2: Extract from conversation with Ben (28.9.88)

His words "It's the wrong reaction, but maybe it wasn't" seem to indicate his realisation that although I may have meant not to be critical, his interpretation of my question as criticism had actually been of value to him.

In Data item 8.3 below, Ben acknowledged the difficulty of what I asked. However, he did tackle the question, and although I have not included his lengthy response, it provided valuable insights for me, and I suspect for him too.

Difficult questions

Ben If today's activity doesn't get them there, I will try and develop a different activity that *will* get them there'.

We were referring to Surface Area, which was 'on the syllabus'.

BJ What does *getting them there* look like, and how will you recognise it?

Ben All I can say is I wish you wouldn't ask such difficult questions.

However, his further response, in terms of assessing 'getting there' was very illuminating.

Data item 8.3: Extract from conversation with Ben (30.11.88)

In data item 8.4, Ben indicates that he has become used to the sorts of questions I ask, and anticipates what I might ask next. I feel here a sensitivity to my distancing questions which suggests that Ben might start to ask himself such questions as part of his own reflection.

Anticipating a question

After Ben's constructivist statement, which I quote in Ch 7 (see p 208), and which he ended with the words,

Ben ... sharing your knowledge with people, which is not possible.

I then asked, "Is it not ever possible, not at all?", and he replied,

Ben Yeah, I suppose it is. Now you're going to pin me down and say 'when' aren't you?

Data item 8.4: Extract (1) from conversation with Ben (24.11.88)

Finally, in data item 8.5, again Ben anticipates what I might ask, and I explore whether he feels threatened by the sort of question which he anticipates. He indicates not so much a problem with our relationship, but that my questions force him to address experiences with which he is not happy and which make him feel negative.

Threatening questions

Ben was referring to aspects of the probability lesson (referred to briefly in Chapter 7) and reasons why it did not go very well. He said:

Ben We never really got going did we? Now you're going to ask me what I mean by 'got going'!

Later, in the same conversation I asked, "Do you find these questions threatening?", and he replied:

Ben Sometimes, yeah.

I asked if it was possible to say when ...

Ben Some of them get very near what I call my professional, err, sometimes I feel that you attack my professionalism by some of the questions. Yes, and when things are not that successful, yeah, they feel a lot more threatening don't they?

I wondered whether I ought to be more careful...

Ben I'm not sure it's between us. Maybe it's something inside as well. Because when you've had a good lesson you feel, 'I'm a great teacher'. When you've had a poor lesson and things have not gone the way you had hoped, you know, I don't know how you feel, but I feel, you know, 'God why am I teaching? I should take up collecting money on the beach for the deck-chairs!'

I asked whether he actually learned more from these negative reflections?

Ben I'm not sure. Because when things have gone well, they're positive things you can write down and things you can use again. When things have gone badly they're negative. You can say, "What am I going to do differently?", but they're things you sort of say, "I'm not going to do that again."

Data item 8.5: Extract (2) from conversation with Ben (24.11.88)

STAGE 3 – CRITICAL ANALYSIS

As a researcher exploring a particular style of teaching and trying to get at the teacher's motivations and beliefs which lay behind what I saw in the classroom, I wanted to become aware of issues which were associated with these. Thus the stage of *critical analysis* which had potential to result

from the *accounting for* stage was a very important one to try to reach. The main feature of this was that we were able to break through the barriers in *accounting for* and jointly inspect what lay behind. I shall turn to an example here to illustrate what I mean by this.

Data item 8.6, which follows, contains an example taken from a conversation with Mike after the lesson on Pythagoras where pupils had been making a poster to convey their thinking from the Pythagoras lessons. The transcript is quite lengthy, although I have edited it where it is possible to do so without reducing the sense. Its essence for me lies in the words 'cognitively dense' to which I referred briefly in Chapter 6. The notion of *cognitive density* was one which Mike and I subsequently explored at length, and which captures notions about the depths of mathematical thinking which can take place in a lesson. I use the example to indicate the teacher's movement through reflection to levels of critical analysis.

Just a cut-and-stick lesson

Mike It was a cut up and stick lesson. It was one of those where they were just making, you know, finishing it off – 'They are just finishing it off, Barbara, don't bother coming in!'.

And I thought, whatever I'm doing, I've got a question in Barbara's (inaudible), I mean, let her come in and see if it is a waste of time, then we'll sit and talk about it afterwards. If we can perceive it as a waste of time, then fine, I've learnt something. If I think it's a useful activity, I've got to be able to justify it and I didn't (inaudible) and I got a lot more out of it today, actually having decided, that's it, we're going to do that. But I felt there was a lot there.

I have omitted a portion where he goes on to talk about particular pupils' work in the lesson.

BJ I want to pull you back to this phrase, 'a waste of time'.

Mike I knew you would!

BJ And think about from whose point of view. I mean, were you thinking about me wasting my time? Or were you thinking of it being a waste of time for the kids; were you thinking about it being a waste of time for you?

Mike Yes, all of those. I think a waste of time for you, yes, in terms of I wondered how much you were going to get out of it in terms of what was going on. And how I was going to use this 'make a poster' idea. I think my initial aim in using it was to make the end result the thing, the end result the poster, a graphic way of showing.

BJ So what you want out of it is a product?

Mike What I wanted out of it was the product. Right. The production of it was a necessary evil.

Um, in a sense it was over long. I think perhaps it was not cognitively dense. It's, as an activity itself is quite good, but it's sort of watered down like orange juice. It takes a long time to do that, but the activity is worthwhile ... and I wanted to make sure the activity itself was useful. And I found it was today, because a lot of them, there were very little demands on my time today, and I could have used that lesson to finish off some reports, or do something else, or tidy the cupboard; because they were all happy. They were all sticking their papers.

BJ Why didn't you?

BJ Why didn't I? Two reasons, One is because you were coming in, and I can't avoid that. And the other is that I actually – last night, I suppose, because you were coming in, and because I want, I put myself on the spot and thinking no, I'm not going to say it's just one of those lessons. I'm making a statement that this lesson has got to be worthwhile, so I'll make this lesson worthwhile. I'll make an effort to get something particular out of it. And if I'm thinking that a poster is a useful tool, then it's got to have another function, not just the production of the poster, and that is to get them to reflect and focus ideas. And also time for me to talk to groups that I might not have time to talk to without having them feeling they want me.

He went on to refer to particular pupils he spoke to, and the value of talking with them; in particular that they had gone through the 'doing' and were at the 'recording' stage of the process 'do, talk and record', (Open University, 1982) but they had not actually gone through the talking stage. He was able to get them to talk, and in doing this revealed some gaps in understanding.

Data item 8.6: Extract from conversation with Mike (20.2.87)

In this example, the anticipation of my presence and the questions I would ask encouraged the teacher to address fundamentally his motivation for offering a particular type of lesson. In doing so he highlighted what he felt to be inadequacies in his objectives, modified his approach, and subsequently was able to point to important results which had emerged from the revised approach. His use of the words 'cognitively dense' along with the phrase, 'a waste of time' highlighted for me significant considerations for different types of lessons, and we were able to discuss this and jointly learn more about how we viewed time spent on different kinds of activity in the classroom.

Mike referred to his own development in the words "if it is a waste of time, then we'll sit and talk about it afterwards. If we can perceive it as a waste of time, then fine, I've learnt something. If I think it's a useful activity, I've got to be able to justify it." Later, he referred to "And if I'm thinking that a poster is a useful tool, then it's got to have another function, not just the production of the poster, and that is to get them to reflect and focus ideas. And also time for me to talk to groups that I might not have time to talk to without having them feeling they want me." He indicated that he had found particular value in getting pupils to talk through their ideas before trying to record them in the poster. I suggest that, as a result of this reflection and critical analysis, Mike learnt a great deal about classroom practice. He would be unlikely to come to future poster lessons in quite the same way again. I suggest that as a result of this experience he gained both in *teaching knowledge* and *teaching wisdom*.

I see *teaching knowledge* in terms of the *theory* of teaching, and *teaching wisdom* in terms of the *practice*. I suggest that Mike's *knowledge* increased in a number of respects which include:

1. The potential in getting pupils to make a poster;
2. The value of using his time, when the class is apparently busy, to talk to pupils who may not need him specifically but who can gain from a talking stage between doing and recording;
3. The cognitive density of a lesson or activity.

It is less easy to say in what respects his *teaching wisdom* might have increased. Teaching wisdom is what a teacher brings to an *in the moment* decision. It involves a moment of choice in which rather than acting instinctively, the teacher can act *knowingly*, can choose to act according to some aspect of teaching knowledge. (See, for example, Cooney, 1988; Davis, 1990)

An example of *teaching wisdom* arose, in circumstances which the above situation reminds me of, in Phase 1. Felicity's class were drawing tiling patterns of quadrilaterals which they had decided would tessellate. Felicity came up to me and said, "I want to tell you now in case I forget later. I'm at a *decision point*. They seem to be happy just cutting and colouring, but I feel not much mathematical thinking is taking place, and I have the urge to move them on. I stopped when I realised it was a *decision point*." The essence of this is her focusing on *decision point* which was a notion which had arisen for us from reflections on previous lessons¹. In the moment of trying to decide whether to 'move them on', she realised that she was about to make a decision, and relayed that noticing to me. I suggest that her teaching wisdom had developed by her ability to stop and recognise that she was making a decision, *in the making of it*. Her awareness of her own practice was more acute, and thus she was more in control of what she wanted from the teaching situation.

As a result of Mike's experience, described above, it is possible that he would similarly find himself in such moments of awareness in the classroom, when suddenly some aspect of his teaching knowledge, perhaps with regard to cognitive density, became available to him in the instant to allow choice of action, greater awareness of purpose, and more control over the teaching act.

My emphasis in writing the above has moved from the *teacher-researcher relationship* into the *reflective teacher*. I shall say more about the reflective teacher below, but shall conclude this section by returning to the researcher's aims with regard to the teacher's reflective activity (see fig. 8.1). In trying to identify aspects of an investigative approach to teaching mathematics I wished to observe not just what occurred in mathematics classrooms but to reach for the underlying principles in and issues arising from investigative practice. In observing the teachers go through reflection and critical analysis, I was able to engage in issues related to their thinking. Often these issues were ones which I distilled through my own thinking, but I should have been unlikely to gain the insights which this required, without the reflective activity on the part of the teachers.

The dashed line in figure 8.1 indicates the possibility of my feeding into the discussion my own interpretations of what had occurred in the lesson. Although I tried to avoid doing this with Clare, for reasons related to objectivity, I found myself drawn into the issues involved where Mike was

¹ For my own reflections with regard to decision points see Appendix 3, section 2.4

concerned². With the realisation that objectivity would not be achieved whatever my level of intervention, and that all interpreting must be related to its history and context, I intervened in Phase 3 as it felt appropriate to do so particular to the situation concerned.

Reflecting on the conceptual model

The model which I present above is my own synthesis from the work which took place. However, aspects of it accord with the work and thinking of others in this area as I shall highlight briefly in this section.

Elbaz (1987) speaks of “a large gap between what researchers produce as reconstructions of teachers’ knowledge ... and teachers’ accounts of their own knowledge”. Inevitably, my analyses of our conversations were different from accounts the teachers themselves would have given. For example, Ben and Mike each said that they would not have regarded themselves as *constructivists*, not having previously encountered this term. However, I feel that I have been able to justify this interpretation, and both teachers responded positively to accounts which I gave. Elbaz (1987) expresses the following hope:

I would like to assume that research on teachers’ knowledge has some meaning for the teachers themselves, that it can offer ways of working with teachers on the elaboration of their own knowledge, and that it can contribute to the empowerment [of] teachers and the improvement of what is done in classrooms. (p 46)

I have suggested that this was the case in my study.

This was supported by Mike, whom I invited to comment on my conceptual model of teacher-researcher relationship. In doing so he questioned my use of the term ‘distancing’ and associated aspects of the model. In his words,

I’m now trying to remember back to how I felt during that time when you were coming in to my lesson. I believe I actually looked forward to it, and there were a number of factors here:

1. I felt valued. If you were coming in to see me then surely there was something of worth there. I supposed I felt “chuffed” in a way.

² See Appendix 4 for a comparison of transcripts relating to my differing intervention in interviews with Clare and Mike.

2. I actually came to like you – from that came a trust that you weren't there to "catch me out"
3. You asked the "right" questions. I can recall being pushed to find answers to questions which I might not have consciously asked myself, but which I felt – on your asking – had been lurking beneath the surface.

I notice that you use the word "hard" and suggest that it had something to do with distancing. I'm not sure I see it that way.

They were "hard" because they were challenging. They were questions I thought I ought to know the answer to but hadn't clearly articulated. I felt the question was important to me.

I'm also not sure your questions *did* "distance" me from my practice. In fact they really took me deeper *into* it. That was part of the reflective process for *me*. To me the two birds metaphor³ doesn't imply distancing but separation. I can be separate, but still very close. (Mike, March 1991)

I find these remarks on *distancing* versus *separating* very valuable in pointing to the teacher's alternative conceptions. Although the term *distancing* was my own, I have since noticed other writers using the same or similar terms. For example, Elbaz (ibid) speaks of a teacher who tackled a problem in her teaching through deliberate comparison with another situation where she felt successful, taking care to control as many aspects of the two situations as possible. Elbaz writes, "the controls had obviously helped her to distance herself sufficiently from the painful aspects of her teaching to make confrontation possible". In this case too, *separating* seems an alternative term to *distancing*. Kemmis (1985) uses distancing to mean *the attitude of self-reflection*. Calderhead (1989) speaks of student teachers having difficulty in "the detachment from their own practice that enables them to reflect critically and objectively". Pearce and Pickard (1987) talk of "standing back from events". *Detachment* and *standing back* are again alternative terms. However, the concept seems well recognised despite the variations in terms used for it.

The model which I offer has similarities with that proposed by Schön (1983 and 1987). Schön described tacit knowing as *knowing in action*. He suggested that the practitioner could move through *reflecting on action* to *reflecting in action*, acts which would result in her knowledge becoming more overt. I suggest, for example, that, in the incident I described above, Felicity was already *reflecting-in-action*, having become aware of her

³ The 'two birds metaphor' (e.g. Mason, 1991) is a concept which Mike and I have discussed at some length, involving one bird *eating* – getting involved in the activity – while another is *looking on* - reflecting on the activity.

decision-making. Joy Davis and John Mason have elaborated a process which they call the *discipline of noticing*, in which a practitioner *notices* and reflects systematically on short incidents, events or *moments* from her practice, in a deliberate attempt to become more knowledgeable about her practice. (See for example Davis, 1990) I have used the terms *giving an account of* and *accounting for*, I believe, consistently with their work.

Teaching knowledge and Teaching wisdom

I have introduced, and to some extent explicated, the terms *teaching knowledge* and *teaching wisdom* above. However, before proceeding to a discussion of reflective practice, I shall say rather more formally what I mean by these terms.

Much has recently been written about our knowledge (or the lack of it) of teachers' knowledge (e.g. Calderhead, 1987). I emphasise that I am talking here of what has been called *pedagogical* knowledge (e.g. Smyth, 1987a), *professional* knowledge, (e.g. Calderhead, 1987) or *craft* knowledge (Berliner, 1987). It is the teacher's knowledge of the practice of teaching. Schön (1983) expresses my perspective well:

... both ordinary people and professional practitioners often think about what they are doing, sometimes even while doing it. Stimulated by surprise, they turn thought back on action and on the knowing which is implicit in action. They may ask themselves for example, "What features do I notice when I recognise this thing? What are the criteria by which I make this judgement? What procedures am I enacting when I perform this skill? How am I framing the problem that I am trying to solve? (p 50)

It is no accident that the above remarks consist largely of questions. Tom (1987) argues that although "pedagogical knowledge is assumed to be useful to practicing teachers", what is needed is "incisive pedagogical questions" which will "not so much tell us what to do as how we might proceed to address our obligations and tasks as teachers". He acknowledges that "these questions, indeed, may at times pull our thinking and acting in differing and perhaps conflicting directions." Tom provides examples of questions which might be asked – interestingly, from my point of view, expressing them in three categories: craft questions, moral questions and questions related to subject matter. He refers to subject matter as "the third element of the teaching triad". I observe that this triadic differentiation corresponds very closely to the one which I have made, where *craft* corresponds to management of learning, *moral* to

sensitivity to students, and *subject matter* to mathematical challenge. My own categories are rather more finely honed to the teaching of mathematics as I have observed it, whereas Tom offers his in more general terms.

It seems clear that the asking of questions will lead to differing answers depending on who asks them. I have also shown that it leads to tensions and dilemmas, as Tom indicates, and have explicated in detail some of those which seemed likely to have general implications.

However, I have not sought to generalise teaching knowledge, rather to express my belief in the value of each teacher's personal explicated knowledge and its overt application. Thus teaching knowledge is what the individual teacher overtly synthesizes from reflection on, or in, practice by seeking to answer the crucial questions which arise. Teaching wisdom involves the ability to draw on this knowledge in practical situations. Shulman (1987) discusses "The wisdom of practice". These concepts are fundamental to an epistemology of practice which I shall express in my final chapter.

The reflective teacher

I have talked extensively about teachers reflecting, within a context of the teacher-researcher relationship. My evidence is that this relationship was very fruitful in encouraging such reflection. The teachers themselves testified to having gained from this in terms of their own professional development. Clare, Mike and Ben referred overtly to aspects of their professional development which were a consequence of our work together. I have included some remarks from Ben and from Mike above. Clare wrote a piece for me entitled, '*Being looked at*' in which she discussed her own reactions to how that work affected her teaching. I have included a significant paragraph from this at the end of the *Clare* section in Chapter 6 (see page 156).

The teachers at Amberley regularly gave up hours at the end of their day in order to talk through with me what had arisen in the day's lessons. This meant that their marking and preparation had to be fitted in at other times. Yet they determined, when our work together came to an end, to continue meeting themselves to talk together similarly. What was rather sad was that this did not take place. I heard from them that other pressures were too great.

I fully acknowledge my debt to the teachers concerned for the time and consideration which they gave me. This study could not have taken place without them, and I should have been very much poorer in terms of the knowledge and wisdom which *I* have gained. However, I recognise that I played a role for the teachers too. Firstly I was *there*. I turned up regularly and had to be fitted in. Whatever the other pressures, the teachers rarely turned me away or were too busy to talk to me. This meant that they spent time reflecting on their teaching, because that was one of the main reasons for my presence. I was therefore an agent of making them give their precious time to reflection on teaching. Or, put more positively, I was a way of enabling them to find time for the important act of reflection, for which they might not have found time otherwise.

The point which I would make is that all teachers should have the opportunity to reflect, and it should not be a luxury only to be afforded at the end of the list of demands. However, there is another consideration which might be much more powerful than the time factor, and that is that reflection without some motivating, supporting, driving, external agent is very difficult to achieve and sustain. Where these teachers were concerned, I was this external force, my questions kept coming, and they had to be addressed. Asking one's own searching questions is very much more difficult.

The *teacher* part of the flowchart in figure 8.1 can be extracted from the flowchart as a whole to give the sequence of activity shown in figure 8.2 on the next page.

I emphasise that a diagram such as this cannot do justice to the process as a whole which involves cycling between the stages as well as subtleties which the simple headings and descriptions of them can not ever totally encapsulate. As in the characterisation of an investigative approach, it is the manifestations of these stages which provide a glimpse of the potential or power of the process.

The teacher alone would have to undertake the functions on the right without external support. However, suppose the teacher is *not* alone. A group of teachers working together could perform these critical functions for each other for their mutual support.

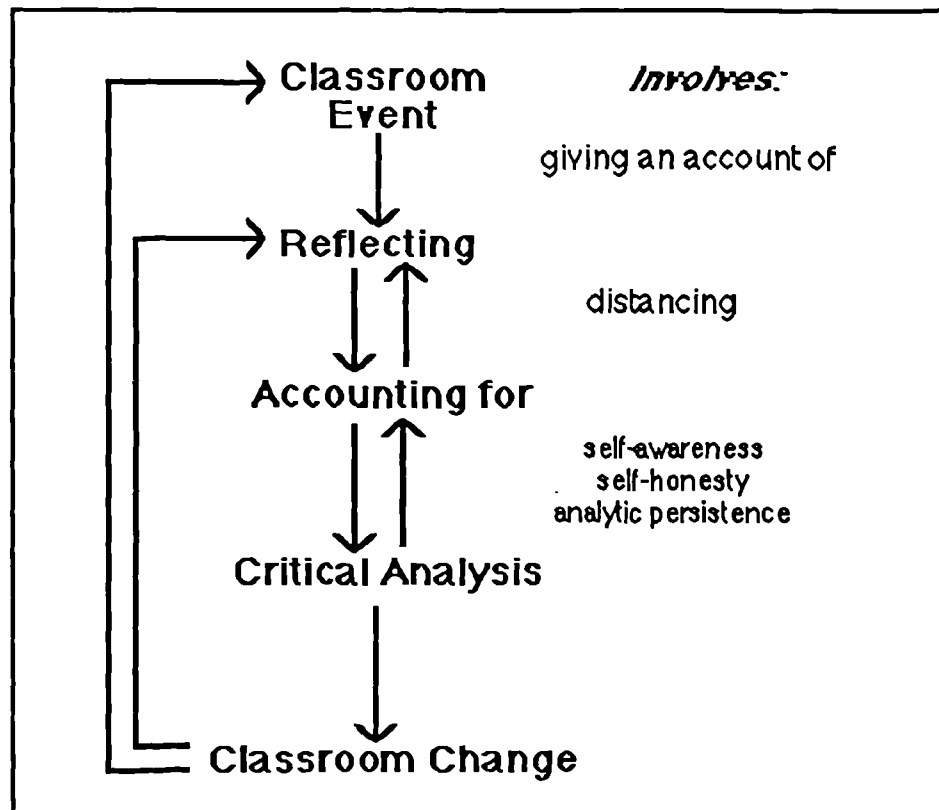


Figure 8.2: The reflective process for the teacher

Gates (1989) speaks of a process of mutual support and observation which teachers in one school engaged in which developed a trusting supportive environment in which teachers could work together to develop their own practice. Others have emphasised the importance of a supportive environment for reflective practice (Zeichner and Liston, 1987), or a supportive ethos for teachers' professional growth (Tabachnic and Zeichner, 1984; Nias, 1989).

This study points to the value of the above sequence of events for a teacher *with support*. It is therefore suggested that teacher development agencies and teachers themselves might consider the levels of support possible to allow the implementation of supported reflective practice. 'Develop your Teaching' (Mathematical Association, 1991), which was written to address some of these ideas from a practical point of view, suggests some such means of implementation.

Developing investigative teaching

I suggested at the end of my section on *Simon*, in Chapter 7, that one approach to teacher development might be to introduce a teacher to the teaching triad, encourage her to use it to start to think about her own practice, and support her in reflection on her teaching and corresponding modifications to her practice. The reflection and modification might be encouraged to fit the stages described above.

The term 'investigative teaching' is one which I introduced to describe a concept of teaching which I envisaged. Although the term has currency in the mathematics education community, I know that my interpretation of it is often different from that of others who use it. Thus, encouraging a teacher to undertake 'an investigative approach' lacks any clear sense of direction, and may be confusing rather than helpful. The confusion, potentially, is bound up in another currency, that of *doing investigations*, which, in my use of terms, does not constitute an investigative approach. Thus, the teaching triad, which I have shown to be closely linked to what I have described as an investigative approach, but which is more clearly defined, is perhaps a better starting point for a teacher who wishes to embark on investigative work.

However, *doing investigations*, is a way in which many teachers have begun to work investigatively. This was true for me, and it was true for the teachers at Amberley. The danger in this is that investigations are seen as an end in themselves, rather than as a vehicle for introducing investigative techniques and processes which can then be used in other mathematical thinking and learning. When I first observed lessons at Amberley, I believe the teachers' aims were in the investigations themselves and their outcomes for the pupils. Little consideration at that stage went into the development of processes for mathematical thinking and problem-solving more generally. However, possibly as a result of our early discussions, I observed a movement towards considering associated ways of working and their application to other teaching. Towards the end of the Phase 1 work, in the *tessellation* lessons, the teachers were exploring how the ways of working in doing investigations could be applied to the teaching of areas of syllabus content. The department also ran an individualised scheme in parallel to the investigative lessons.

At Beacham, the teachers did not claim to be using an investigative approach, although what I saw was a more advanced form of what I had seen begin at Amberley. Investigative processes such as specialising and

conjecturing were in evidence. Ways of working were pupil-centred – pupils' views were respected and ideas valued. Questioning was encouraged. Exploratory tasks were a common part of lessons. At the same time, in parallel to this work, the department ran an individualised scheme in which pupils worked at their own pace on graded workcards. The two types of work, running in parallel, provided a balanced approach for pupils between direct consideration of syllabus work, in the scheme, and exploratory work supporting and extending the scheme and producing coursework material in anticipation of GCSE. The Beacham situation seemed to be a more mature version of that at Amberley. The ways of working were more established, teachers had made their aims more explicit, and there was greater confidence in the enterprise as a whole. I should qualify this slightly. There were members of the department at Beacham who were not as secure in these ways of working as Clare and Mike whom I observed. These teachers were much closer to those at Amberley.

The two paragraphs above are designed to provide a flavour of what I see as a progression in development across teachers in the two schools. They involve different teachers, but the progression observed might be paralleled by the development of a single teacher, starting off by doing investigations in the classroom, and gradually developing this into an explicit way of working which can be applied to the teaching of mathematics in syllabus areas. Thus the development from *early Felicity* to *late Clare* might be possible for a single teacher. The next stage would then be to move on to *Ben*.

Ben had spent two years as an advisory teacher, during which time he had worked with teachers in their classrooms and run courses for teachers. He then developed a teaching approach which was a synthesis of this experience, and owed much to his own thinking during this time. The approach, he claimed, was designed to be investigative, and, as I have said, his meaning of this term was likely to be close to mine. Yet, some of his lessons he labelled *didactic*. I explored the tensions in this in some detail in Chapter 7, but its relevance to this chapter lies in its indications for levels of teacher development. I suggest that teacher development does not have an end, and that there is no such thing as a fully fledged investigative teacher. An investigative teacher continues to investigate through the reflective process. I see Ben still exploring what it means to teach investigatively, and in particular relating ideals in this to the practicalities with current working pressures and the demands of the National Curriculum. However, I feel that Ben has reached a plateau of

development, in which he is now refining his ideas and exploring issues more deeply rather than modifying his practice in major ways. I feel the state which Ben is now in is one to aim for. It involves a mature sense of purpose, an explicit working practice, and a natural reflective attitude to everyday work.

My own development as a reflective practitioner

In producing a reflexive account of my research study I have included, throughout, details of my own thinking and its development. This was particularly overt in the two interludes between the research chapters.

I claim that my own development as a researcher parallels the teacher development of which I speak above. Although this thesis as a whole charts my development, for the purpose of this chapter, I shall outline it in terms which explicate the parallelism.

1. Throughout my study I have recognised *significant* events.

For example Clare's 'hands down think' was significant enough for me to record it at the time she first uttered it, and I subsequently fitted it into a pattern of Clare's overtly encouraging thinking in her pupils

In the early stages of my study I noticed and recorded classroom events. Recording involved *giving an account of* what I had noticed. I did not ask *why* I noticed it until after the event. Later, in reflecting on what had occurred I tried *to account for* my observation. This *accounting for* included the purpose of the event as I perceived it in teaching and learning terms, and my own reasons for noticing it.

2. I engaged in critical analysis in
 - interrogating my own theoretical perspective, looking for some *fit* with the event;
 - placing the event alongside others, seeking for common significance which could lead to patterns, and possibly to an emergent theory.
3. I took action to validate my resulting thinking in some or all of the following ways:

- I read through transcripts of interviews related to the event, seeking resonance with my own perceptions;
- I sought perceptions of other participants, comparing their account with my own both in reporting on and interpreting the event;
- I tried out my own account on other participants, again seeking resonance with my own perceptions.

This process had the following outcomes in terms of the event labelled *hands-down-think* from the example above:

- It fits into what I have described as *Management of Learning*; it can be seen as an attempt by Clare to indicate to students that their thinking is important, and that giving time to thinking is valuable.
- It fits a constructivist perspective: Clare pays overt attention to pupils' construal of what they have encountered by emphasis on thinking, and provision of time.
- It provides an example for linkage between ML, which is a synthesis from classroom observations, and constructivism – my epistemological base. Thus it is an agent for linking theory with practice.

From the above description, it might appear that the process which I describe is linear, finite and straightforward. In practice the process was, and is, cyclic and complex. The stages of *accounting for* and *critical analysis* typically involved seeking generality in some form. They were demanding in terms of awareness and persistence. They required a capacity for distancing, seeing events from both inside and outside in terms of my own theoretical perspective as participant in the event and my view as external observer. Events did not come singly, so the amount of information processing was high. Reflection on events from one lesson overlapped with observation of subsequent lessons. Interviewing often came before time for reflection and could involve simultaneously discussion of past, present and future lessons. Identification of characteristics of one event led to heightened awareness in perceiving subsequent events.

One particular difficulty, as I have expressed before, was the recognition of significance soon enough for validation with other participants, particularly pupils. Seeking perceptions of pupils some time after an event rarely proved fruitful. Either they could not remember the event, or they could not re-enter it sufficiently to summon up the images which it created for them at the time. The most helpful remarks from pupils came when I was able to ask for their perceptions very close to the event. For an event such as Clare's *hands-down-think*, I should have liked to solicit pupils' views when it occurred. Recognising its significance when I reflected after the event was too late. Ideally I should be able to spot it, in the moment, and ask about it there and then if possible. This requires a high level of awareness/consciousness of the generality or underlying issues which motivate the recognition of significance.

The recognition of significance *in the moment* was neither natural nor easy; neither was it obvious how it might be pursued. Two indications of my own development were firstly when recognition started to happen, and secondly when I noticed that it was happening. An example is the recognition of *didactic tension*. I became aware of didactic tension through reading Brousseau (1984) and Mason (e.g. 1988b, c), in resonance with my own experience and in discussion with colleagues. I could look back to my records of lessons and recognise instances of didactic tension. However, in one transcript of conversation with Ben after a lesson, I observe that I referred to didactic tension as part of our discussion. This indicates that didactic tension had reached a high level of consciousness, so that I could notice manifestations of it when they occurred, which gave me the chance instantly to seek resonance with the teacher. When subsequently I recognised that this is what I had done, the process became available to me to use overtly.

The attribution of significance in the *hands-down-think* example was in terms of my construal of teaching in constructivist terms. What I have just described is an alternative level of significance which may be thought of as the significance of *the attribution of significance*. This might be seen as analogous with the mathematical concept of 'function of a function'. At a simple level, a function operates on some variable, and values of the function can be obtained from particular values of the variable. At a more complex level the function can be seen to operate on other functions, and in particular on itself. Perception of this operation demands a clear understanding of the function and its levels of application. Students' difficulties with such application often arise from confusing the levels.

In coming to terms with attribution of significance and its relation to rigour in this research, I have had to struggle to become clear about levels of attribution and subsequent validation. Nolder (1991) quotes a teacher who said “when you undergo change yourself you don’t really see it – it’s bit by bit every lesson.” The essence of being a reflective practitioner is in becoming aware of change, or the possibility for change, in order to influence its direction. This lies crucially in the act of *distancing*. It involves deliberately and determinedly interrogating one’s own experience, asking *what* (giving an account of) and *why* (accounting for). I believe that change is discrete, whereas development is continuous. Change has a sense of being done *to*, whereas development includes the possibility for continuous involvement and influence. An epistemology for practice might be seen as knowingly influencing one’s own development through controlling its directions.

Reflective practice is ‘critical’ and demands ‘action’

Much of the literature in the area of reflective practice emphasises the importance of *critical* reflection. For example Van Manen (1977) defines reflection at three different levels the third of which, *critical reflection*, concerns the ethical and moral dimensions of educational practice. Boud, Keogh and Walker (1985) refer to “goal directed critical reflection” which concerns reflection which is “pursued with intent” (p 11). They cite Mezirow (1978, 1981) who talks of ‘perspective transformation’ – “the process of becoming critically aware of how and why our assumptions about the world in which we operate have come to constrain the way we see ourselves and our relationships” (Boud, Keogh and Walker, 1985, p 23). I also pointed to Smyth’s (1987b) usage above (see page 276). However, it is Kemmis’s (1985) use of *critical*, closely allied to social action, which I feel is closest to the way I have used it. I shall therefore refer to this in some detail.

Kemmis writes:

We are inclined to think of reflection as something quiet and personal. My argument here is that reflection is action-oriented, social and political. Its product is praxis (informed, committed action) the most eloquent and socially significant form of human action (p 141)

He advocates *critical* reflection, in which reflection is concerned with thought itself, transcending strictly technical or practical reasoning to “consider how the forms and contents of our thoughts are shaped by the

historical situations in which we find ourselves". He claims that reflection, like language, is a social process, and *exemplifies these* thoughts by referring to his own activity in reflecting about reflection:

... consider my (critical) reflection about reflection itself in this chapter: again, I depend upon language and upon the significance of the issue to readers, but I also aim to place the idea of reflection in a context of history and social theory ... (p 144)

Kemmis examines Habermas's (1972) notion of interests guiding the search for knowledge, and in particular the emancipatory interest which "aimed at emancipating people from the dictates of taken-for-granted assumptions, habits, tradition, custom, domination and coercion, and self-deception". He goes on to speak of reflection in this context:

Critical reflection aims to discover how criteria have come to be accepted, to analyse their historical and social formation, and to organise social action towards emancipation; ... (p 146)

Kemmis claims that reflection is shaped by ideology, and itself shapes ideology. We reflect from our own ideological standpoints, and these ideologies change as a result of reflection. We make choices which influence our actions and affect our subsequent experience:

In reflection we choose, implicitly or explicitly, what to take for granted and what to treat as problematic in the relationships between our thought and action and the social order we inhabit ...

Reflection is a process of transformation of the determinate 'raw material' of our experiences given by history and culture, and mediated through the situations in which we live) into determinate products (understandings, commitments, actions), a transformation effected by our determinate labour (our thinking about the relationship between thought and action, and the relationship between the individual and society), using determinate means of production (communication, decision-making and action). (p 148)

Kemmis claims that the scientific study of reflection aims to improve reflection, and that this should explore the double dialectic of thought and action, of the individual and society. He examines the place of both empirical-analytic studies, and interpretive studies in this exploration, finding them both wanting in some respects, and proceeds to define forms of study which overcome these objections. These, he claims, must be conducted through self-reflection and must engage individuals and groups in ideology critique and participatory, collaborative and emancipatory action research (Carr and Kemmis, 1983).

Kemmis suggests that such study converges with critical social science, which:

requires the separate but simultaneous development of scientific discourse, the development of understanding and insights, and the development of practical action ... [and] presupposes a community of participants-researchers committed to the critical development of their own social life: their practices, their understandings of these practices, and the institutions and situations they inhabit and constitute through their action. Such an approach to social science is emancipatory in the sense that it is aimed at overcoming felt dissatisfactions, unjust processes of social control and decision-making, and irrational processes of communication. (p 155)

and that these criteria are met by *emancipatory action research*, which is a form of critical social science, and is increasingly being employed in educational settings including those involving professional development. It involves participants in:

planning action (on the basis of reflection); in implementing these plans in their own action (praxis); in observing or monitoring the processes, conditions and consequences of their action; and evaluating their actions in the light of the evidence they collect about them (returning to reflection) as a basis for replanning and further action. This is the spiral of self-reflection composed of cycles of planning, acting, observing, reflecting, replanning, further action, further observation, and further reflection. (p 156)

I believe that this describes quite closely both my own activity in conducting this study, and the model which I have presented above to describe the reflective activity which is likely to form a part of investigative teaching. Although Kemmis speaks of the wider social scene, most of his remarks can be seen to relate to the social environment in which the mathematics teacher operates.

Conclusion

The research in which I have engaged has involved a deep level of enquiry into the motivations and beliefs of the teachers concerned. In order to get at these, it has been necessary both to engage myself, and encourage the teachers to engage in, deep levels of reflection. The teachers have indicated that this has been valuable to them in learning about teaching, increasing their own teaching knowledge, and developing their classroom practice. It seems that I have played a supportive role in their development in provision of opportunity for, and encouragement to

sustain, reflection, and pressure to inspect sensitive areas. The implications of this for teacher development more generally are,

1. that the stages of reflection which I have discussed have potential to be of value to teachers beyond the bounds of this study in working on and influencing the direction of their own practice;
2. that some form of support is necessary. This could be in the form of support from colleagues.

I have discussed the progression in development which I have observed across the three phases of my research, and have suggested that this progression might be relevant to a single teacher working on developing an investigative approach to teaching mathematics. Because the language of 'investigative teaching' involves a set of diverse meanings, I have suggested that the *teaching triad* might be a starting point for any teacher in questioning classroom practice and developing more aware and explicit approaches to teaching.

However, the term *investigative teaching* itself takes on new meaning as a result of this chapter. It can be seen in terms of the teacher actively investigating the teaching process along with colleagues within the social context of the school. This could result in teachers increasing their teaching knowledge and teaching wisdom, and, in Kemmis's terms, becoming *emancipated* within this social environment. Emancipation might be seen in terms of controlling their teaching with confidence from a sound knowledge base, rather than responding intuitively from what Schön has expressed as their knowledge-in-action.

CHAPTER 9

CHARACTERISTICS OF AN INVESTIGATIVE APPROACH TO MATHEMATICS TEACHING

In Chapters 5, 6, and 7, I discussed my analysis of classroom observation and conversations with teachers and pupils in the three phases of this study. In this I described many classroom situations, which I analysed with respect to my own theoretical perspective regarding the teaching and learning of mathematics through an investigative approach. I have explained how I see this approach embedded in a radical constructivist philosophy of knowledge and learning.

In this chapter, I wish to distil from these classroom manifestations characteristics and issues which seem pervasive. This is not to say that investigative teaching, or teaching according to a constructivist philosophy, will invariably have these characteristics, but rather that these have seemed to be significant in this study where I recognise that my sample of teachers is very small and selective. My purpose is to draw on the common themes which I perceive in the classrooms which I have studied. Delamont and Hamilton (1984), who were quoted in Chapter 4, claim that *some* degree of generalisation makes sense:

Despite their diversity, individual classrooms share many characteristics. Through the detailed study of one particular context it is still possible to clarify relationships, pinpoint critical processes and identify common phenomena. Later abstracted summaries and general concepts can be formulated, which may, upon further investigation be found to be germane to a wider variety of settings.

My purpose is thus to highlight common themes, which may subsequently be tested against 'a wider variety of settings'. I shall point to manifestations which seem to be in some sense germane to the investigative approach and to issues which have been commonly raised.

The classrooms

I shall begin by setting the common scene as I saw it in the classrooms I observed. In all cases throughout the three phases, the teachers worked with classes of 25–32 pupils. They set tasks which involved mathematics and on which pupils worked. These tasks and the way in which they were

set varied considerably both from teacher to teacher, and from lesson to lesson for any one teacher, depending on the particular objectives, both declared and undeclared, for any lesson. However, there were common features ...

1. Type of tasks which teachers set for pupils to work on.

I saw these in the main as inviting inquiry, raising questions, encouraging conjectures and requiring justification.

Typical tasks in this respect were Clare's *packaging*, Felicity's *tessellating quadrilaterals*, Ben's *Kathy-shapes* (see Appendix 5), Mike's *billiards*. In some cases, the focus of the task was particular mathematical content, as in *packaging* (volume and surface area) and *tessellating quadrilaterals* (properties of quadrilaterals); in some cases, as in *Kathy-shapes*, focus was both on mathematical content (area and perimeter of shapes) and on the processes of problem solving; in other cases the focus was the processes of problem solving, as in *billiards* which had no required mathematical content.

Issues for the teacher included: how to get pupils to work in this way; how to design a task to suit the way of working and the experience of the pupils; how to foster desired ways of working mathematically and on mathematics; how to enable pupils to reach particular mathematical conclusions; what to constrain or leave open within a task; how to balance such tasks against use of established mathematics schemes.

2. Introduction of a task by the teacher to the pupils

I saw this as designed to get pupils involved in thinking: sufficiently closed to enable all pupils to make a start, and sufficiently open to allow all pupils to extend their work according to their ability and interest.

For example, Clare invoked pupils' imagery by asking them to imagine lines crossing and to count their intersections. Mike asked pupils to measure the flow of water from a tap for certain angles of turn of the tap, in order to introduce variables and graphs. In the $\frac{x}{y} = \frac{x+y}{y+2x}$ lesson, Ben asked pupils to give him particular values for x and y and, as a class, to work out the values of each side of the equation and decide if they were the

same. In each case, pupils were given time to think, to make suggestions, to try out ideas for themselves and to become familiar with what was being introduced so that it became less threatening and possible for them to make a start with it..

Issues for the teacher included: how much to constrain; how constraints affect outcome; how desired outcome affects introduction of a task.

3. Organisation of the classroom – groups – discussion.

I saw informal arrangements of furniture. In classrooms at Beacham and Compton this was the norm. Pupils sat in groups around tables for all of their mathematics lessons. Often the arrangement of tables changed, sometimes to suit a particular task or activity, sometimes to suit the pupils who wanted to sit together or separately. At Amberley, classrooms were usually organised in rows of desks. For investigative work pupils were encouraged to move desks together and sit around them facing each other. For work on their individualised scheme they tended to sit in rows.

Groups were sometimes constructed by the teacher, and sometimes by pupil choice. Clare often constructed groups by asking certain pupils to work together. Mike and Ben mainly allowed pupils to choose where to sit and with whom to work. Talking to each other and working together within a group were usually encouraged overtly by the teachers.

Issues for the teacher included: of what such talk usually consisted; how the teacher could monitor what was being said and done; what the discussion actually contributed to learning; whether groupwork was actually more than just sitting together to work.

4. Use of apparatus or equipment. Practical work.

Many activities offered benefited from the use of some physical objects. In some cases the teachers provided, or asked pupils to provide, the apparatus which they wanted pupils to use.

For example, in *moving squares* Ben gave out counters, in *tessellations*, Jane gave out plastic and cardboard shapes. In

packaging, Clare asked pupils to bring in as many different shaped bottles and packets as they could find. In other cases, pupils decided that some form of apparatus would be useful to them and either got it from the cupboard or asked the teacher to provide it. Scissors, glue, and different types of paper were widely available. Pupils in all the classrooms were used to working with apparatus.

Sometimes tasks were *based* on some type of practical activity. For example, Ben asked pupils first to make certain three-dimensional shapes in order to introduce notions of *surface area*. Clare asked pupils to use pieces of string to construct various formations and investigate which ones formed a *knot*.

Issues for the teacher included: availability and distribution of apparatus; the particularity of the apparatus and encouraging pupils to abstract from it; the consequences of the lack of precision in the use of apparatus.

5. Mode of operation of teacher

When not engaged with the whole class, the teachers I observed were to be found moving around the classroom listening to and talking with groups or individuals. Some interactions might take only seconds, whereas others involved the teacher in sitting with the pupils for 10 – 15 minutes. Clare in particular often signalled to the class that she would spend extended periods of time with certain pupils, and planned overall that pupils could expect periodically to have her concentrated attention on their thinking.

Issues for the teacher included: where to spend time; differing demands on time; the nature of an interaction; lack of access to thinking of pupils when the teacher was not present.

6. Pupil activity and behaviour

I saw pupils in the main settling down to tasks set and working throughout a lesson mainly 'on-task'. Behavioural problems were occasional, not regular, and teachers dealt swiftly with them. They did not seem to upset any of the lessons.

In many of the lessons, the way pupils tackled a task was left up to them. For example in looking for *Kathy-shapes*, Ben's pupils first decided *what* shape they would focus on, e.g. a rectangle, and then decided how they would go about *seeking* Kathy-rectangles. In the first *Pythagoras* lesson, Mike's students were given two brief statements and it was left up to them *how* they would tackle the statements. In *packaging*, Clare's pupils had produced a list of questions in a brainstorming session, and it had been up to each group to decide which questions they would tackle and how.

Issues for the teacher include: pupils asking themselves 'what does the teacher want from us?'; self evaluation by pupils of their work; what teacher intervention is appropriate; how directive teacher involvement should be.

7. Teacher evaluation of learning, feedback for future planning

I saw teachers making judgments constantly regarding the way they interacted with pupils or advised or required pupils to work. My conversations with the teachers indicated that most interactions involved on the spot evaluation of pupils' thinking. Interventions were often geared to such evaluation, leading to an immediate form of feedback. Sometimes decisions were not clear cut and there was evidence of the teacher's struggle in making responses appropriate to the needs of the pupils.

Teacher's knowledge of pupils was based on these evaluations and this fed subsequent planning for the class as a whole.

For example, in the *Kathy-shapes* lesson, Ben was surprised by some pupils' apparent misconceptions in finding heights of triangles, and he subsequently based a whole lesson on triangles so that these misconceptions could be widely addressed. Clare quite often pinpointed areas of mathematics in which certain pupils seemed to have difficulty, and then used her KMP lessons to allow pupils more experience in these particular areas.

There are tensions for me in trying to express general features of the classrooms as I have been doing above. The most immediate is that no sooner do I set out to express a commonality than I come up against nuances of difference. As I have offered the particular examples in each

section, I have been struck by the *differences* in the particular manifestations and issues, as much as by the similarities which I am trying to express. This emphasises the complexity of the teaching act, its dependence on particular circumstances, on the objectives of the teacher, and on the prevailing classroom ethos.

A second tension is that some of the commonalities seem almost too obvious to express. There is, in the UK, some cultural currency in which certain aspects of mathematics classrooms are generally regarded as 'good practice'. Particularly since the Cockcroft report, *working in groups*, *encouraging practical work* and *classroom discussion*, for example, have been seen as part of this 'good practice'. It therefore seems that where 'good practice' is observed, such features should be evident. However, it is in the particular manifestations of these features that the issues for teaching arise, and sometimes these issues present such difficulties for the teacher that a return to more traditional teaching seems preferable to tackling the issues.

As I discussed in Chapter 3, Desforges and Cockburn (1987) report, as a result of working with teachers extensively over ten years, that they have seen no evidence of classrooms where *higher order skills* are seen to be operational consistently over substantial time periods. According to their research, even so-called *good* teachers are so bound by the pressures, constraints and demands on a teacher's time and energy that they cannot sustain enquiry methods, draw on the spontaneous skills and interests of children, and have the capacity to monitor each individual child, seeing when to intervene and when to leave alone (cf p 142). Their conclusion includes the following statement:

We set out on this investigation with the suspicion that the teacher's job is more complex than that assumed by those who advise them on how to teach mathematics. Put bluntly we have found what teachers already know: teaching mathematics is very difficult. But we feel we have done more than that. We have shown that the job is more difficult than even the teachers realize. We have demonstrated in detail how several constraining classroom forces operate in concert and how teachers' necessary management strategies exacerbate the problems of developing children's thinking. (p 155)

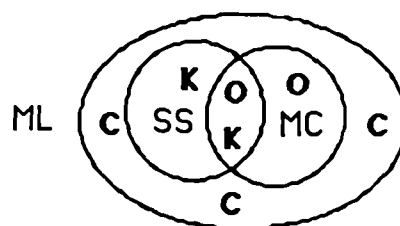
Their claims are that the teachers concerned, although espousing belief in notions of good practice and striving to achieve the development of higher order skills in pupils, nevertheless were unable to succeed within the current system.

In response to this, while recognising all the limitations of my own study, I feel I can say otherwise. In looking at 'an investigative approach to teaching mathematics' I have been focusing on teaching whose chief objective is the higher order skills of which Desforges and Cockburn speak. I selected my very few teachers in order to study the characteristics of such an approach, so I do not claim to speak of teachers more generally. However, these teachers did give evidence of offering high cognitive demands and achieving higher level thinking from their pupils over a sustained period of time. Remarks from their pupils suggested that this was not just something which happened when I was in their classrooms, and conversations with the teachers themselves revealed belief structures and reflective practices which could not have been invented for my purposes. I worked with secondary teachers, whereas the teachers studied by Desforges and Cockburn taught infants. These secondary teachers had in the main strong mathematical backgrounds, and their own mathematical thinking was well developed. This may have contributed to my different findings.

One respect in which my findings support those of Desforges and Cockburn is that of the *complexity* of the teaching task. Any attempt to generalise this results in over-simplified statements which seem to deny the importance of the particularities of their interpretation. I came up against this as I tried, above, to present some overview of the general features of the classes I observed. As a result of this, I now attempt to express the commonality which I perceived from a more global perspective, by considering *the teaching role* in the classrooms which I observed. This brings me back to the teaching triad and its relationship to the particular features of the classrooms detailed above.

The teaching role

I have suggested that the teaching role may be characterised by the teaching triad of *Management of Learning*, *Sensitivity to Students* and *Mathematical Challenge*, related as in the following figures reproduced from Chapter 7:



Letters O, C and K refer to aspects of teaching as in the next figure, and these in turn relate to the two principles of constructivism offered in Chapter 2.

Radical Constructivism	Teaching	The Teaching Triad
1. Knowledge is actively constructed by the learner, not passively received from the environment.	Offering mathematical challenges appropriate to the learner (O)	Mathematical challenge
2. Coming to know is an adaptive process which organises one's experiential world. It does not discover an independent, pre-existing world outside the mind of the knower.	Creating opportunities and an environment for mathematical thinking and exploration (C)	Management of learning
	Knowing the learner well in order to perceive both appropriateness of challenges and their fit with a learner's past experience (K)	Sensitivity to Students

The teacher's *Management of Learning* involves creating the classroom ethos within which higher level cognitive demands can be made (Mathematical Challenge) in a way in which pupils will be not only receptive to the demands but able to act on them to achieve higher-level thinking (Sensitivity to Students). I shall attempt now to analyse this teaching role as I saw it in the classrooms which I observed, and as it might be more generally applicable to teaching which arises from a constructivist perspective.

The teaching role may be seen as *controlling* the learning situation. I used the term *control* in terms of Mike's teaching, and to some extent it could fit the teaching of either Clare or Ben. Clare talked of 'training' students, and Ben talked of 'gaining control in order to give freedom'. However,

'control' carries with it many negative connotations, for example, of 'directing', 'telling', or 'demanding', and in general of *restricting* freedom. These terms would mostly be inappropriate to the work of the teachers I observed. Alternative words which have been used in education more widely are *fostering*, *encouraging*, *enabling* and *facilitating*. *Facilitating*, for me, carries too much of a sense of 'anything goes'. This was not usually the case, in my observations where teachers had very firm objectives. *Encouraging* is a positive term, and I certainly saw teachers encouraging pupils, although I felt in some cases that the degree of 'push' was more than encouragement. *Enabling* seems also positive, but can be seen as veering towards being directive. *Fostering* – Chambers Dictionary gives 'to *encourage*, to *promote*, to *cherish*' – seems closest to what I observed. It carries with it an element of caring, but at the same time of a clear promotion of values. I shall therefore express the control which I observed as a *fostering* of activities and attitudes within the classroom.

These activities and attitudes may be seen to be directed towards *establishing meaning*, *mutual respect*, and *responsibility for own learning* in the classroom.

ESTABLISHING MEANING

A constructivist philosophy places great emphasis on individual meaning-making and communication as a product of sharing meanings. Perhaps the most important aspect of the teaching and learning of mathematics is the development of meaning, its communication, and its reconciliation with shared meanings which have developed over centuries. Learning about, for example, *Pythagoras' theorem* and *vectors* must be seen first of all in terms of individual development of meaning, but secondly in terms of the reconciliation of individual perceptions with established conventions.

I saw the teachers promoting strategies and processes for establishing meaning and encouraging its communication. For example, I saw them invoking *imagery* (e.g. Clare's introduction to *lines crossing*, Mike's *racetrack*); I saw them encouraging pupils to *recall/reflect* (e.g. Ben's recalling understandings from the last *vectors* lesson; Mike encouraging recording in *red books*); I saw *negotiation* of ideas and methods (e.g. generalising in *moving squares* (Ben) and in *Pythagoras* (Mike)). I saw processes of *expressing* – saying what you see (e.g. in *vectors* and *lines crossing*), *questioning* (overtly required by Mike, and by Ben in *vectors*), *sharing* (e.g. in *packaging*, and in *moving squares*), and of *pattern spotting* (overtly required by Ben, and by Mike in *Pythagoras*).

Reconciliation of individual perceptions with established conventions was essential in the case of, for example, *fractions*, *vectors*, *Pythagoras' theorem*. I saw the teachers promoting established meanings while valuing pupils' individual perceptions. This took a variety of forms from exposition (Clare's *fractions*, Ben's *vectors*), through Mike's offering students the benefit of his own experience, as in the girls working on *square sums*, to Clare's use of cognitive dissonance in '*It's a cuboid*'.

ENGENDERING MUTUAL TRUST AND RESPECT

Two factors militate vitally against the *individual* nature of establishing meaning. The first is its potentially sterile, or at best narrow and limited, learning outcome. Communication allows meanings to become broader, richer, better reasoned and creatively extended (e.g. Bishop, 1984). The second is that classrooms typically have about 30 pupils. Even if it makes sense for the pupils to work independently of each other, the sharing of the teacher's time among them becomes ridiculous. They might each get only two or three minutes per week after administrative duties have been done.

Strategies and processes such as *negotiation*, *expressing* and *sharing* all demand cooperative activity and others such as *recall/reflect*, *invoking imagery* and *pattern seeking* benefit from pupils having access to the perceptions, images and patterns of others.

Such cooperation and beneficial sharing requires classroom harmony that, in my own experience, cannot develop without a requirement for mutual trust and respect in the classroom. I saw this in the classrooms of Clare, Mike and Ben, and gained some clues to its establishment. It was manifested in good relationships between teacher and pupils and among pupils themselves, which I felt was related to a requirement for a sensitivity to the needs of others. This was most overt in the teachers' obvious sensitivity to their pupils, which was manifest, for example, in Clare's awareness of gender issues and special needs of many students, and Ben's attention to the quieter members of his classroom, requiring *listening* when someone was speaking. Mike used effectively the technique of reporting back, in which students reported on their activity and ideas while others listened and later questioned. *Listening* was explicitly valued by being required in all the classrooms.

These three teachers all organised their classrooms in groups which enabled sharing and cooperation. Often the groups were self-selecting, but all teachers occasionally directed particular pupils to particular groups or

to individual work. The group organisation did not mean that pupils always worked cooperatively or that there was never individual work. Certain pupils elected to work alone at times, or were required to do so by the teacher. Sometimes, although a group was ostensibly working together, much individual thinking took place between acts of sharing. Occasionally, a teacher overtly suggested cooperation as being appropriate to the successful completion of a given task.

There were expectations of responsible behaviour. In the three classrooms, pupils usually moved freely about the room and spoke freely to one another. There was potential for disorder, noise and chaos. However, this was rarely the case. There were periods which were very noisy. Some were tolerated by the teacher, perhaps were necessary to allow or encourage everyone to talk about an idea. Others were restrained by the teacher asking for, and usually obtaining, quieter or silent work. Where pupils were overtly disruptive, the teachers dealt firmly with them as individual cases.

ENCOURAGING RESPONSIBILITY FOR OWN LEARNING

The establishing of meaning and mutual respect led to a classroom ethos in which pupils could work together on the development of mathematical concepts. Although the above sections have said little of mathematics, it has been *mathematical* activities which have formed the basis of meaning-making – negotiation and expression of *mathematical* ideas, *mathematical* pattern seeking, raising of *mathematical* questions, invoking of *mathematical* images. In the classrooms of Clare, Mike and Ben there was overt emphasis on *thinking*, which was *mathematical* thinking. However, I felt that this went beyond the subject to a meta-level of thinking about the act of learning itself. I felt that each of these teachers actively encouraged pupils to take responsibility for their own learning.

This could be seen in pupils' own recognition and use of the strategies which the teachers promoted. For example, Mike's students recognised *questioning*, Ben's *pattern spotting*. In *moving squares*, pupils talked about *conjecturing* and *deriving a formula*. There was a sense of freedom to explore situations and pupils followed diverse directions according to their own interests and abilities. In Mike's reporting back sessions, students actively questioned each other's results. Some pupils naturally extended their activity to new areas – for example Lesley's extending to rectangles in *moving squares*, and Nicky's extending to a cube in *Kathy-shapes*.

Many of the teachers' strategies overtly promoted pupils' reflection on their learning. Mike's use of red books asked students to reflect on and record their thoughts from a lesson, then to use this recording to help their recall in the next lesson. Clare had a variety of instructions which urged pupils to think about what they were doing and to plan ahead. Ben created situations to encourage pupils to make their own decisions. One statement from Ben is worth quoting here in this context:

Did I tell you about the interesting incident which I had there? One was explaining to the other about trig – it was Rachel to Pat, and I was sort of talking with them and I went away, and then suddenly realised what I'd been saying. I was not talking about trig – I wasn't even talking about that. I was talking about the role of the teacher and the learner, and their responsibility. And that's a really peculiar position for a maths teacher to get into in some ways isn't it? You know, I've left my subject, in effect, for other people to teach, and I'm there teaching how to take on different roles. It's a funny situation. I didn't talk about any maths at all. Pat was saying, 'I don't understand', and Rachel was getting really annoyed about this, and I said to Pat – "*As a learner* you've got to think about what she's saying and say, 'Stop – this is where I don't understand.' – that's your responsibility, and if you can't do that, Rachel can't help you. And I said to Rachel, "She's having problems with what you're saying – can you say it in a different way?" Then I walked away. I didn't talk about the real problem with the maths. (Ben 1.3.89)

Ben spoke here of leaving the maths 'for other people to teach'. The other people were the pupils. I referred to the higher-level thinking processes which I felt were evident in all three classrooms. These involved pupils in thinking through situations for themselves, deciding on their own questions, making and testing their own conjectures, being critical of their results. One question which arose poignantly in Phase 1 was *how a teacher could get pupils to be critical*; for example to ask *why* quadrilaterals all tessellate, rather than be happy to accept that they just do, or worse to believe that one still might find some which do not. In Phases 2 and 3, pupils of the teachers more experienced in an investigative approach were often critical in this way.

Thus the higher-level cognitive demands in these classrooms manifested themselves not just in demanding pupils to think through a problem for themselves, but in encouraging pupils to make decisions about their *way* of working on the problem, and in many cases about the problem on which to work.

ESTABLISHING AN INVESTIGATIVE APPROACH

I spoke to Clare in March 1991 about her new work as head of mathematics in a large comprehensive school, a job which she moved to in 1988. She read my latest version of Chapter 6, and one of her remarks was that she was no longer threatened by what I had written because she now felt that it described her work in the new job (see also p 159). It had taken her three years to feel so confident. Ben changed schools soon after my Phase 3 observation. He made clear that he found it difficult initially in the new school to work in the way which I had seen at Compton. All three teachers made clear that there were other classes in which they had not achieved the relationship which they wanted with the pupils. There were various reasons for this, but the main reason given was insufficient time to induct students into their way of working. Making their expectations overt to pupils was not something which could be achieved easily and quickly, especially with students who had very different expectations. The Phase 1 teachers began to explore an investigative approach with year 7 students (11–12 years) whose expectations of secondary school were not yet established, and they were having some success in setting up an ethos of respect and mutual trust. On the contrary, Simon, trying to work investigatively with year 11 (15–16 years), came up against long established expectations and attitudes towards mathematics lessons.

I have not tried to research the process of setting up an investigative approach with a new class of pupils, but a longitudinal study of this sort would be worth while.

I tried to represent the characteristics of an investigative approach, to which I have referred in the sections above, as a linked network. This, however, was limited by the two dimensionality of the page. Encouraging *responsibility for own learning* seems to build on *meaning-making* with the *thinking* and *critical* dimensions relating to processes of *expressing*, *questioning* and *pattern seeking*, for example. The engendering of *mutual trust and respect* seems to be overarching, since without the *good relationships* and *sensitivity to the needs of others*, *negotiation* and *sharing*, for example could be unlikely to succeed. The best representation, which I could make of this, was the triangular form expressed in figure 9.1 on the next page. The three outer triangles can be drawn (by imagined linking threads) to produce a three-dimensional figure – a tetrahedron – which goes some way to embodying the relationships which I have expressed.

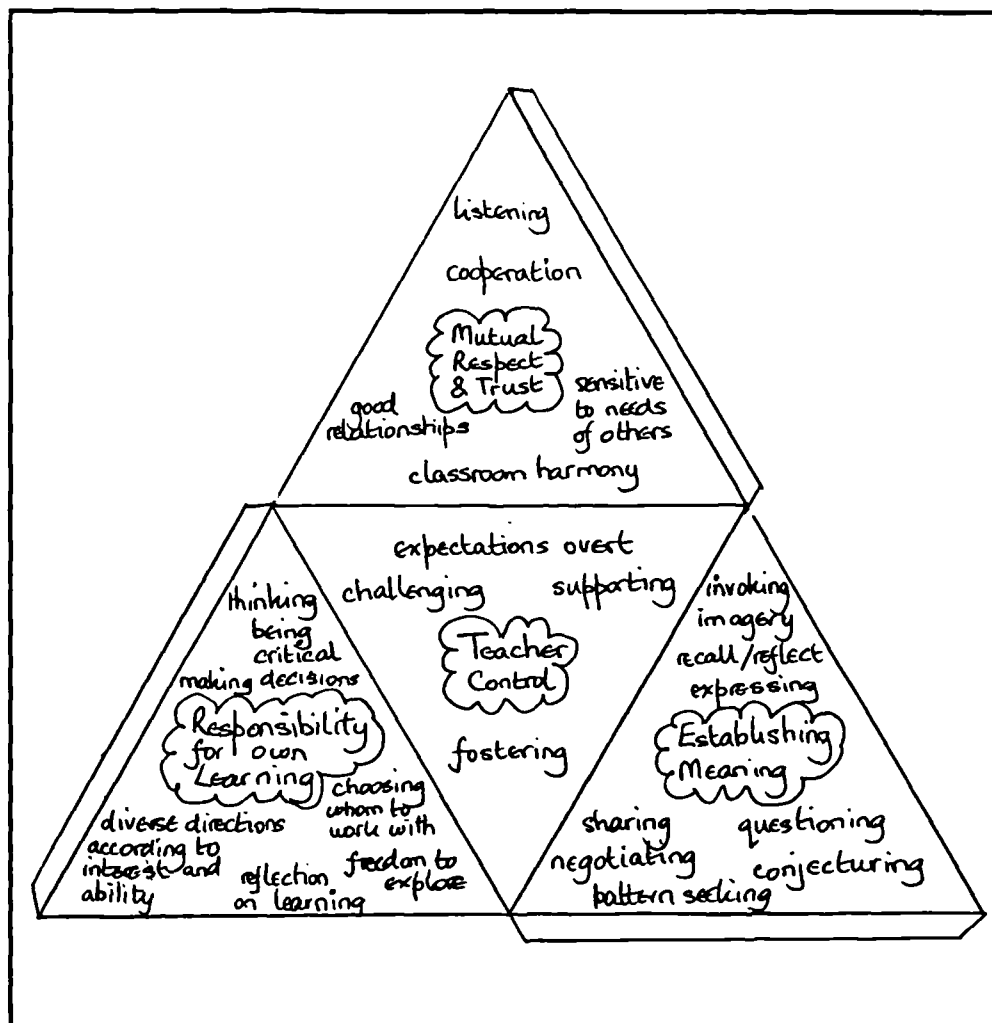


Figure 9.1: Characteristics of an investigative approach

The wider issues

My study has pointed to the close links which exist between the social structure of the classroom and the fostering of high-level mathematical thinking where the role of the teacher is concerned. What I have not so far addressed explicitly, in terms of my own observations and analysis, is the building of mathematical concepts of pupils involved, and the wider social environment in which the mathematics classroom was embedded.

BUILDING OF MATHEMATICAL CONCEPTS

My research in this study has been overtly into the *teaching* in the classrooms which I observed. I have inevitably come into contact with

pupils' mathematical work and thinking, but this has not been the principal focus of my study. I have not been able to say that the teaching which I observed was effective in terms of its *learning outcomes*, although it was without doubt effective in producing high level mathematical thinking in a high proportion of pupils. Learning outcomes are difficult to assess in any short term study, and I made no attempt to make judgments about learning.

In terms of my discussion of concept-building in Chapter 3, I saw teachers striving to foster principled, or relational understanding of mathematical concepts by pupils. I saw evidence of the success of this for individual pupils, as in Luke's principled understanding of Pythagoras' theorem in Ben's *vectors* lesson. I saw the fostering of schematic linkage, for example in Clare's focus on different aspects of *fractions*, and in Mike's two *Pythagoras* tasks. I gained little sense of teachers' perceptions of concept hierarchy. This was not something which we discussed overtly, and my study with any single teacher was too short to gain any fine sense of progression across topics. At Beacham, the following of the KMP scheme meant that each student had an individual route through the scheme. The project work served to highlight areas of strength and weakness, and I saw the teachers choosing students' KMP routes to complement this. I saw Ben's class moving from work on area and perimeter in the *Kathy-shapes* lessons, to work more explicitly on triangles, leading to Pythagoras theorem and vectors. Subsequent work on surface area, and later on trigonometry, built on the earlier work. This is broadly hierarchical, but I felt that the freedom of direction inherent in lessons on a day-to-day basis allowed for a flexibility of movement between topic areas and levels which adherence to a stricter hierarchy might have denied.

SOCIAL ISSUES

Traditionally, aspects of academic work and social interactions in classrooms have been separated by educational researchers (Doyle, 1983, cited in Desforges and Cockburn, 1987, p 21), but my study points towards the value of making links between them. In Chapter 3, I referred to the writing of, for example, Bishop (1988) and Jenner (1988), suggesting what links need to be made, and to some studies in which this linkage is seen to have begun, for example, Emblen (1988) and Walkerdine, (1989).

The social scene in the classrooms which I observed was very largely determined by the teachers' fostering of a classroom ethos. Clare, Mike and Ben particularly were all overtly concerned to foster respect and trust

and to encourage all pupils to participate. In the case of the Beacham teachers, this was within a school which explicitly promoted these values, and a department which worked together to implement them. However, both teachers indicated that their relationship with other classes was not always as good as in the classes which I observed, and that they were working hard to develop the ethos with other classes. This suggested that the ethos I observed was particular to the teacher rather than to the school. Although Compton felt a friendly school, and pupils were mainly well behaved, there was not the emphasis on mutual respect which Beacham overtly espoused. My experience with Simon's fifth year class made evident to me the difference which was possible between classes within the same school.

In the classes which I studied, it seemed mainly the case that girls took as active a part as boys, and I saw high-level thinking from both sexes. Clare and Ben both gave indication of being alert to gender issues and taking related action. There were only a minority of pupils of Asian or Afro-Caribbean origin in any of the classes, and I saw no obvious difference in the way they were treated or they behaved. They took an integral part in the lessons which I observed. I did not see any work designed to emphasise cross-cultural aspects of mathematics. The social environment beyond the school was only discussed in the case of particular pupils, where this was relevant to issues which arose from the classroom. For example, Jaime's family did not speak English at home, and Clare was aware of this in her interactions with him and in judging his response to classroom tasks, although he seemed reasonably fluent in English.

Conclusion

Characteristics of an investigative approach to mathematics teaching may be seen in the overt philosophy of the teachers in encouraging mathematical-meaning-making through classroom strategies and processes which encouraged pupils to take responsibility for their own learning. This involved making overt pupils' own mathematical constructions and relating this to pupils' individual experience. By encouraging expression of individual images, and their negotiation with those of other pupils, and emphasising the importance of following directions according to interest and ability, teachers succeeded in persuading pupils to engage with high-level cognitive demands and to think critically and creatively. The fostering of mutual trust and respect ensured that the environment was supportive of, and not threatening to, this thinking.

CHAPTER 10

BRINGING THEORY CLOSER TO PRACTICE

Introduction

It is possible to trace a number of developments through this study.

The first is the development of an investigative approach to mathematics teaching through the three phases of research, from early experimental beginnings to an established form. Whereas the teachers in Phase 1 were just starting to consider what an investigative approach might mean for them, the teacher Ben, in Phase 3, was working explicitly according to an investigative approach in his terms. The teachers in Phase 2 were working in a way which could be regarded as investigative, although it was never labelled as such. Throughout the three phases, I have offered examples of classroom interactions, as manifestations of aspects of an investigative approach, which chart this development. Associated with these manifestations have been issues which they raise for the teacher in working investigatively.

The second is the development of the teaching triad. I see this as a device developed through characterising Clare's teaching, which has been shown to be valuable in describing the teaching of Ben and of Mike, and which could have more general applicability. I believe it to have potential for describing investigative teaching more generally, and also for the teacher in planning and evaluating teaching. Both of these are suggested as areas for further research.

My perception of *investigative teaching* has grown beyond the offering of investigative approaches to mathematical thinking and learning to pupils in the classroom. The third development is that of my coming to view teachers as reflective practitioners who develop their teaching through active reflection on their practice – i.e. they *investigate* their teaching. Moreover, I have come to believe that any teacher who endeavours to implement a constructivist perspective of mathematics teaching will be such a reflective practitioner who actively investigates the process of teaching and its outcomes.

The fourth is the development of my own thinking throughout the research, from early theoretical notions of an investigative approach, through an exploration of practical manifestations of such an approach, to a view of investigative teaching which fits closely with a constructivist perspective of knowledge and learning. It has become important to recognise my own role as a reflective practitioner in the developing of this perspective, and the consistency between the different levels of reflective practice which contribute to the rigour of this study.

As a result of these developments I now propose what may be called (a) a constructivist pedagogy; and (b) an epistemology for practice, which together are a synthesis of the thinking which this thesis presents.

A constructivist pedagogy

In suggesting what the concept of a constructivist pedagogy might involve, my theoretical starting point is in the two principles of constructivism quoted in Chapter 2 (von Glasersfeld, 1987a; see p 13) which I shall now paraphrase as follows:

1. Knowledge is constructed by the learner. It is not received from an external source.
2. Learning is a process of comparing new experience with knowledge constructed from previous experience. The result is the reinforcing or the modification of existing knowledge.

For mathematics teachers starting from these principles, overtly or implicitly, I claim that the following question is fundamental to their construction of teaching:

What sense are the pupils making of the mathematics they encounter in my classroom?

In order to answer this question, for one or all of the pupils in a class, the teacher has to gain access to their construal. The fruit metaphor was offered (see p 42) as an image to describe the nature of the transparency of pupil construal for a teacher. It is the teacher's construal of pupils' construal of mathematics which will motivate the designing of activities and the offering of mathematical challenge within a classroom.

A teacher's operation is seen, traditionally, in terms of planning and decision-making (Clark and Peterson, 1986). I saw the teachers in my

study planning and implementing ways of working in mathematics lessons to

- a) enable and enhance pupils' *mathematical construal*;
- b) develop an environment conducive to enhancing mathematical construal.

In *any* mathematics classroom, some construal of mathematics by pupils takes place. The negative images which many adults have of mathematics (Sewell, 1981; Howson and Kahane; 1990, Civil, 1990) suggests that their construal was not *effective* mathematically. I saw teachers striving for *effective* construal, which I see as the development of *relational* understanding of mathematics rather than *instrumental* understanding (Skemp, 1976). Alternatives to the term *relational*, pointed out in Chapter 3, are *conceptual* (Brown, 1979) and *principled* (Edwards and Mercer, 1987). The three terms capture essences of this understanding – it involves *relating* mathematical ideas, possibly by making explicit the links between them; it involves a sense of concept, a wholeness rather than a sum of parts; it involves a knowledge of the rules or principles, knowing *why* as well as *what*. In the classes which I observed, pupils were encouraged to develop mathematical *process* alongside mathematical *content* (Bell, 1982) and to take responsibility for their own learning. Thus pupils engaged in *high level thought processes* (Desforges and Cockburn, 1987). I believe that this was only possible because of (b) above. It was through the sensitive managing of 30 pupils within the learning environment, the creating of a social ethos of mutual trust and respect, and the harnessing of social possibilities for enhancing learning that effective mathematical construal was enabled. Thus (b) might be seen as a subset of (a).

It is not my purpose to provide a recipe for classroom organisation or planning for teaching, as this would be inconsistent with the theory which I offer in this chapter. I have, for example, pointed to the investigative nature of activity, the importance of asking questions, of pattern spotting and conjecturing, the working in groups, the emphasis on listening and on discussion. Chapter 9 discusses common characteristics of classrooms which I observed, in terms of such factors. Although it is likely that any teacher working from a constructivist perspective will include such features in the design of teaching, I am concerned with the process motivating this design.

THE TEACHING TRIAD

I introduced *the teaching triad* to characterise one teacher's practice, and subsequently proposed it as an analytical device through which to regard teaching and to characterise an investigative approach more generally. Moreover, I suggested that the teaching triad could be seen to be linked to a constructivist view of learning. I propose further that the teaching triad can be seen as a device to enable the planning of teaching, and through which teaching can be evaluated.

1: MATHEMATICAL CHALLENGE

In fostering effective mathematical construal, teachers have to create tasks to introduce their mathematical agenda. Such tasks incorporate some degree of *Mathematical Challenge*, or *cognitive demand*. Higher-level cognitive demand (Doyle, 1986) may be associated with higher-level thought processes in pupils. I noticed repeatedly the high level demands which teachers in my study made of their pupils. This was evident in the tasks which were set for a class as a whole (cf Chapter 6, p 133, Clare's students seeking generality in patterns of sevenths; Chapter 7, p 236, Ben's pupils' working on their *own* vector questions), and in the challenges which were directed at individual pupils (cf Chapter 6, p 177, Mike's challenges to Phil, and p 138, Clare's to the girls in '*It's a cuboid*'). Discussion with teachers revealed that creating this demand was high on their planning agenda. Thus mathematical tasks were inventive, with a variety of form and style of presentation designed to interest and motivate pupils. Creation of such tasks raises many questions regarding, for example, how the task is related to the teacher's desired mathematical outcome; how suitable the degree of challenge is to the particular pupils to whom it will be offered; how the proposed style and form will relate to experience and expectations of pupils in the class.

2: SENSITIVITY TO STUDENTS

Decisions about tasks, and their form and style, must be related to the pupils with whom they will be used. The success of such activities in promoting the mathematical thinking which the teacher desires depends on the pupils' involvement with the activity. Despite common starting points, responses in terms of pupils' images and emphasis will vary, since each pupil must fit the activity to their own particular experience (cf Chapter 6, pp 144 and 148, Rebecca and Jaime). A task needs to have scope for varying depth and direction of involvement. Narrow, stereotyped activities are likely to fail to involve some pupils, or be too trivial to require much involvement

(cf Simon's lessons, Ch 7 p 254). Thus the teacher has to be prepared to follow directions which individual pupils take, and gauge responses to fit the thinking of a pupil. This lies within the domain of *Sensitivity to Students*. The teachers in my study gave evidence of knowing pupils well and using this knowledge to decide on appropriate responses (cf Chapter 6 p 144, Clare and Rebecca; p 177, Mike and Phil; Chapter 7, p 233, Ben and Luke). Questions arise concerning how the teacher gains such knowledge, and organises the learning situation to foster significant pupil involvement.

3: MANAGEMENT OF LEARNING

The teachers in my study, like most teachers in the U.K, worked with classes of about 30 pupils, with the multiple pressures and constraints of organising a classroom to which Desforges and Cockburn, (1987), refer. Rather than seeing 30 pupils as a limiting factor, I perceive value in organising the classroom to capitalise on the number of people within it, firstly because of the impossibility of the teacher spending enough time with each pupil separately, and secondly due to the value for learning of human interaction (e.g. Vygotsky, 1978). Such classroom organisation falls within the domain of *Management of Learning* which has been shown to be crucial to both Mathematical Challenge and Sensitivity to Students. In this domain, I saw teachers deciding on appropriate organisations to suit particular activities and mathematical objectives; on how to offer learning strategies which pupils can use to facilitate their involvement and engage with mathematics; on how to develop good relationships between people in the classroom which foster trust and respect in which involvement can take place without threat. Related questions concern the degree of making expectations overt to pupils; how pupils can be included in management decisions or invited to make these decisions themselves, and how this relates to the maturity of the pupils; what messages are conveyed to pupils by the teachers' own emphasis in the course of activity.

4: A DEVICE FOR PLANNING AND EVALUATION

I suggest that one basis of a constructivist pedagogy is the two principles at the beginning of this chapter and the teaching triad, together with the associated questioning. I have given examples of the sort of questions which might be tackled, but the essence of such a pedagogic approach is that the questions will be related to the particular circumstances of the teacher and any class of pupils. Chapter 9 highlights patterns in the resolution of some of these questions for teachers and classes in my study. Chapters 5 to 7 elaborate aspects of the pedagogy of these teachers. Planning for teaching involves tackling such questions, and this is the first

layer of decision making. Subsequent layers are necessary in the intricate interactions among teachers and pupils within the classroom. Examples of this are offered in Chapter 5 to 7, and there will be further elaboration below.

Evaluation of teaching follows naturally from the form of planning proposed. If planning has consisted of identifying questions and resolving them in terms of particular operations, then evaluation would involve relating the outcome of an operation to the questions from which it arose. Elaboration of such evaluation falls within an *epistemology for practice*, and so will be delayed until that section.

TEACHER-PUPIL INTERACTIONS

The introduction of tasks, and the whole class involvement which is often associated with this, is important to the creation of productive mathematical activity, i.e. that which is effective in terms of pupils' mathematical construal. However, it is arguable that the main teaching acts are those which take place between the teacher and pupils as pupils pursue their own thinking¹. A concept which cropped up frequently in my analysis of such teacher-pupil interactions was that of the *zone of proximal development*, ZPD, (Vygotsky, 1978). I have shown that the ZPD may be seen as a useful analytical device, and have claimed that it is very closely linked to notions of Mathematical Challenge and Sensitivity to Students. In any interaction, the teacher has to decide what level of challenge it is most appropriate to offer, and this depends upon a high degree of sensitivity to the particular pupil. The ZPD, seen as the difference between the pupil's potential in continuing alone and in continuing with intervention from the teacher, provides a way for the teacher to regard that intervention. Firstly, the way ZPD is expressed (in Vygotsky, *ibid*) suggests that there is *more* potential *with* intervention than without. As there are often times when intervention seems inappropriate, ZPD is relevant only when a teacher has judged that intervention could be valuable. The teacher has to make a judgement about the pupil's potential in order to decide upon the nature of the intervention and its degree of challenge. What the teacher says or does can be regarded as *scaffolding* across the ZPD (Bruner, 1985). The ZPD is not an external measure. It is not therefore a device which the teacher can use without close knowledge of the pupil and her immediate cognitive processing (Brown et al, 1989, expand on this in terms of 'situated cognition'). However, it has potential as a mental construct which might

¹ This is not to deny the value of pupil-pupil interaction to which Vygotsky also referred. However, the *teacher's* teaching has been my main focus in this study.

form the basis of a teacher's thinking in making the criteria of intervention more overt. In this respect it can be seen as a tool of *an epistemology for practice*.

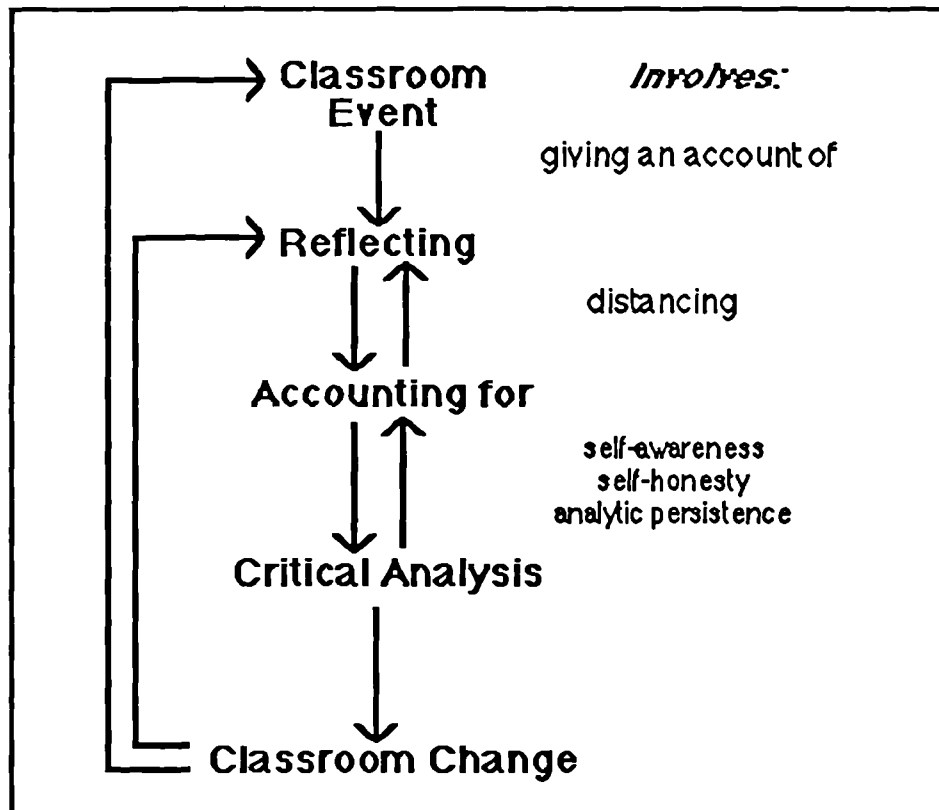
TENSIONS AND ISSUES

Throughout my writing, I have referred to tensions and issues which arose for the teachers whom I studied (cf Chapter 6, p 158, Clare's *gender* considerations, p 183, Mike's *cognitive density*). However, over and over again, I found myself linking questions, issues or tensions which arose in particular situations to *the teacher's dilemma* (Edwards and Mercer, 1987) or to *the didactic tension* (Mason, 1988b,c). Associated with these, but arising particularly from work with Ben in Phase 3, was *the didactic/constructivist tension*. Although there may seem to be differences between these three tensions, I see them globally being a part of the overriding issue which working from a constructivist perspective raises for the teacher. This is manifested in statements like, 'when to tell', or 'to inculcate while apparently eliciting knowledge' or 'if I leave pupils to construct for themselves how can I be sure they will construct what I want them to construct?'. It is polarised for the teacher in terms of leaving pupils free to pursue their own thinking, yet in having particular mathematical goals which should form the focus of their construal. It may be seen in practice, on the one hand, in terms of pupils' aimless wandering in investigations whose purposes are vague, and on the other as the teacher's implicit or overt pushing of pupils in directions which fit the teacher's own thinking. However, such polarisations caricature rather than characterise the tensions. The tensions seem most acute when a teacher struggles, in a moment of decision, for a response which is appropriate to the pupil concerned (cf Chapter 6, p 184, Mike's response to Phil who had two conflicting methods for finding area; Interlude B, p 193, Clare's 'prodding and guiding' dilemma'). This is the essence of the challenge/sensitivity balance. It is reflected in teachers' moves towards taking pupils explicitly into their confidence with regard to their purposes in intervention (cf Jaworski, 1991, Clare's words to the boy who had been working on the nature of crossing in string in forming a knot; Ben's discussion of his intervention with Jenny and Lesley, in Appendix 5). It is related to questions of how far pupils take form for substance, responding to what they perceive teachers as wanting, rather than behaving naturally in the perceived form (cf Chapter 7, p 251, Colin's response to Ben's invitation to comment on an activity).

I believe that the essence of a constructivist pedagogy lies in tackling this issue, and that the tackling of this issue leads to an epistemology for practice.

An epistemology for practice

From my observations of the teacher-researcher relationships which developed in the course of my study, and the spontaneous remarks of teachers in each phase with regard to the influence which our work together was having on their thinking, arose the following sequence of activities which constitute reflective practice, originally introduced in Chapter 8.



These stages of *reflection*, *accounting for* and *critical analysis* lead to the active exploration of the teaching process which involves the raising of issues and the modification of practice. This might be regarded as *investigative teaching*. These words were initially an abbreviation of *an investigative approach to mathematics teaching* and referred to the incidence of investigative activities in the classroom through which mathematical thinking could be approached. Their linking to the exploration of teaching itself, creates a level of consistency between the approach to mathematics

which the teacher could be seen to endeavour to create, and the approach to such creation. The teacher-pupil relationship might be seen to parallel the researcher-teacher relationship, with the teacher acting as *distancer* for the pupil – encouraging pupils to reflect on the doing of mathematics and so develop mathematical awareness – rather than as a source of knowledge.

As I pointed out in Chapter 8, a problem for the teacher in trying actively to engage in this reflective process, is the difficulty of doing it alone, without some external source of *distancing*. I suggested there that the active support of colleagues might serve such a function. However, I suggest that focusing devices could help the individual reflective process, and that two such devices are the fruit metaphor (see Chapter 3, p 42) and the ZPD.

These devices might be seen to enhance awareness where it is most needed. In the case of the fruit metaphor, it is in the awareness of there being something behind the more overt behaviour of the pupil. It is all too easy under pressure to respond to this behaviour as it appears in the immediacy of the classroom. However, being aware of what might lie behind the behaviour in terms of pupil construal may enable a more sensitive response to be made, or a more sensitive post hoc evaluation which will influence future responses.

Use of the ZPD as a focusing device is rather more active. It involves overt consideration of the level of response which could be made and its likely and desired effect. It requires the teacher to consider what the pupil might be expected to achieve *without* teacher involvement, and *with* some form of involvement, and what, if any, resulting sacrifice ensues. It is very difficult to achieve at the moment of an interaction. However, a post hoc analysis of an interaction in terms of ZPD can lead to the teacher becoming more aware of what involvement would be appropriate at a later stage, and simultaneously develop a more disciplined approach to involvement.

Thus, awareness of the fruit metaphor and ZPD can lead to a more knowledgeable approach to interactions with pupils. In Chapter 8 I referred to this knowledge as *Teaching Knowledge*. I believe that Clare, Mike and Ben demonstrated an overt awareness of their teaching knowledge. Indeed Clare referred to this knowledge: “Sometimes I know and sometimes I don’t know, and the ways that I know, I know because they apply in lots of different situations”. The application of this knowledge *in* the teaching situation, I have called *Teaching Wisdom*. I believe that here lies the closest link between theory and practice. Teaching wisdom involves recognising a situation by accessing teaching knowledge, deciding on what is required in

the situation and retrieving and applying a form of action which is appropriate.

Much of what I offer above is embryonic theory. It might be tested against the reflective practice of other practitioners by seeking resonance with their experience. It might be researched by setting up a study of practitioners who would be willing to engage in the stages of reflective practice discussed above. I suggest that such research could contribute significantly to the development of teaching.

A critical appreciation of this study

This study has been based on observations of the classroom practice and associated thinking of six teachers, and on my own thinking and experience as a teacher, teacher-educator and researcher. It has been overtly interpretive, and I have endeavoured to make levels of interpretation clear. I have seen it as my task to weave a story of reflective practice as it has emerged from what set out to be a study of an investigative approach to mathematics teaching. Unifying the various themes has been an underlying theory of the construction of knowledge and its modification through experience. This has forced a continued questioning of consistency of operation throughout.

I believe that the essence of the study lies in bringing theory close to practice. This is partly in recognition of the practitioner as a theoretician who implicitly or explicitly interprets theory into practice; the *explicit* interpretation being what is referred to as *reflective practice*. However, it is also in the importance of the classroom manifestations of aspects of theory. However persuasive a theory, it is the manifestations of the theory in practice which elucidate its practical interpretation; i.e. which ultimately authenticate it in practical terms. When a theory is validated through resonance with the experience of others, it is actually such manifesting, or embedding in practice, which takes place.

A part of this study was the embedding in practice of *the teaching triad*. Firstly it was seen to resonate with Ben's experience. I was then able to identify manifestations of it in Ben's teaching. Finally I was able to reinterpret Mike's teaching in its terms. Throughout the study, I have interpreted theoretical notions of an investigative approach in terms of the teaching which I have observed. Through these manifestations, I have

become convinced of the nature of an investigative approach and have been able to make links with a constructivist philosophy of teaching and learning.

I make no claims for either the teaching triad, or an investigative approach beyond my own beliefs, strengthened and supported by this study. Whether they have the wider applicability which I propose must be left to their validation through others' experience, or to their further exploration through research.

Methodological implications

This study has employed a number of well-documented techniques in the collection of data on teaching and on teacher perceptions and beliefs, and its analysis. I refer in particular to data-gathering techniques of participant observation and informal interviewing, and to validation techniques of triangulation and respondent validation. Alongside my analysis, I have provided a research (auto)biography which elaborates contexts and decisions behind the interpretations which I have made.

It is in its *overt* recognition of interpretation within a constructivist theoretical perspective that I feel the study breaks new ground methodologically. Thus the study does more than recognise patterns which form the basis of theory for further research. It proposes that theory can only be validated through experience, that it is the resonance of theory with experience which leads to its authentication. Practical manifestations of theory play a crucial part in this. They elucidate the theory, making it available for the construal of others in terms of their experience. For example, the teaching triad may instantly resonate with the experience of another teacher because its terms have meaning within that experience. However, it is more likely that manifestations of the triad, such as those offered in earlier chapters, are more powerful in triggering response from others in terms of 'that reminds me of ...'.

I have relied on respondent validation throughout the study in seeking resonance between my own interpretations and those of the teachers, and sometimes the pupils. The reader of this text is also in the position of respondent. I have offered both theory developing from my interpretations, and practical manifestations on which those interpretations were based. The ultimate test of what I offer in this thesis will be the extent to which the reader is convinced of its arguments through resonance with her own experience.

Future directions

I feel that what has been synthesised as a constructivist pedagogy and an epistemology for practice has potential to be of value to three groups of practitioners – teachers, all those involved in the professional development of teachers, and those involved in research into teaching – with particular emphasis on the teaching of *mathematics*.

For teachers, a constructivist pedagogy offers a way of viewing pupils' learning (of mathematics) which can influence their teaching, and an epistemology for practice offers a supportive mechanism for the development of this teaching. The aspects of teaching which are documented, and issues which these have raised for the teachers in this study, provide manifestations of theory to which teachers can relate in terms of their own practice. The teaching triad provides a potential framework through which to develop and evaluate classroom approaches and strategies. The sequence of reflective activity suggests how the overt development of teaching might take place, preferably within an environment of mutual support with colleagues

For those involved in the professional development of teachers, which clearly includes teachers themselves, an epistemology for practice offers a framework through which development can be approached. The most difficult part of such development might be seen to be in *distancing* and *analytical persistence* for the lone practitioner. Thus, those involved in enabling teacher development might focus on support mechanisms which can enable growth through reflective practice. One such mechanism is elaborated in 'Develop *your* Teaching' (Mathematical Association, 1991).

Where research into teaching is concerned, there are a number of areas arising from this study which could valuably be subjected to further research. They are:

1. Do teachers see the teaching triad as a helpful device for describing teaching, for teacher planning, and for the evaluation of teaching? If so, in what ways is it helpful? What links may be perceived between the triad and a constructivist view of mathematics teaching?

2. This study has highlighted a number of practices (elaborated in Chapter 9) which seem to relate to a constructivist perspective. May any of these practices be seen to be fundamental to such a perspective?
3. This study suggests that the teaching of mathematics from a constructivist perspective requires reflective practice. This has been described through the reinterpretation of *an investigative approach to mathematics teaching* in terms of *investigative teaching*. What links are there between investigative activity in the teaching and learning of mathematics and the investigation of the teaching act itself? Further study of manifestations of reflective practice could elucidate its links with a constructivist view of teaching and learning.

Conclusion

This thesis has charted a journey. It has been a personal journey. What set out to be a characterisation of investigative, and subsequently constructivist, practice has ended as a perception of the *construction* of practice which is as follows: Constructivism deals with the construction of knowledge. Theories may be seen to be the basis of knowledge from many epistemological standpoints. An epistemology for practice suggests that the practice of teaching derives directly from knowledge or theory, and that the development of practice only makes sense in correspondence with development of associated theory. It is in the explicating of this theory that we can learn about practice, and it is the manifestations of the theory in practice which lead to its explication.

REFERENCES

- Antaki C. & Lewis A. (1986) *Mental Mirrors. Metacognition in Social Knowledge and Communication*. London: Sage
- Austin, J.L. and Howson, A.G. (1979), 'Language and mathematical education', *Educational studies in mathematics*, 10(3), p 161-97
- Ball S.J. (1981) *Beachside Comprehensive* Cambridge: Cambridge University Press
- Ball S.J. (1982) 'The verification and application of participant observation case study' in *Case study methods 5*, 'Perspectives on case study : ethnography', 141-164, Deakin: University of Deakin Press
- Ball S.J. (1990) 'Self doubt and soft data : social and technical trajectories in ethnographic fieldwork'. in *Qualitative Studies in Education*, Vol 3 no 2, 157-171
- Bauersfeld, H. (1985) 'Contributions to a fundamental theory of mathematics learning and teaching' invited lecture to *Canadian Mathematics Education Congress*, Quebec.
- Becker, H.S. (1958) 'Problems of inference and proof in participant observation', *American sociological review* 23, 6, 652-660
- Bell, A.W. (1982), 'Teaching for combined process and content objectives', *Proceedings of the Fourth International Congress on Mathematical Education*. Boston: Birkhäuser p 587-590
- Bell A.W. and Bassford, D. (1989) 'A conflict and investigation teaching method and an individualised learning scheme - a comparative experiment on the teaching of fractions' in *Proceedings of the 13th International Congress on Mathematical Education*. Paris: PME 125-132
- Bell, A.W., Rooke, D. and Wigley, A. (1978-9), *Journey into Maths*. Glasgow: Blackie
- Bereiter, C. (1985) 'Towards a solution of the learning paradox', Review of Educational) 'Ways of thinking about students and classrooms by more and less experienced teachers', in Calderhead, J. (ed.), *Exploring teachers' thinking* London: Cassell
- Bishop, A J. (1984) 'Research problems in mathematics education II' in *for the learning of mathematics*, vol 4 no 2 pp 40-41
- Bishop, A.J. (1988), 'Mathematics education in its cultural context' in *Educational Studies in Mathematics* 19 179-191

- Bishop, A.J. and Nickson, M. (1983), *A review of research in mathematical education. Part B: Research on the social context of mathematics education*. Windsor, Berks: NFER & Nelson
- Bittinger, M.L.A. 1968, 'A review of discovery', *The Mathematics Teacher* 61 140-146
- Bloor, D. (1976), *Knowledge and social imagery*, London: Routledge and Kegan Paul.
- Blumer, H. (1956) 'Sociological analysis and the variable.' *American Sociological Review*, 21: 683-690
- Blumer, H. (1966) 'Sociological implications of the thought of George Herbert Mead', *American Journal of Sociology*, 71, 5, 535 - 544
- Boud, D., Keogh, R. and Walker, D. (1985) *Reflection: turning experience into learning*. London: Kogan Page
- Brissenden, T.H.F. (1980) *Mathematics Teaching*. London: Harper and Row
- Brousseau, G. (1984) 'The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics.' in H.G. Steiner et al *Theory of mathematics education (TME)* Bielefeld, Germany: Universität Bielefeld/IDM
- Brown, B. B. (1968) *The experimental mind in education*, New York: Harper and Row
- Brown, M. (1979), 'Cognitive development and the learning of mathematics', in A. Floyd (ed.), *Cognitive development in the school years*. London: Croom Helm
- Brown, J.S., Collins, A. and Guguid, P. (1989) 'Situated cognition and the culture of learning', *Educational researcher*, January-February, 32-42
- Bruner, J.S. (1961) 'The act of discovery', *Harvard Educational Review*, XXXI 21-32
- Bruner, J.S (1985) 'Vygotsky: a historical and conceptual perspective'. In J.V.Wertsch (ed.), *Culture Communication and Cognition: Vygotskian Perspectives*. Cambridge: Cambridge University Press.
- Bruner, J.S (1986), *Actual minds, possible worlds*. Harvard University Press, Cambridge, Mass.
- Burgess, R.G. (1985a) *Field methods in the study of education*. London: Falmer
- Burgess, R.G. (1985b) *Strategies of educational research*. London: Falmer

- Calderhead, J. (1984), *Teachers' classroom decision making*. London: Holt Rinehart and Winston
- Calderhead, J. (ed.) (1987) *Exploring teachers' thinking* London: Cassell
- Calderhead, J. (1989) 'Reflective teaching and teacher education', *Teaching and Teacher Education*, Vol. 5 No. 1, 43-51
- Carr, W. and Kemmis, S. (1983) *Becoming critical: Knowing through action research*. Geelong, Victoria: Deakin University Press
- Chomsky, N. (1975) *Reflections on language*. New York: Pantheon Books
- Cicourel A.V. (1973) *Cognitive Sociology*. London: Penguin
- Civil, M. (1990) "'You only do math in math": A look at four prospective teachers' views about mathematics' *For the learning of mathematics* 10, 1, 7-9
- Clark, C.M. and Peterson, P.L. (1986) 'Teacher's thought processes'. In M. Wittrock (ed.) *Handbook of research on teaching* 255-296, New York: MacMillan
- Claxton, G. (1989) *Being a teacher: A positive approach to change and stress*. London: Cassell
- Claxton, G. (1990) *Teaching to learn: A direction for education*. London: Cassell
- Cobb P. (1988) 'The tension between Theories of Learning and Instruction'. In *Educational Psychologist*, 23(2) 87-103. Lawrence Erlbaum Associates Inc.
- Cobb P. and Steffe, L.P. (1983) 'The constructivist researcher as teacher and model-builder' *J R M E* 14 p 83-94
- Cobb, P., Yackel, E., & Wood, T. (1988), 'Curriculum and Teacher Development: Psychological and Anthropological Perspectives', in Fennema, E. Carpenter, T.P. & Lamon, S.J. (eds.) *Integrating Research on Teaching and Learning Mathematics*, Madison WI: Wisconsin Center for Education Research, University of Wisconsin-Madison, pp 92-131
- Cohen L. and Manion L. (1989) *Research methods in Education* (Third edition) London: Routledge
- Collins, A. (1988), 'Different goals of inquiry teaching', in *Questioning Exchange*. Vol 2, no 1, 39-45
- Cooney, T.J. (1984) 'The contribution of theory to mathematics teacher education' in H.G. Steiner et al *Theory of mathematics education (TME)* Bielefeld, Germany: Universität Bielefeld/IDM

- Cooney, T.J. (1988) 'Teachers' decision making' in D. Pimm (ed.), *Mathematics teachers and children*. London, Hodder and Stoughton
- Crombie, A.C. (1952) *Augustine to Galileo*. London: Mercury
- D'Ambrosio, U. (1986), 'Socio-cultural bases for mathematics education', in M. Carss (Ed.), *Proceedings of the Fifth International Congress on Mathematics Education*, Boston, Mass.: Birkhauser
- Davis, J. (1990) 'The role of the participant observer in the discipline of noticing', in F. Seeger and H Steinbring (eds.) *Proceedings of the fourth conference on the systematic cooperation between theory and practice in mathematics education : Overcoming the broadcast metaphor*, Institute für Didaktik der Mathematik der Universität Bielefeld. Germany: Universität Bielefeld/IDM
- Davis, J. Gates, P. Griffin, P. James, N. Jaworski, B. and Mason J. (1989) *The study of teaching moments*. Milton Keynes: Centre for Mathematics Education, Open University
- Davis P.J. and Mason J.H. (1989) 'Notes on a radical constructivist epistemology applied to didactic situations', *Journal of structured learning*, Vol 10 157-176
- Davis, P.J. and Hersh, R. (1981), *The mathematical experience*. Brighton: Harvester Press
- Delamont S. and Hamilton D.(1984) 'Revisiting classroom research: a cautionary tale' in Delmaont S. (Ed) *Readings on interaction in the classroom*, Methuen, London pp 3-24
- Denzin, N. K. (1978) *The research act: A theoretical introduction to sociological methods*. New York: McGraw Hill
- Denvir, B. and Brown M. (1986) 'Understanding number concepts in low attaining 7-9 year olds: Part II The teaching studies' *Educational Studies in Mathematics* 17 143-164
- DES, (1982) *Mathematics Counts*. (The Cockcroft Report) London: HMSO
- DES, (1985a) *Education for all* (The report of the committee of inquiry into the education of children from ethnic minority groups, under the chairmanship of Lord Swann) London: HMSO
- DES, (1985b) *Statistics of Education 1984*. London: HMSO
- Desforges C. (1985) 'Training for the management of learning in the primary school', in H. Francis (ed.) *Learning to teach: Psychology in teacher training*, Brighton: Falmer Press
- Desforges C. and Cockburn A., (1987) *Understanding the Mathematics Teacher*, Brighton: Falmer Press
- Dewey, J. (1933) *How we think*. London: D.C. Heath and Co.

- Dewey, J. (1963) *Experience and Education* New York: Collier Books (The original appeared in KAPPA DELTA PI, 1938)
- Dowling, P. and Noss, R. (eds.) (1990) *Mathematics versus the National Curriculum*. London: Falmer
- Doyle, W. (1983) 'Academic work', *Review of Educational Research*, 53, 159-200
- Doyle, W. (1986) 'Classroom organisation and management' in Wittrock, M. (ed.) *Handbook of research on teaching*. New York: Macmillan
- Driver, R. (1983) *The pupil as scientist*. Milton Keynes: Open University Press
- Eales, A. (1986) 'Girls and mathematics at Oadby Beauchamp College', in L. Burton (ed.) *Girls into maths can go*. London: Holt, Rinehart and Winston
- Edwards, A. D. and Furlong, V.J. (1985) 'Reflection on the language of teaching', in R.G. Burgess (ed.) *Field methods in the study of education*. London: Falmer
- Edwards D. and Mercer N. (1987) *Common Knowledge*. London: Methuen
- Edwards, A. D. and Westgate, D. P. G. (1987) *Investigating classroom talk*. London Falmer
- Eisenhart M.A. (1988) 'The ethnographic research tradition and mathematics education research', *Journal for research in mathematics education*, Vol 19, no. 2, 99-114
- Elbaz, F. (1987) 'Teachers' knowledge of teaching: strategies for reflection', in J. Smyth (ed.) *Educating teachers*. London: Falmer
- Elbaz, F. (1990) 'Knowledge and discourse: the evolution of research on teacher thinking,' in C. Day, M. Pope and P. Denicolo (eds.) *Insight into teachers' thinking and practice*. London: Falmer
- Elliott, J. and Adelman C. (1975) 'The language and logic of informal teaching' in *The Ford teaching project, Unit 1, Patterns of teaching*. Norwich: University of East Anglia, Centre for Applied Research in Education
- Emblen, V. (1988), 'Asian children in schools', in D. Pimm (ed.) *Mathematics, teachers and children*, London: Hodder and Stoughton
- Fensham, P., Power, C., Tripp, D. and Kemmis, S. (1986) *Alienation from schooling*. London: Routledge and Kegan Paul
- Flanders N.A. (1970) *Analysing Teaching Behaviour* Reading, Mass.: Addison Wesley
- Fodor, J.A. (1975) *The language of thought*. New York: Crowell

- Fodor, J.A. (1980) 'Fixation of belief and concept acquisition.' in M. Piattelli Palmerini (ed.) *Language and learning: the debate between Jean Piaget and Noam Chomsky*. Cambridge: Harvard University Press
- Freire, P. (1972) *Pedagogy of the oppressed*. London: Penguin
- Furlong V.J. & Edwards A.D. (1977) 'Language in classroom interaction: theory and data', in *Educational Research*, Vol 19, No 2, pp 122-8
- Gardiner, P. (1967) 'Vico, Giambattista (1668-1744)', in P. Edwards (ed.) *The Encyclopedia of Philosophy*, Vol 7 and 8, London: Collier MacMillan
- Gates, P. (1989) 'Developing conscious and pedagogical knowledge through mutual observation', in P. Woods (ed.) *Working for teacher development*. London: Peter Francis
- Gates, P. (1991) 'The National Curriculum on trial - the case for the prosecution', *Mathematics Teaching*, 134
- Giroux, H. (1983) *Theory and resistance in education: a pedagogy for the opposition*. Amherst, Mass: Gergin and Garvey
- Glaser B.G. and Strauss A.L. (1967) *The discovery of grounded theory*, London: Weidenfeld and Nicolson
- Gold, R. (1958) 'Roles in sociological field observation', *Social forces*, 36, 217-223
- Habermas, J. (1972) *Knowledge and human interests*, trans J. Shapiro, London: Heinemann
- Hamilton D. & Delamont S. (1974) 'Classroom research: a cautionary tale?' *Research in Education*, 11, 1-15 (reprinted in Stubbs M. & Delamont S. 1976, (Eds) *Explorations in Classroom Observation*, Chichester: John Wiley)
- Hammersley M. (1986) *Controversies in classroom research*, Milton Keynes: Open University Press
- Hammersley M. (1990) *Classroom ethnography*. Milton Keynes: Open University Press
- Hargreaves, D. (1967) *Social relations in the secondary school*. London: Routledge and Kegan Paul
- Hart, K. (1981) *Children's understanding of mathematics 11 - 16*. London: John Murray
- Hemmings, R. (1980) 'Multi-ethnic mathematics: Part II Secondary', *New approaches to multicultural education*, 8, 4, 1-4

- Howson, A.G. and Kahane, J. -P. (1990) *The popularisation of mathematics*. Cambridge: Cambridge University Press
- Hoyles, C. (1982) 'The pupils' view of mathematics learning', *Educational Studies in Mathematics*, 15 (2) 105-127
- Hughes, M. (1988), *Children and Number*. Oxford: Basil Blackwell
- Inhelder, B. and Piaget, J. (1958), *The growth of logical thinking from childhood to adolescence*. London: Routledge and Kegan Paul
- Isaacson, Z. (1988) 'The marginalisation of girls in mathematics: some causes and some remedies', in D. Pimm (ed.) *Mathematics, teachers and children*, London: Hodder and Stoughton
- Jackson, P. (1968) *Life in classrooms*. New York: Holt
- Jaworski, B. (1985a) 'A poster lesson', *Mathematics Teaching*, 113, December, 4-5
- Jaworski, B. (1985b) *An investigative approach to teaching and learning mathematics*. Milton Keynes: The Open University
- Jaworski B. (1986) 'A constructivist view of mathematics teachers', in Underhill R. and Jaworski, B. 1991, *Constructivism and Mathematics Education: A discussion*. Milton Keynes: Centre for Mathematics Education, Open University
- Jaworski, B. (1987) 'A question of balance'. *Mathematics in School*, 16 (2)
- Jaworski B. (1988a) 'Investigating your own teaching'. *Unit 5 of ME234 Using mathematical thinking*. The Open University, Milton Keynes.
- Jaworski, B (1988b) 'Is versus seeing as: Constructivism and the Mathematics Classroom', in D, Pimm (ed), *Mathematics Teachers and Children*, London: Hodder and Stoughton
- Jaworski, B. (1988c) 'One mathematics teacher', *Proceedings of the Twelfth PME Conference*. Veszprém, Hungary: Genzwein
- Jaworski, B. (1989) 'To inculcate versus to elicit knowledge', in *Proceedings of the 13th International Congress on Mathematical Education*. Paris: PME 125-132
- Jaworski, B. (1990) "'Scaffolding" - a crutch or a support for pupils' sense-making in learning mathematics?' in *Proceeding of the Fourteenth PME conference*, Mexico
- Jaworski, B. (1991) "Knots", in E271 *Curriculum and Learning*.Part 3.1 Milton Keynes, The Open University

- Jaworski, B. (in press) 'Mathematics Teaching, what is it?' *For the learning of mathematics*.
- Jenner, H. (1988) 'Mathematics for a multicultural society', in D. Pimm (ed.) *Mathematics, teachers and children*, London: Hodder and Stoughton
- Kelly, G.A. (1955) *The psychology of personal constructs. 1 and 2*, New York: Norton
- Kemmis, S. (1985) 'Action Research and the politics of reflection' in Boud, D., Keogh, R. and Walker, D.(eds.) *Reflection: turning experience into learning*. London: Kogan Page
- Kilpatrick J. (1987) 'What constructivism might be in mathematics education', in PME-XI proceedings, Montreal
- Kochman T. (1972) *Rappin' and stylin' out: communication in urban black America*. Urbana, Illinois: University of Illinois press
- Lacey, C. (1966) 'Some sociological concomitants of academic streaming in a grammar school', *British Journal of Sociology*, XVII, 3, 245 - 262
- Lacey, C. (1970) *Hightown Grammar*. Manchester: Manchester University Press
- Lerman, S. (1989) 'Constructivism, Mathematics and Mathematics Education', *Educational Studies in Mathematics* 20, 211-233 Kluwer, The Netherlands
- Love, E. (1988) 'Evaluating mathematical activity' in D. Pimm (ed.) *Mathematics, Teachers and Children*. London: Hodder and Stoughton
- Lutz F. W. (1981) 'Ethnography: the holistic approach to understanding schooling', in Green J. & Wallat C. (Eds)) *Ethnography and Language in Educational Settings*, Ablex, (Vol V in the series *Advances in Discourse Process*, ed. Freedle R.) reprinted in M. Hammersley (ed.) (1986) *Controversies in classroom research*. Milton Keynes: Open University press.
- Mason, J. (1984) 'Towards one possible discipline for mathematics education', in H.G.Steiner et al, *Theory of mathematics education*, Occasional paper 54, Bielefeld, Germany: Universität Bielefeld/IDM
- Mason, J. (1988a), *Learning and doing mathematics*. London: Macmillan
- Mason, J. (1988b) 'Tensions', in D, Pimm (ed.), *Mathematics, Teachers and Children*, London: Hodder and Stoughton
- Mason, J. (1988c) 'What to do when you are stuck', in *Using mathematical thinking*, ME234, Unit 3, Milton Keynes: Open University

- Mason, J. (1990) 'Reflections on dialogue between theory and practice, reconciled by awareness', in F. Seeger and H Steinbring (eds.) *Proceedings of the fourth conference on the systematic cooperation between theory and practice in mathematics education : Overcoming the broadcast metaphor*, Institute für Didaktik der Mathematik der Universität Bielefeld. Germany: Universität Bielefeld/IDM
- Mason, J. (1991) 'Reflection and the psychology of possibilities'. Invited talk to conference *Conceptualising reflection in teacher education*. Bath: University of Bath
- Mason, J., Burton, L. and Stacey, K. (1984) *Thinking mathematically* . London: Addison-Wesley
- Mason J.H. and Davis, P.J. (1988) 'Cognitive and Metacognitive Shifts', in *Proceedings of the Twelfth PME Conference*. Veszprém, Hungary: Genzwein
- Mathematical Association, (1991) *Develop your Teaching*, Cheltenham, UK: Stanley Thornes
- Maxwell, J. (1985) Hidden Messages, in *Mathematics Teaching*, 111, 18-19
- McIntyre D. & Macleod G. (1978) 'The characteristics and uses of systematic classroom observation', in McAleese R. & Hamilton D. (Eds) *Understanding Classroom Life*, NFER, 111-129
- McNamara, D.R. (1980) 'The outsider's arrogance: the failure of participant observers to understand classroom events', *British Educational Research Journal* Vol 6, No 2,
- Measor, L. (1985) 'Interviewing: a strategy in qualitative research', in Burgess, R.G. (ed.) *Strategies of educational research*. London: Falmer
- Mezirow, J. (1978) *Education for perspective transformation: women's reentry programs in community colleges*, New York: Centre for Adult Education, Columbia University
- Mezirow, J. (1981) 'A critical theory of adult learning and education', *Adult Education* 32,1, 3-24
- Nias, J. (1989) *Primary teachers talking*. London: Routledge
- Nickson, M. (1989) 'What is multicultural mathematics?', in P. Ernest, (ed.) *Mathematics teaching. The state of the art*. London: Falmer
- Nolder, R. (1991) *Bringing teachers to the centre stage: a study of secondary school teachers' responses to curriculum change in mathematics*. Thesis submitted to London University for examination for PhD

- Nuffield mathematics project, (1973) *Checking up III*. John Murray and W.R. Chambers, London
- Open University, (1982) *Developing mathematical thinking*, EM235, Milton Keynes: The Open University
- Open University, (1985), *Secondary Mathematics : Classroom Practice* PM644 (Video). Milton Keynes: Open University
- Open University, (1988) *Using mathematical thinking*, ME234, Milton Keynes: The Open University
- O'Reilly, D. (1990) 'Hierarchies in mathematics: A critique of the CSMS study', in Dowling, P. and Noss, R. (eds.) *Mathematics versus the National Curriculum*. London: Falmer
- Pask, G. (1976) *The cybernetics of human learning and performance*. London: Hutchinson
- Pearce, J. and Pickard, A. (1987) 'Being a teacher: Towards an epistemology of practical studies.' in J. Smyth (ed.) *Educating teachers*. London: Falmer
- Peterson, P.L. (1988) 'Teachers' and Students' cognitional knowledge for classroom teaching and learning', *Educational researcher* June-July, 5-14
- Piaget, J. (1937) *La construction du réel chez l'enfant* Neuchatel: Delachaux et Niestlé
- Piaget, J. (1952) *The child's conception of number*. London: Routledge and Kegan Paul
- Piaget, J. (1967) *Biologie et Connaissance*, Paris: Gallimard
- Piaget, J. (1970) 'Piaget's theory'. In P.H. Mussen (ed.), *Carmichael's manual of child psychology*. New York: Wiley
- Piaget, J. (1972) 'Intellectual evolution from adolescence', in *Human development* 15, 1-12
- Pimm, (1987) *Speaking mathematically*, London: Routledge
- Pimm, D. (in press) ' "Why are we doing this?" Reporting back on investigations', in D. Sawada, (ed.) *Language in the learning of mathematics*. Edmonton, Alberta, Canada: Alberta teachers Association/Barnett House
- Plowden Report, (1967) *Children and their primary schools*, London: Central Advisory Council for Education.
- Polanyi, M. (1958) *Personal Knowledge*. London: Routledge and Kegan Paul

- Pollard, A. (1984) 'Coping strategies and the multiplication of differentiation in infant classrooms', *British Educational Research Journal*, 10, 1, 33-48
- Polya, G. (1945) *How to solve it*. New Jersey: Princeton University Press
- Polya, G. (1962) *Mathematical Discovery*. New York: John Wiley
- Popper, K. (1963) *Conjectures and Refutations*, London: Routledge
- Popper, K. (1982) *Quantum Theory and the Schism in Physics*, London: Unwin Hyman
- Richardson, K. (1985) 'Learning Theories', *Unit 8 of E206, Personality, Development and Learning*, Milton Keynes: The Open University
- Rommertveit, R. (1974) *On message structure: a framework for the study of language and communication*. New York: Wiley
- Schoenfeld, A. H. (1985) *Mathematical problem solving*. New York: Academic Press
- Schön D.A. (1983) *The reflective practitioner*, London: Temple Smith
- Schön D.A. (1987) *Educating the reflective practitioner*, Oxford: Jossey-Bass
- Schutz, A. (1964) *Collected papers II: Studies in social theory*, A. Brodersen (ed.) The Hague: Nijhoff
- Scott-Hodgetts, (1986). 'Girls and mathematics: the negative implications of success', in L. Burton (ed.) *Girls into maths can go*. London: Holt, Rinehart and Winston
- Sewell, B. (1981) 'Use of mathematics by adults in daily life' (Advisory Council for Adult and Continuing Education) Quoted in DES 1982 op. cit. 6-10
- Sharrock W. & Anderson R. (1982) 'Talking and Teaching: reflective comments on in-classroom activities' in Payne C.G.F. & Cuff E.C. (Eds) *Doing Teaching: The Practical Management of Classrooms*, Batsford. Reprinted in M. Hammersley (ed.) (1986) *Controversies in classroom research*. Milton Keynes: Open University press.
- Shayer, M., Kuchemann, D.E. and Wylam, H. (1976) 'The distribution of Piagetian stages of thinking in British middle and secondary school children', *British Journal of Educational Psychology*, 44, 2, 155-164
- Shuard, H. and Rothery, (1984) *A. Children reading mathematics*. London: Murray

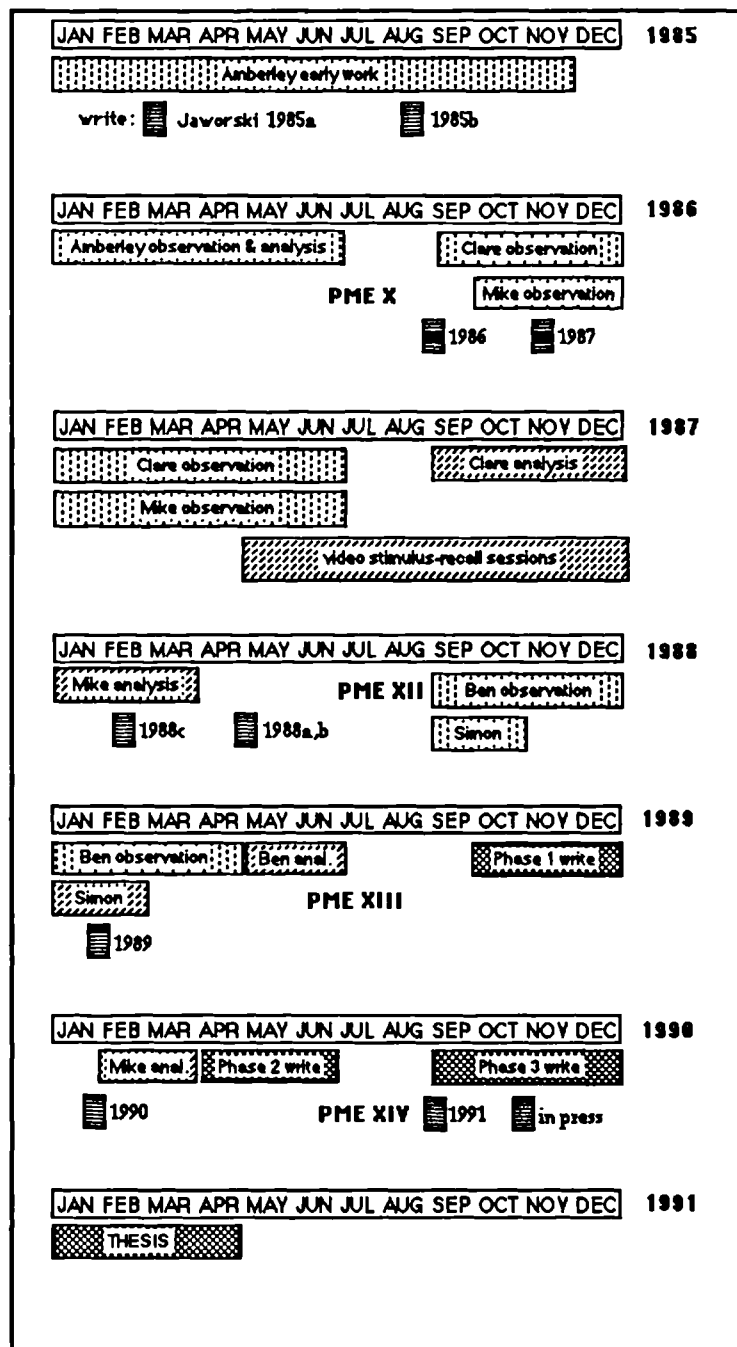
- Shulman, L.S. (1975) *Teaching as clinical information processing*. (Report of Panel Six, National Conference on Studies in Teaching) Washington DC: National Institute of Education
- Shulman, L.S. (1987) 'The wisdom of practice: Managing complexity in medicine and teaching.' in D.C. Berliner and B.V. Rosenshine (eds.), *Talks to teachers*. (369-386) New York: Random House
- Sinclair, J. McH. and Coulthard, R. M. (1975) *Towards an analysis of discourse: The English used by teachers and pupils*. London: Oxford University Press
- Skemp, R. R. (1971) *The psychology of learning mathematics*. London: Penguin
- Skemp, R. R. (1976) 'Relational understanding and instrumental understanding' *Mathematics Teaching*, 77.
- Skemp, R. R. (1989) *Mathematics in the primary school*. Routledge, London
- Smedslund, J. (1977) 'Piaget's psychology in practice', *British Journal of Educational Psychology*, 47, 1-6
- Smith, S. (1986) *Separate tables?* London: HMSO (For the Equal Opportunities Commission)
- Smyth, J. (1987a) (ed.) *Educating teachers*. London: Falmer
- Smyth, J. (1987b) 'Transforming teaching through intellectualising the work of teachers.' in J. Smyth (ed.) *Educating teachers*. London Falmer
- Sperber, D. and Wilson D. (1986) *Relevance*. Blackwell, Oxford
- Steffe, L.P. (1977) *Constructivist models for childrens' learning in arithmentic*. Durham N.H: Research workshop on learning models.
- Stone, A. (1989) *What's missing in the metaphor of scaffolding?* Paper presented at the annual meeting of the American Educational Research Association, San Francisco, 1989
- Sutherland, R. (1991) 'The changing role of algebra in school mathematics: the potential of computer-based environments', in P. Dowling and R. Noss, (eds.) *Mathematics versus the National Curriculum*. London: Falmer
- Tabachnick, B.R. and Zeichner, K.M. (1984) 'The impact of the student teaching experience on the development of teacher perspectives.' *Journal of teacher education*, 35, 6, 28-36
- Tahta, D. (1970) 'Idoneities', in ATM members (eds.), *Mathematical reflections*, Cambridge: Cambridge University Press

- Tom, A.R., (1987) 'Replacing pedagogical knowledge with pedagogical questions', in J. Smyth (ed.) *Educating teachers*. London: Falmer
- Underhill, R. (1986) 'Mathematics Teacher Education: A Constructivist Perspective', in Underhill R. and Jaworski, B. 1991, *Constructivism and Mathematics Education: A discussion*. Milton Keynes: Centre for Mathematics Education, Open University
- Underhill R. and Jaworski, B. (1991) *Constructivism and Mathematics Education: A discussion*. Milton Keynes: Centre for Mathematics Education, Open University
- Van Manen, M. (1977) 'Linking ways of knowing with ways of being practical' *Curriculum Inquiry* 6(3), 205-228
- von Glasersfeld E. (1982) 'An interpretation of Piaget's constructivism'. In *Revue internationale de philosophie* No 142-143 Pgs.612-635
- von Glasersfeld E. (1983) 'Learning as a Constructive Activity'. In PME-NA Proceedings, Montreal, September-October, 1983, 41-69
- von Glasersfeld E. (1984) 'An introduction to radical constructivism'. In P. Watzlawick (ed.) *The invented reality*. London: W.W. Naughton & Co.
- von Glasersfeld E. (1985) 'Reconstructing the concept of knowledge'. *Archives de Psychologie*, 53, 91-101.
- von Glasersfeld E. (1987a) 'Constructivism'. In T Husen & N. Postlethwaite (eds) *International encyclopedia of education: Supplement Vol 1*. Oxford: Pergamon .
- von Glasersfeld E. (1987b) 'Learning as a Constructive Activity'. In C. Janvier (Ed) *Problems of representation in the teaching and learning of mathematics*. Lawrence Erlbaum, Hillsdale, NJ.
- Vygotsky L.S. (1962) *Thought and Language*, Cambridge, Mass: M.I.T. Press
- Vygotsky L.S. (1978) *Mind in Society. The development of the higher psychological processes*. London: Harvard University Press
- Walden, R. and Walkerdine, V. (1985) *Girls and Mathematics. From primary to secondary schooling*. Bedford Way Papers 24. London: University of London Institute of Education.
- Walkerdine, V. (1984) 'Developmental psychology and the child-centred pedagogy: the insertion of Piaget into early education' in Henriques et al *Changing the subject*, London: Methuen
- Walkerdine, V. (1989) *Counting girls out*, London: Virago
- Wheeler, D. (1982) 'Mathematization matters', *For the learning of mathematics*, vol 3, no 1, 45-47

- Wood, D. (1988) *How children think and learn*. Oxford: Basil Blackwell
- Wood D.J. Bruner J.S. & Ross G. (1976) 'The Role of Tutoring in Problem Solving' *Journal of Child Psychology and Psychiatry*, 17, 2, 89-100
- Wood, D.J., Wood, H.A. and Middleton, D.J. (1978) 'An experimental evaluation of four face-to-face teaching strategies'. *International journal of behavioural development*, 1 (2), 131-47
- Woodrow, D. (1989) 'Multicultural and anti-racist mathematics teaching', in P. Ernest (ed.) *Mathematics teaching. The state of the art*. London: Falmer
- Woods, P. (1985) 'Ethnography and theory construction in educational research', in R.G. Burgess (ed.) *Field methods in the study of education*. London: Falmer
- Zaslavsky C. (1979) *Africa Counts*, London: Lawrence Hill
- Zeichner, K.M. and Liston, D.P. (1987) 'Teaching students teachers to reflect.' *Harvard Educational Review*, 57 (1), 23-48

APPENDIX 1 CHRONOLOGY AND CONVENTIONS

1. Research Chronology



2. Transcript Conventions

Much of the data with which I worked consisted of the words of teachers and of pupils. These were represented as words on paper in the form of transcriptions of audio-tapes, of summaries of the words on audio-tape, or of field notes which contained words or summaries. Re-presenting these for the reader required decisions to be made as to the most suitable forms which would

- be faithful to the data,
- reflect my thinking and analysis, and
- be reader-friendly.

I have tried to present the words of others in a way which carries with it the sense which I made from the recording. For example, where there were pauses or emphasis I have tried to indicate where these occurred. I was influenced by the work of Edwards and Mercer (1987), in which they say,

Our aim has been to present these sequences of talk as accurately as possible, using some conventions for the transcription of discourse, but at the same time ensuring that they remain easily readable and comprehensible. Our purpose has not been to produce an analysis of linguistic structure, but to provide the sort of information that is useful in analysing how people reach common understandings with each other of what they are talking about. (p ix)

People do not speak with formal punctuation, so the transcriber has to make decisions about where sentences begin and end, where commas are appropriate, and where quotes occur. I have used all the usual punctuation to fulfil the objectives expressed above. However, it is my punctuation and I recognise that it will influence interpretations which are made by those reading what I have written. The following conventions are also used:

...	Words omitted, either because they were irrelevant to the issue being discussed, or because they were inaudible. Where it is important to distinguish inaudibility I use the word 'inaudible'.
/	Pause of less than 2 seconds
//	Pause of more than 2 and less than 5 seconds
///	Pause of more than 5 seconds
<i>italics</i>	Emphatic speech

I have not used a special convention for two speakers who are talking simultaneously. Where this seems relevant, I mention it particular to the individual circumstances.

In order to avoid inclusion of very lengthy transcripts, I often include the transcribed speech in the relevant parts with a summary of that which occurred between them, in order to try to present a more complete picture without the length, and to emphasise the parts on which I want to focus.

In order to refer to lines of speech in a transcript extract, I have numbered the *statements* made. Numbers are normally provided at the beginning and end of one extract, and every five statements between.

APPENDIX 2

CONSTRUCTIVISM — A HISTORICAL PERSPECTIVE

According to von Glasersfeld (1984, 1985, 1987b), the place of constructivism in the history of epistemology dates back to the pre-Socratics. Xenophanes is reported to have said, “for if he succeeds to the full in saying what is completely true, he himself is nevertheless unaware of it.” A dichotomy regarding the nature of knowledge has been in existence since man began to question the basis of his own knowledge. This polarises between those thinkers who seek truth, or objective reality, and those who are prepared to go some way to the notion that we cannot ever know such truth – so other means have to be found to account for what we observe. Epistemologists through the ages may be seen as *metaphysical realists* on the one hand – who believe in the existence of a real world and think in terms of striving to know it – and *sceptics* on the other, who question the ability to know it. However, it has been known of sceptics themselves being drawn into the trap of acting as if a reality exists, if not overtly seeking for it. The belief in fundamental truth is very seductive.

It is von Glaserfeld’s claim that the majority of thinkers between the pre-Socratics and the later middle ages were metaphysical realists. The church had great influence on such thinking, believing itself guardian of *truth* about the nature of existence. However, with the advent of theories which threatened the picture of the world which the church held to be unquestionable, such as those of Copernicus and of Galileo, an alternative scenario was proposed for the pursuit of scientific knowledge. Popper (1963) writes of the dilemma which Galileo had in wishing to promote a theory that questioned what the Church held to be a true description of the world. According to Popper, “There was no objection to Galileo’s teaching the mathematical theory, so long as he made it clear that its value was *instrumental* only; that it was nothing but a ‘supposition’ ... or a ‘mathematical hypothesis’ – ‘a kind of mathematical trick, invented and assumed in order to abbreviate and ease the calculations’.” He quotes Cardinal Bellarmino who wrote of Galileo:

Galileo will act prudently ... if he will speak hypothetically ... : to say that we give a better account of the appearances by supposing the earth to be moving, and the sun at rest, than we could if we used eccentrics and epicycles is to speak properly; there is no danger in that and it is all that the mathematician requires. (Popper, 1963, p98)

Galileo supported the Copernican, heliocentric, theory of the universe. He had observed the moons of Jupiter through his telescope and supported the hypothesis that the Earth and other planets similarly revolved around the sun. Popper suggests that there would have been no problem had Galileo been able to fall into line with Andreas Osiander who wrote a preface to Copernicus’ controversial treatise *De revolutionibus*, in which he said,

There is no need for these hypotheses to be true, or even to be at all like the truth; rather one thing is sufficient for them — that they yield calculations which agree with the observations. (Popper, *ibid*, p 98)

The point being made was that a theory could be justified in terms of its ability to explain observations which were made. If this were all that was claimed, if the theory was not offered as a description of the true state of the world, then there would be no controversy. However, Galileo wanted to go further than this. He “conjectured, and even believed, that it (the Copernican system) was a true description of the world.” (Popper, *ibid*)

A hundred years later, the Copernican system had developed into Newton’s Theory of gravity and developed a following which made it a serious competitor to religion. Bishop Berkeley, in criticising Newton, was convinced that “this theory could not be anything but a ‘mathematical hypothesis’, that is a convenient instrument for the calculation and prediction of phenomena of appearances; that it could not be taken as a description of anything real”. (Popper, *ibid*, p 99)

Popper claims that it was the philosophers who took up this view leading to, in Hume’s hands, “a threat to all belief – to all knowledge, whether human or revealed”, or in Kant’s, a science of “mere phenomena, the world as it appeared to our assimilating minds”. Most physicists, however, like Galileo, continued in their search for truth, until recent times when the *instrumentalist* view (as Popper calls it) has become an accepted dogma. Popper himself believes that the instrumentalists, both philosophers and scientists have much to reexamine: “For at least in the eyes of those who like myself do not accept the instrumentalist view, there is much at stake in this issue.” (Popper, *ibid*, p 101)

Instrumentalism involved a movement away from the expression of theories which in some way tried to describe the world as it *is*, to theories which tried to describe the world as it *is observed*. Thus the validity of a theory in instrumentalist terms depends on its success in fitting the observations. However, the observations themselves are made by individual human beings, and so von Glasersfeld claims that since the *observations* are made by an experiencing subject, they depend on the subject’s way of perceiving and conceiving. They are no more objective than any other experience. In response to Cardinal Bellarmine’s advice to Galileo, von Glasersfeld comments that a radical constructivist, being under no obligation to defend the church’s *truth*, would make the same suggestion to any scientist, since what the scientist concludes can *only* be based on personal observation and experience. von Glasersfeld’s view seems to be that instrumentalism is movement towards constructivism, while yet trying to maintain some semblance of objectivity.

Popper himself, although providing an account of the history of instrumentalism, yet claims that instrumentalism is an unsatisfactory theory, because,

What we are seeking, in science, are *true theories* — true statements, true descriptions of certain structural properties of the world we live in. These theories or systems of statements may have their instrumental use; yet what we are seeking in science is not so much usefulness as *truth: approximations to truth; explanatory power, and the power of solving problems*; and thus, *understanding*. (Popper, 1982, p 42)

and von Glasersfeld comments,

There is no doubt that Popper intended an *objective* world, ie a ready-made world into which we are born and which, as explorers, we are supposed to get to know. This is the traditional realist view and Popper does his best to defend it. (1983)

The first person explicitly to formulate a *constructivist* theory of knowledge, according to von Glasersfeld, was Giambattista Vico, in 1710, in his treatise *De antiquissima Italorum sapientia*. (On the most ancient knowledge of the Italians) Vico had become profoundly critical of Descartes' theory of knowledge. He argued that, "the Cartesians misapprehended the nature of the physical sciences themselves, wrongly regarding these as capable of affording us the same kind of certitude that is to be found in the field of geometrical demonstration, the Cartesian paradigm of true knowledge". (Gardiner, 1967) Vico coined the phrase "Verum est ipsum factum" Gardiner translates this as, 'the true (verum) and the made (factum) are convertible', thus, 'we can know for certain only that which we ourselves have made or created'. Gardiner translates Vico's rejection of Descartes' proof of his own existence:

... the clear and distinct idea of the mind not only cannot be the criterion of other truths, but it cannot be the criterion of that of the mind itself; for while the mind apprehends itself it does not make itself, and because it does not make itself it is ignorant of the form or mode by which it apprehends itself." (Vico Opere, Vol I p136¹)

Vico appeared to claim that the Cartesians confused the nature of mathematical theories and scientific theories. Gardiner describes Vico's thinking,

although propositions of the type exemplified in mathematics unquestionably satisfy the Cartesian criterion of self evidence, the ground of their certitude is to be sought not in self-evidence but in the fact that mathematical systems are systems which men themselves have constructed. The truths of mathematics are irrefutable because the rules and conventions governing the symbols or concepts used in such systems are created by man and, in the final analysis, are arbitrary. ...

¹According to Gardiner, Vico's complete works, under the title *Opere complete*, Benedetto Croce, Giovanni Gentile, and Fausto Nicolini (eds), 8 vols in 11, was published in Bari from 1914 to 1941.

Physics is necessarily "less certain" than mathematics, for in physics we have to do with that which we have not created. Only God can know in the full sense the nature and workings of the universe, since it was he who made it. (Gardiner *ibid*)

Thus according to von Glasersfeld, Vico's belief that "objective ontological reality may be known to God, who constructed it, but not to the human who has access only to subjective experience" (1983), showed Vico to support a constructivist epistemology.

In present times, Piaget's thinking might be regarded as constructivist, although his work, possibly due to being misunderstood in translation, has indicated contradictory messages. Von Glasersfeld claims of Piaget, "There are many places where Piaget indicated that his theory of cognition was different (to one of realism or only of trivial constructivism) and that the results of cognitive activity could never be a replica of ontological structures." (1984) I consider Piaget's contribution to the analysis of mathematical thinking and learning in Chapter 3.

I find in interpreting von Glasersfeld's accounts of the thinking which was precursory to present day constructivism that the *truth* dichotomy is ever present. By this I mean not only the polarisation between those who seek objective truth, and those who deny the possibility of knowing it; but also the apparent seeing of *degrees* of objectivity. For example, in how far was the thinking of Vico more constructivist than that of the early instrumentalists? Rather than apologising for his review of history being 'both sketchy and biased', von Glasersfeld makes a point about ontology. Considerable research into the original works could yield no more than the researcher's interpretation of what they contain. He points out,

But for constructivists, all communication and all understanding are a matter of interpretive construction on the part of the experiencing subject, and therefore in the last analysis, I alone can take responsibility for what is being said in these pages. (1984)

Von Glasersfeld emphasises the impossibility of knowing objective truth, even by degree. He said, of Popper's putting aside instrumentalism in favour of a continued search for objective truth, "The realists and the sceptics are once more in familiar deadlock". I see current manifestations of Vico's view of mathematics, and the contrary view that mathematics exists in the world around us, in mathematics classrooms and in views expressed by mathematics educators. Current educational research and practice shows evidence of the truth dichotomy between the realists and the sceptics. This should not imply, however, that there are necessarily two camps. I see in myself, in the teachers I observe and in the research methods which I and others employ the two sides of truth in coexistence. Even in the overt espousal of a radical constructivist belief, the nature of truth continues to raise its head. I say more about such ontological implications in Chapter 2.

APPENDIX 3

THE PHASE 1 LESSONS AND RELATED THINKING

CONTENTS	PAGE:	
1	Summaries of the six lessons which I taught	10A
	Lesson 1 — Describing shapes — 28.1.86	10A
	Lesson 2 — Fly on Wall video — MI10 — 4.2.86	10A
	Lesson 3 — $2 + 3 = 5$ etc — 11.2.86	10A
	Lesson 4 — Red and White rod patterns— 4.3.86	11A
	Lesson 5 — Cuisenaire-rod algebra 11.3.86	11A
	Lesson 6 — Test sheet and framing problem 18.3.86	11A
2.	Five items relating to the lessons above	12A
	2.1 Lesson plan for the first lesson — 28.1.86	12A
	2.2 Test sheet for the sixth lesson — 18.3.86	13A
	2.3 Framing problem for the sixth lesson — 18.3.86	14A
	2.4 Noticing decision points – an anecdote – Feb, 1986	15A
	2.5 Some thoughts on my current research roles and activities – February, 1986	17A

1. ***Summaries of the six lessons which I taught***

In what follows, I have reproduced the notes which I wrote in 1986 to summarise these lessons, adding only extra information [in square brackets] where I now feel it would clarify these notes.

LESSON 1 — DESCRIBING SHAPES — 28.1.86

I described a shape to the class. They had to draw. Discussion of results.

Establishing class atmosphere — need for being precise about what you mean — mental images.

Work in pairs, describing own shape to partner, who draws it.

Discussion of results.

[A lesson plan for this lesson is provided below.]

LESSON 2 — FLY ON WALL VIDEO — MI10 — 4.2.86

Showed video, discussed fly's route in *pause-for-thought*.

Brainstorming on questions to ask — list on board. Pairs choose own question to work on. Difficulties with 'apparatus'.

[MI10 is a BBC series of 10 minute programmes for mathematical investigation, incorporating a *pause-for-thought* to encourage discussion of what has been seen in the first part of the programme. This programme involved a fly's route across a box, and way of fitting string around a box.

Difficulties with 'apparatus' involved cutting out and fitting together boxes and fitting string. I felt at the time that the practical difficulties got in the way of pupils' thinking.]

LESSON 3 — $2 + 3 = 5$ etc — 11.2.86

Wrote up on board: $2 + 3 = 5$ }

$4 + 6 = 10$ } and invited possible third lines.

Conjecturing, describing orally and in written form.

[This lesson is discussed in sections 2.4 and 2.5 below]

Hwk — describing red and white patterns of Cuisenaire rods— in preparation for next lesson.

LESSON 4 – RED AND WHITE ROD PATTERNS– 4.3.86

Follow-up from hwk. Black Cuisenaire rod + 3 red plus a white.

Blue rod = 3 reds and 3 whites. Different arrangements. How do we describe them — in words, in shorthand?

Discussion of different notations.

[I wrote about this lesson, and the following ones in Jaworski 1987]

LESSON 5 — CUISENAIRE-ROD ALGEBRA 11.3.86

Wrote up algebraic descriptions of a few patterns e.g. $B = Y + P$; $N = 3xR + 3xW$ ($9 = 3x2 + 3x1$).

Invited class in 6 groups to find as many equivalent patterns to *orange* as possible and to symbolise them and to insert their colour numbers (e.g. red = 2)

Very noisy, but most people involved.

LESSON 6 – TEST SHEET AND FRAMING PROBLEM – 18.3.86

Test sheet to allow evaluation of individual response to pre-algebra work. Problem to allow application of ideas. [Copies of both are in sections 2.2 and 2.3 below.]

Second lesson of the pair more successful than first — gave more direction towards looking at total length of Cuisenaire rods in frame. Lots of conjectures; spent time persuading them to be convincing then to express conjectures algebraically.

2. Five items relating to the lessons above

2.1 - LESSON PLAN FOR THE FIRST LESSON 28.1.86

Lesson 1

T. I'm thinking of a triangle which has two angles the same, and the third angle is larger than the others. Can you visualise such a triangle? (Possibly ask someone what I said)
Try drawing on paper the triangle which you visualise.

Class discussion of results - same?, different? why?

Activity ① In pairs 1st person draw shape on paper in secret.
describe shape in words (no hands)
2nd " draw shape which is described.
Negotiate result if necessary

Class discussion - invite 3 or 4 couples to talk about their activity and what they felt - what was going on in this?

In pairs - same again, reverse roles

Whole class - further discussion; possibly get one pair to do the activity publicly

Activity ②

T. I have a shape in my mind - I've drawn it on paper, so I won't be tempted to change it - You have to find out what it is by asking me questions and building up your own mental image. When you think you've got it, you can draw your image on the board for me to see. I'll tell you if it's the one I'm thinking of.

2.2 – TEST SHEET FOR THE SIXTH LESSON ~ 18.3.86

KEY	
Orange	O (10)
Navy Blue	N (9)
Black	B (7)
Purple	P (4)
Light Green	C (3)
Red	R (2)
White	W (1)

EXAMPLE

O			
C	C	C	W

$$O = C + C + C + W$$

$$(10 = 3 + 3 + 3 + 1)$$

or

$$O = C \times 3 + W$$

$$(10 = 3 \times 3 + 1)$$

or

$$O = 3C + W$$

$$(10 = 3 \times 3 + 1)$$

Draw a sketch for each of the following:

1. $B = W + R \times 3$
2. $N = (W + R) \times 3$
3. $N = C \times 3$
4. $O = 2(C + R)$
5. $O = 2P + 2W$

Write in symbols for each sketch

6.

N		
C	C	C
7.

N		
P	P	W
8.

O			
P	W	P	W
9.

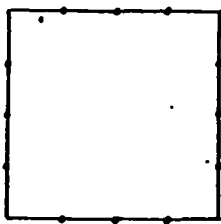
O					
R	R	W	W	R	R
10.

O				
R	C	W	C	W

On the back of the sheet write the number versions of sketches 1 to 10.

2.3 – FRAMING PROBLEM FOR THE SIXTH LESSON – 18.3.86

Investigate - ways of making a frame with Cuisenaire rods to fit around a square picture - the frame should be one rod deep. Start with the one below:



Frame this picture using Cuisenaire rods.

- 1) What colours can you use?
- 2) What is the minimum numbers of rods possible? How many ways can you achieve this?
- 3) Which frames use only one colour of rods?
 " " " two colours " " ?
- 4) What is the total length of rods in the frame?
 Try to write down some symbol equations for this length.

2.4 – NOTICING DECISION POINTS – AN ANECDOTE – FEB, 1986

I noticed a tension during my work in a classroom last week.

It was my third lesson with the group of (30+) 11-year-old pupils and I was in the process of encouraging communication, both verbal and written, and establishing a conjecturing atmosphere.

On the blackboard was:

$$(1) \quad 2 + 3 = 5$$

$$(2) \quad 4 + 6 = 10$$

$$(3)$$

Pupils were in the process of justifying possible line-3's. We'd had conjectures of:

$$8 + 12 = 20 \quad \text{and} \quad 6 + 9 = 15 \quad \text{and} \quad 5 + 10 = 15$$

Some of the justifications were:

$8+12=20$ is the right one because it's produced by 'doubling' each row.

$6+9+15$ is what we want because it produces the 2,3, and 5 'times' tables.

$6+9=15$ is the one because you just 'times' the column number by the row number. (column numbers interpreted as those in the first row.)

$5+10=15$ is right because you're just adding them up.

It could be anything because 'she' (me!) didn't say it had to be a sequence.

Some of this was well articulated, much of it was not. I tried to write on the board what was said, and then get them to modify it.

There was no great response to modification. Clearly they didn't see a need for it, although they were beginning to see the point of saying clearly in words. This stage could have gone on for some time.

I was conscious of a need to move on mathematically. Ending the lesson with this debate somehow seemed to leave things in the air, and I had no follow-up lesson to finish it off.

So I suggested voting on one of the three forms; $6+9=15$ won, and the lesson progressed to further conjecturing of lines of the sequence and rules for generating the sequence.

The lesson had a mathematical outcome, but would more have been gained by pursuing explanation and written communication rather than satisfying my desire to forge ahead with the mathematics?

A consequence of this experience which was very potent for me occurred in a lesson that I observed later the same day. The regular teacher of the above class who had been present during my lesson with them was teaching another of her groups. She was having a session of writing pupil's ideas on the blackboard after a brief activity. Several ideas had been written up, and one boy was giving a lengthy, but coherent account of his images from the activity. At the end of this the teacher said something like 'yes, that's very interesting isn't it', although she didn't write anything up. There had been some lively behaviour from others in the class during the last exchange and she commented 'It's bad enough when you don't listen when I'm speaking, but it's even worse when someone else is speaking'. Then she moved on to the next stage of the lesson. I wondered whether she had been pressured by the behaviour, or had made a conscious decision that it was time to move on to the next stage for some other reason. When I asked her, she said that she hadn't noticed making the decision to move on, but on reflection realised that she had felt some need to halt the class contributions then and move onto group or individual work. She felt that she would have done the same regardless of the behavioural aspect.

I felt that noticing my own behaviour during the earlier session had made me more aware of potential decisions that she had to make.

2.5 – SOME THOUGHTS ON MY CURRENT RESEARCH ROLES AND ACTIVITIES - FEB, 1986

February 1986

Some thoughts on my current research role(s) and activities

I am working in School (1) with teachers A and B as:

- (1) Teacher of two first year groups (of broadly higher ability), one each of A and B, with teacher present (mostly non-participant) in the classroom, for 1 double lesson per week each.
- (2) Observer of A's 3rd year group (middle ability) for 1 double lesson per week.
- (3) Participant in dialogue with A and B on shared (practice) classroom experience and issues arising from it.

I am aware of operating at a number of different levels, all closely related, and 'playing' (sometimes simultaneously) various different roles.

Role 1 Classroom Teacher

My concerns here are with the content and processes that I want to work on with pupils, and my aims in doing this.

For example:

'Communication of mathematical ideas' is my main theme for a number of lessons. The 'content' of the lessons is chosen to provide a medium or context for the process of communication to take place. The content is also related to areas of syllabus which A and B specify. Activities are designed so that pupils communicate their ideas concerning the content on which the ideas are based.

In a particular lesson pupils were given:

1. $2 + 3 = 5$
2. $4 + 6 = 10$
- 3.

and invited to contribute a third line. Conjectures of

$$8 + 12 = 20, 6 + 9 = 15, 5 + 10 = 15$$

- 2 -

were forthcoming, and pupils were asked to explain why each of these was suggested. The mathematical content concerned the investigation of some agreed sequence of statements of the form $x + y = z$, and generalisation of the generating process.

This mathematics provided a context for communication in that pupils had to explain to the satisfaction of others any conjecture which they made of a particular item of the sequence or rule for generating the sequence. From previous lessons they had learned that all their ideas are valuable, but can only be appreciated by others if adequately explained or expressed. Mostly they were willing to offer ideas and they were gaining experience in negotiating meaning with each other, although there was still reliance on 'the teacher's opinion'. In this lesson I aimed to progress into the requirements of written communication by inviting pupils to write down their 'rules' as they thought of them, and to contribute forms of words to write on the blackboard for common consideration. The words produced were mostly verbal statements written down. Opportunity for discussing ambiguous or ill-formed language was slight as there was more concern with negotiation of the ideas which it expressed. What was required was the concentration of the group on an agreed idea, so that refining of its clear expression could be a common concern. This became an aim for a future lesson.

Role 2 - Analyses of my own way of working and issues arising from it.

My concern here is to identify tensions between theory and practice, and between conflicting ideals, in working with pupils in the classroom where I am the teacher, and to reflect on their resolution.

For example:

Mutual respect of ideas is a worthy ideal in getting pupils to work co-operatively. In practice this involves giving fair consideration to all seriously offered ideas from pupils and not imposing unsubstantiated judgements. In the context of the lesson described above, pupils were at one stage offering justifications for a variety of ways of continuing the sequence

- 3 -

1. $2 + 3 = 5$
2. $4 + 6 = 10$
3. \vdots

The third entry should be

- (1) $8 + 12 = 20$ since this is produced by 'doubling' on each row.
- (2) $6 + 9 = 15$ since this produces 2's, 3's and 5's 'times-tables'.
- (3) $6 + 9 = 15$ since you just 'times' the column number by the row number (column numbers interpreted as those in the first row).
- (4) It could be anything as she (me) didn't say it had to be a sequence.
etc

At least half the class had something to contribute at this level. Some of their statements were different expressions of the same idea, some were rather obscure, some were very difficult to understand and needed a great deal of negotiation and rephrasing. Clearly discussion here could have continued for the rest of the lesson. However, inevitably, pupils held allegiance to their own ideas and became restive when the discussion stuck with someone else's, possibly unrelated, contribution. Then subsidiary conversations sprang up, or individuals started to look bored.

A serious possibility here was to invite small group discussion (perhaps in fours) on their chosen aspect of interest at this level. However, I felt a strong imperative to focus the discussion, so that pupils could get a sense of their mathematics 'going somewhere'. So, I invited a vote on the alternatives under discussion, from which ' $6 + 9 = 15$ ' was selected. Further terms of the sequence followed quickly and consideration of the n^{th} term resulted. I was very conscious of having chosen to 'direct' the lesson in this way, but, at the same time, of conflicting motives, and a sense of regret for what was lost in taking this direction. (David Pimm would call it a gambit, but this is too strong a word implying conscious motivation and sacrifice, whereas it was much more a sad recognition that two ideals were mutually incompatible and of the need to choose one of them.)

- 4 -

On reflection I think the decision was the right one under the circumstances of my seeing the pupils only once a week. However, had an early follow-up lesson been possible, it would have been valuable to stay with small group discussion for development of communication skills, leaving mathematical development for the next lesson.

Role 3 - Observer of another teacher's lessons for research purposes.

My concern here is to help the teacher concerned identify aspects of their own practice which are of particular interest in the context of an investigational approach to learning mathematics.

For example:

Whilst observing one of A's 3rd year lessons I was conscious of A's subtle changing of a pupil's language as she wrote the pupils idea onto the blackboard. I was curious as to whether this was a conscious act on her part (perhaps of clarification) or whether she was doing it subconsciously.

Another observation, which was particularly pertinent for me as it followed the lesson I described earlier, was A's 'moving-on' of the lesson from one stage to the next. She had been in the process of writing pupils' ideas onto the blackboard. One boy (Glen) had described his idea (which I thought particularly interesting) at some length. A acknowledged the idea but did not write it up. Instead she moved the class on to a further activity. I wondered at her decision there,

- a) to not write up Glen's idea
- b) to move on to the next activity.

How conscious a decision was this? Did Glen's idea have anything to do with it?

Role 4 - Co-analyser/Researcher with another teacher of aspects of practice of particular interest or concern.

This role is more difficult to specify as it involves working towards a situation where the teacher analyses her own practice and reflects on its development, and where I as researcher observe and learn from these acts.

- 5 -

However, it is also a consciousness-raising role of helping the teacher first of all to be aware of acts and decisions which she makes in the classroom, to reflect on them and work on their development where appropriate.

For example:

In A's lesson just discussed, I mentioned that I had noticed her subtle altering of pupils' language when writing it onto the blackboard, and was curious as to how consciously she had done this. From her response it appeared that she had been doing it chiefly subconsciously, but on having it pointed out realised that she was aware that she did it, and that it was an instinctive desire to clarify what had been said. It is now an aspect of her practice which we are both aware of and can discuss her conscious application or suppression of it.

A bonus of her also observing my lessons with her pupils is that I can air the observations that I make about my own practice for her consideration. When it is an issue that relates to us both it is particularly fruitful in that it is a common concern which we can work on together.

Role 5 - Researcher into classroom practice

This is the most difficult role to define. It suggests that there must be a substantial degree of objectivity in my classroom observations which at the moment is certainly not present. In fact I am not just an observer in A's lessons as I am explicitly trying to raise her consciousness of her own acts and decisions in those lessons. By using our shared experiences of teaching and observing I can introduce examples from my practice that may act as a 'trigger' to her noticing examples from hers. In using a particular language to describe my concerns, I can help her to express her own. Only when she is noticing and talking freely can I hope to become at all close to objectivity.

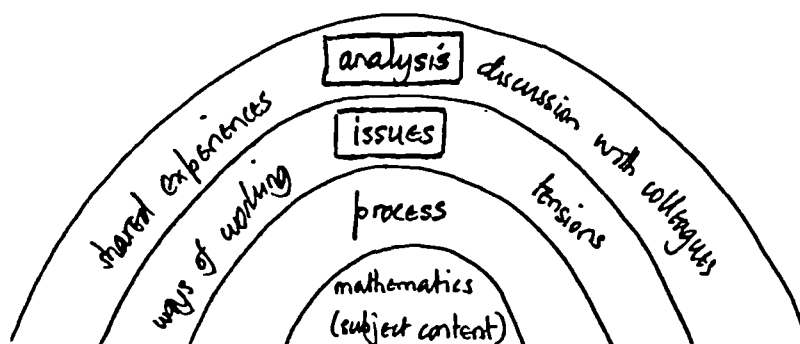
However, if teachers are to be helped to be conscious of their classroom practice in order to develop it, some appreciation of what such help involves is important. So in fact working on this constitutes important

- 6 -

research. Perhaps what is needed is a second researcher to observe my working with A? (Or, I need someone else to play my current role, so that I can become the observer.)

Levels of operation

These various roles seem to fit within a number of levels of operation, or perception, in which a certain hierarchy is present:



At the lowest level is the mathematical content of a lesson:

eg ratio - what aspects of ratio to work on - what activities to employ
- what texts to use etc.

Next is process: eg specialising and generalising. Helping pupils to recognise what they are doing and what they need to do in making the mathematics meaningful for them.

Then the chosen way of working and the tensions resulting from this
eg choosing to work as a full group rather than in small groups; choosing to push on the mathematical ideas rather than taking an opportunity to give more time to individual expression of ideas.

Observation of tensions here is important as this leads to the identification of the important issues to pursue.

- 7 -

Finally the analysis of all above where I look back on what happened in the classroom and try to make sense of it, pulling out the important observations - deciding what was important. Being strict in putting aside the rest. Very important here is the joint analysis of 'shared' experiences with colleagues - the observer's v. the teacher's view of the lesson. Asking why did I/you do that, what did I/you feel at that point? Identifying the issues. Getting colleagues to recall similar experiences in their other lessons.

What am I aiming for?

My official title for research is "To determine and work on the important issues involved in working investigatively in mathematics lessons" with the intention that this should be teacher focussed. My particular interest is in a teacher's identification of issues and subsequent action in consequence of this. Implicit in such observation is the establishing of a means of communication with the teachers on whom case studies will be based. This may require means of raising their awareness first, so that they notice aspects of their practice which are significant and on which they want to work.

This is already starting to happen with A; less so with B, although probably because I haven't been observing her lessons and am certainly less close to her. A question here about this 'closeness': It implies a level of understanding and communication that could be either a result of talking and sharing ideas, or a natural affinity. It is probably both, but how much of the poorer level of communication with B is due to a lesser affinity?

I am starting to 'pull out' some issues from my communication with A, and have to now agree with her about how she wants to work on them (if she does). What I really want is for her to start pulling out some issues herself and deciding how to pursue them.

I think I ought to be learning from the development with A, so that hopefully this process might be easier with the next teacher.

The stages which I can identify are:

1. Getting to know A. Observing her classroom. Penetrating my own threat to her. Starting to talk about her classroom and way of working. Teasing out her thoughts and opinions - insinuating a few myself (!) eg. the pamphlet.

- 8 -

2. A level of communication where we're not quite equal yet - but she can perceive my vulnerability in the classroom and so I become less of a threat. We have experiences to share now so a common concern. As I talk about my own tensions and issues she can start to identify her own.
3. We're here now. I'm aware of a sort of 'readiness' - an almost 'make or break' potential. What happens now could be crucial. This is where I need to get my act together. I can sense that I'm being accepted more as a colleague and that it is an opportune time to put some pressure on A to start thinking and operating. Is this too imposing? Should I be simply observing how things go? Is it legitimate to push? This is the objectivity thing in extreme - a lack of objectivity is one thing, but being interventionist is surely another?

APPENDIX 4

THE PHASE 4 LESSONS AND RELATED THINKING

CONTENTS	PAGE
1. The early analysis of Clare's lessons.	26A
2. Further manifestations of Clare's <i>sensitivity to students</i> (SS).	33A
3. Extracts from video-stimulated-recall sessions.	39A
4. Comparison of transcripts of interviews with Clare and Mike.	41A
5. Pythagoras episode — William <i>tells</i> the group.	50A
6. The <i>strange billiard table</i> lesson— introductory sheet.	52A

1. *The early analysis of Clare's lessons*

CLASSIFYING ATTRIBUTES FROM FIELD NOTES AND RECOLLECTIONS

This analysis resulted from working on my field notes, from the eight lessons which I observed in the autumn term, in an attempt to identify what I had regarded as significant and in some way to categorise it. I started by reading through a set of notes several times, and then numbering, in sequence from the beginning, episodes *which seemed to be of significance*.

The phrase 'which seemed to be of significance' is a crucial one. I was very aware of my selectivity in the episodes on which I chose to comment, at two levels. Firstly I had chosen, implicitly, to make a note of these episodes when recording in the classroom, and simultaneously omitted to remark on others. Secondly as I read through the notes, some of what I had written became more important than the rest when I allocated numbers prior to categorisation. I subsequently wrote the piece on 'significance' which I include in Chapter 4, p 80, in recognition of the issues which had been raised.

I then extracted these significant episodes, from the field notes, and tried to put some description onto them. Choosing descriptors was not easy to begin with, as I wanted them to be specific to the individual episode, yet to be able to group episodes in terms of a descriptor. I therefore needed something which would be generalisable at some level.

Data item 1 is a copy of one page of my field notes from Lesson 1, *Lamp, child and shadow*, showing my subsequent numbering of significant episodes, 6,7,8, and 9 in this case. Data item 2 is a list of significant episodes from that lesson, together with my first attempt at coding. Episodes 6 to 9 on the second of these items correspond to those marked on the first. The coding A,M,ML,Q etc., my first attempt at descriptors, is expanded in Data item 3.

These items are typical of this level of analysis for all of the eight lessons. However, the coding categories evolved. For example M stood originally for Classroom Management (*and* learning management). I subsequently introduced ML to be Management of Learning. Finally ML subsumed classroom management, as it did some other categories. I, for Imagery, was a very particular technique which Clare used, but it was not clear why I should differentiate it from S for Strategies. Were there other strategies that I should want to emphasise by noting them separately? Some of the numbered episodes seemed to defy an obvious classification, numbers 1 and 15 in Data item 2, for example. Then others seemed to fit possibly into two or more categories, e.g. 2 and 8. What did this have to say about the categories concerned?

	26.9.86.	3
12.25	<p>2 boys enter - + leave.</p> <p>T - trying to convince Francis you're right you're trying to conv. Joanne that you're right</p> <p>A - Francis.</p> <p>O - Joanne</p> <p>2 waiting boys ask q'n - T - "yes" - return to seats.</p> <p>T - talks to girls C. + B</p> <p>"Have you got this graph ... What are you going to do to find out if that's true?"</p>	<p>6</p>
12.30	<p>(I'm following the teacher round - what about others in room?)</p> <p>One boy at window - sighting sun with a ruler</p> <p>Tp (B) - ... could you show me</p> <p>A lot of mvk. in room - some boys come back in - girls C + B go out.</p> <p>Tp (boys who came in) talk about what they've just been doing - 'sun 27 mill miles away'</p> <p>T - tells 2 girls to talk to each other about what they've been doing</p>	<p>7</p> <p>How do class feel with me making + listening</p> <p>How do <u>renew</u> this classroom.</p>
12.35	<p>It's about shadows + the sun - measuring - do pupils have a clear picture of what this is about</p>	<p>8</p> <p>9</p> <p>I don't know what any pupils are doing or thinking.</p>

4th Year lesson 26.9.86 (~20 pupils mixed ability)

TOPIC - Lamp Child & Shadow - Last or penultimate lesson on this project

- 1) Making a suggestion, but giving students choice. - *pointing out possibilities*
- A 2) M Working in groups or pairs (how constructed?)
- 3) M "Tell me what you're going to do."
- 4) Q Questioning (probing? pushing?)
- 5) Q "So, what're you going to do?"
- 6) S "You try to convince --- that you're right" (twice)
- 7) Q "What're you going to do to find out if that's true?"
- A 8) P Pupils moving freely around room.
- 9) S Telling 2 groups to tell each other about what they've been doing.
- 10) Q What're you going to do next?
- 11) Q Questioning (why? trying to get pupil to see something?)
- 12) M Remonstrating for bad behaviour.
- 13) Q "What're you going to do with that?"
- 14) R Questioning motives (social relationship?)
- 15) Telling what she wants from pupils. - *making expectations explicit?*
- 16) S "Spend last few minutes recapping what you've been doing in this lesson." *encouraging reflection*
- 17) M Telling students that they can only have more time if they convince her they need it.

Data item 2 - Significant episodes from Lamp child and shadow lesson, 26.9.86

Key to symbols used in Analysis Current at 9-10-87

A	Class Atmosphere, and contributions to atmosphere
DP	Decision Point
E	Exposition
M	Classroom Management (& learning management?)
I	Use of Imagery
P	Pupil interaction of note
Q	questioning
R	Relationship with Pupils
S	Strategies

Data Item 3 – Key to coding categories 9.10.87

I tried to draw links between categories, listing different attributes or classification headings. Data items 4 and 5 show two of my attempts at coming to terms with the multiplicity of what I was seeing. Every episode recorded seemed to have degrees of significance which I wanted to address, and I asked, “Is this overdoing it?”. I continued in my diary,

Not sure that I am getting closer to an objective sense of Clare's operation — it's still very intuitive, Classification and categorising is proving more difficult than I expected — a defining vocabulary is what I think I need, but it's not coming easily. (Diary, 15.10.87)

I recognise now that the many hours spent on this were essential although it seemed at the time as if the process would never end and I felt that I was not actually getting anywhere.

At this point I wrote Jaworski (1988), a research report for PME XII¹, based on Clare's teaching, in which the discipline of having a limit of 8 pages forced me to select categories on which to report. I had somehow to convey the essence of what I was seeing in this teacher's operation in a few categories. I tried to choose those categories which seemed most important in that they characterised this operation.

There seemed to be three areas to consider – what I had begun to refer to as Management of Learning, although it was less broad at this stage than it became subsequently, Clare's extreme knowledgeability about her students and her attempts to tailor her teaching of them to their particular needs and characteristics, and finally her approaches to mathematics, developing students mathematical thinking and knowledge of mathematics. I realised that these three areas were not distinct, and I drew a diagram of interlocking circles to represent the links which I saw, fitting some of the individual attributes or classroom instances into the various sections of the diagram to help myself visualise the effectiveness of the

¹ Meeting XII of the International Group for the Psychology of Mathematics Education, held in Hungary

Class Headings for consideration of her operation

Ways of organising classroom

- construction of groups

- level of freedom given to pupils

Ways of talking to / with pupils

Use of questions

Convergence of mathematics

Advising on ways of working

Keeping control (disciplinary)

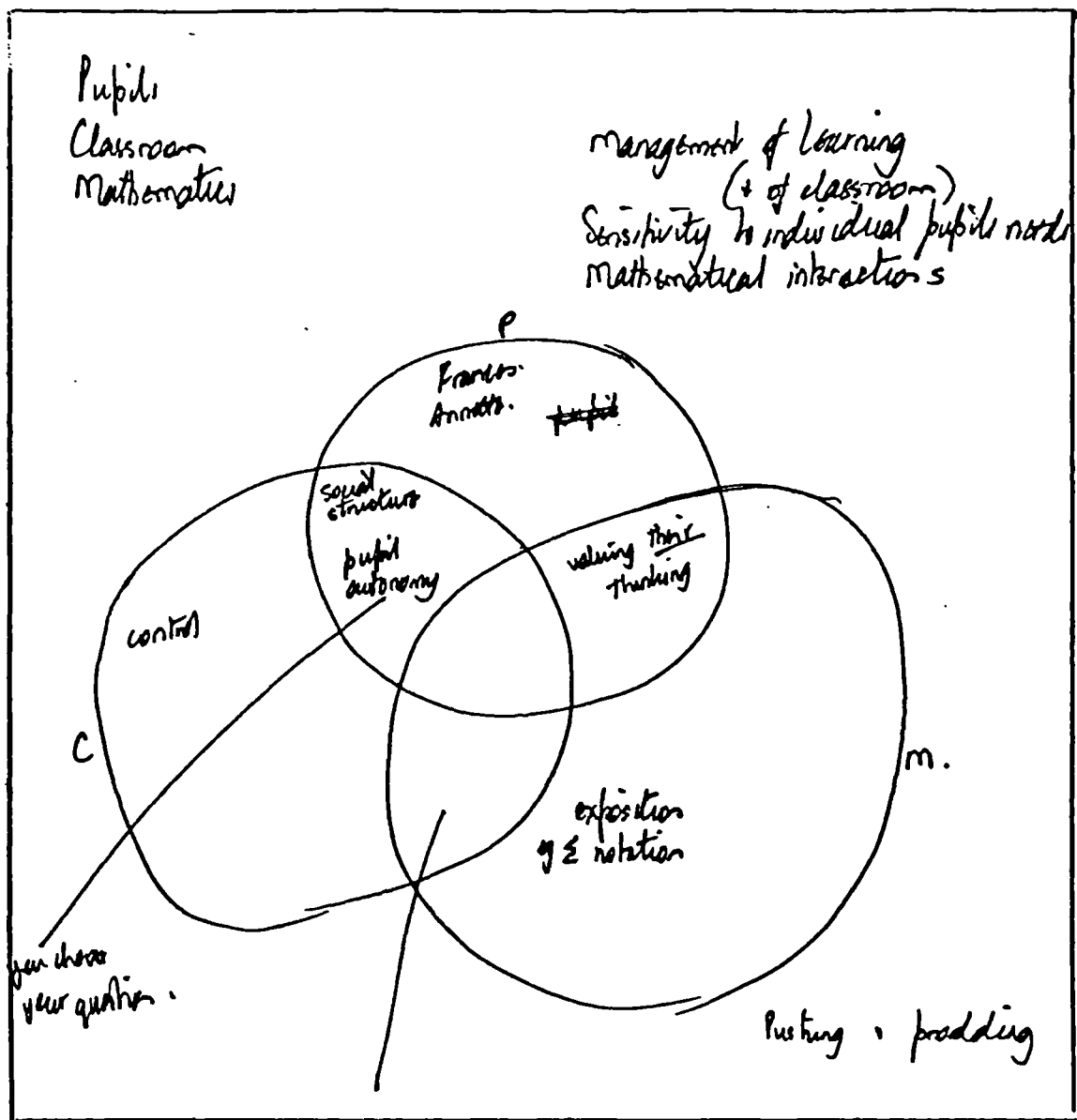
Encouraging pupils own thinking

Encouraging pupils to take responsibility for own learning

Discussing assessment with pupils

Relationship with pupils

Concern for pupils
Respect



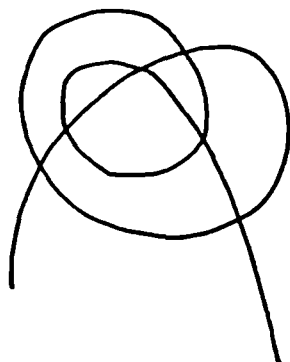
Data item 6 – First diagrammatical attempt at the teaching triad

2. *Further manifestations of Clare's sensitivity to students (SS).*

In Chapter 6 I gave some evidence of Clare's sensitivity to students. Further evidence is provided in the following excerpts.

KNOTS — A DISCUSSION

After Clare's whole-class introduction to the first knots lesson (described in Jaworski, 1991), the whole-class discussion merged into activity and exploration in pairs. Students were given a piece of paper and asked to 'jot down ideas' and 'any comments, any patterns you want to keep'. Somewhere in the process of this Clare asked the question, "How many ways are there of getting that arrangement in the piece of string?"



She referred to the diagram which she had drawn on the board at the beginning of the lesson, and to which initial discussion had been directed. There had been replies of 36, 14, 7, 99, etc and Clare cautioned them to make sure that the arrangements were indeed all different. There was some discussion about recording results and the form this might take. Ultimately the class settled down to exploring for themselves, in pairs, using pieces of string. Clare moved around the room from pair to pair listening, observing, and sometimes involving herself in the students' thinking.

Timesing or adding?

The following excerpt is taken from Clare's conversation with one pair of students:

- | | | |
|---|---|---|
| 1 | C | What are you doing here? What were you just saying?
... |
| | S | We start from there. You could go either over or under there, |
| | C | Well it may not be, but if you have two choices at each place, are you adding those to get your estimate? |
| | S | No, we're times-ing them. |
| 5 | C | Oh, what happens if you multiply? |
| | S | 12 times 12, it's 144. |

- C Where's the 12 come from?
- S Well, no, no, how many junctions have we got? 6 junctions. And we say, well there's 2 choices at each junction. that's 12 there ... yeah, 6 times 2 ... 12.
- C What are you going to multiply?
- 10 S The number of junctions by the number of choices.
- C Why?
- S To make the total number of ways you can do it.
- C This is a bit of a side-track .. imagine you've got two things you can choose from, OK? That's a bad example .. three things you can choose from. Does that mean that if it was both over and under, over and under, over and under, that it would be 6. Because I don't think it would. You see, the first two would be 4, but then for the third one it could be one way or another, so that's two ways you could have that .
- S I see.
- 15C Do you?
- S That's the over all pattern, because if you do like that, that was under, you've got another junction there which is over, ...
- C Can you jot down your reason there, because there is something, ...I was actually talking about string, but I don't want to keep you away from the string, so can you jot down where that 12 came from, because you've made ...
- S We're adding rather than times-ing.
- 19 C Yes, you're adding, you're not multiplying. You told me you were multiplying — That is adding. that is adding 6 lots of 2.

Data item 7 – Extract (1) from transcript of *Knots 1 – 13.1.87*

In the beginning Clare's questions seemed to be of genuine inquiry into what the students were doing and thinking. However, with the question, "Oh, what happens if you multiply?", she appeared to start probing, questioning their thinking, possibly because she suspected some misunderstanding, possibly because she was trying to sharpen up their thinking, possibly because she suspected that their imprecise articulation

indicated intuitive perceptions lacking in clarity. With the comment, “This is a bit of a sidetrack”, she took the plunge and got involved in their thinking herself. She then seemed to realise that her interjection may not have been helpful to the students, and urged them to record their own thoughts. In working on this piece of transcript I recognised issues which arose many times with Clare and during my wider study to do with communication, evaluation of students’ responses and the most appropriate teaching strategy. I perceived her to be struggling with understanding where they were at, handling what appeared to be an incorrect operation, and yet not devaluing their thinking.

Their ultimate response seemed serendipitous in defusing the teaching tension. Was their recognition that their multiplication should have been addition the result of anything which the teacher said or simply of her offering them opportunity to think aloud?

KNOTS — THE VALUE OF APPARATUS

Although hesitancy and questioning may have been often evident in Clare’s interactions, there were times when she was confident and sure that a particular approach was appropriate. In the next excerpt she came to two boys who had put their hands up:

‘A theory’		
1	P	We’ve got a theory.
	C	Do I want to hear a theory on the basis of how many bits of string that you’ve ... No, I don’t, you’re quite right, I don’t . considering you did your first bit without touching your string, I don’t want to hear your theory.
	P	We did touch it once,
	C	Yeah. What do you have to do to theories?
5	P	You have to prove them.
	C	The amount of crosses ...
	P	Or disprove them, that’s a lot more fun.
	C	What are you trying to do?
	P	Well, Clarke knows how to do something, but I don’t so he’s telling me how ..

- 10 C Clarke, oh of course you do, Clarke was on my summer course, yes, But I seem to remember we spent about half an hour on this, didn't we? So Clarke won't be very much further ahead, and we don't refer to people by their surnames in this school, thank you very much! OK, what Clarke will have learnt on my summer school course, that this is an extremely important part of the work, not just this and this, OK?

Data Item 8 – Extract (2) from transcript of *Knots 1* – 13.1.87

She appears to be very off-hand, unsympathetic and unsupportive. The excerpt lacks context, and this contributes to its terseness. A significant statement is, "Considering that you did your first bit without touching your string, I don't want to hear, your theory". The pair of boys to whom she was speaking were quite bright but very casual about their way of working. Her tone was part jocular and part hectoring, and the interchange was conducted with smiling faces. As I observed the episode, I perceived what the words here cannot convey, that there was a very well-developed adult relationship between them, and Clare did not need to be polite when she felt that the students were ignoring something which she valued and which they knew she valued. She said later:

The people I'm most conscious of pulling back and putting onto something else are people who I feel need to do things with string. And I actually feel quite strongly, because of not having an apparatus-based education myself, and knowing what I'd missed from it – like not being able to visualise an octahedron and this kind of thing – and I think it's important that they do realise that it's not babyish to use apparatus. ... Knowing Simon and Rob I'm fairly certain that they wouldn't have got very far without actually working with pieces of string. (Clare 13.1.87)

Her belief in the value of apparatus was strongly founded and I saw many occasions of her indicating where students might gain from its use.

COMMENTING ON PARTICULAR STUDENTS

Clare's knowledge of students

Ann — ... just copies other people's work. ... She just coasts along, picking up whatever other bits there are. Now I don't know about getting her to work on her own, because I think she'd just crumble. I actually think it's the best we're going to get, and it's an awful judgement, at this stage. I think the best we're going to get from Ann is competent writing up of other people's thoughts which she has made an effort to understand, and in fact her written work this year has been in excess in quality of what I thought she could do. So I'm quite pleased. She's confident working in that group and she feels

she's contributing things. Um, I mean, in her write-ups she says 'we did this' and 'we did that', so Ann does feel that she's involved, but she's very very low ability, and I think it would be awful actually at this stage to make her do something on her own.

James — He does have very good ideas, very maverick ideas in maths. But he has great difficulty following them through and producing anything concrete at the end. [I asked, "Does he know that, I mean, have you said that to him?"] Oh yes, he knows that. He's a constant disappointment to himself because he has the ideas but he doesn't do what's involved. ...

I think it's partly because he feels that if he can have these good ideas he shouldn't have to ask for help in following them up. But also, when I have given him advice and suggestions, he hasn't followed those up either. So there's an element of not being bothered. ...

His write-up consisted of ways in which this project [on *Statistics*] might be extended, ever so many of them. So he said things like, 'it would be nice to look at what would happen if the ratio of boy-babies to girl-babies was 51 to 49, and it would be nice to look at some other thing, and some other thing, and he even produced something that said, 'if I had the figure for so-and-so I would do this with them', and produced a series of mock figures, when in fact producing the real figure wouldn't have been that difficult. [These remarks on James extended to 3 sides of A4 transcript]

Tracey — Tracey's project was interesting. I put him up a whole grade. He has special learning difficulties. He's dyslexic, and I went to see his — the woman who runs the reading group — last week because it struck me that getting unders and overs [in *Knots*] in the right order, because they were the component parts of the whole thing and that the order they were in was important, it struck me that that was probably very like getting letters in a certain order and recognising a word, and that he might actually have had special difficulties in tracing around the string and doing the overs and unders. So I upgraded him and put a long note at the bottom for the examiners and so on when the time comes because they have to be specially moderated.

[I asked, "Does Tracy himself see that?"] Oh, yes. I wrote it all out and I let him read it, and he's going to take it down and discuss it with ... as well. What is the sad thing though is that there are other children in that room with the same difficulties, but because they have never been picked up by the system there isn't really anything one can do about it. I mean, Ann can't. The more I teach her the angrier I get, because she couldn't follow the string round at all. She had to touch it to make sure she wasn't leaving it. That child has such difficulties, and I get jolly irate.

**Data item 9 – Extracts from transcripts of discussion with Clare
13.1.87 & 10.2.87**

3. *Extracts from video-stimulated-recall sessions.*

I include here extracts from transcripts of stimulated recall sessions as evidence to support remarks made in Chapter 4 and Interlude B. In Data item 10, Clare, Mike and I had watched an excerpt in which Clare had been working with Ann, in a KMP lesson, on area of plane shapes. Clare recalled the event from the video, and her reflection resonated with Mike's experience causing him to recall an event from one of his own lessons.

Teachers recalling teacher-student interactions

- | | |
|-------|---|
| Clare | This is a lovely conversation. It's all about chopping up. It's an L-shape, chopped into two rectangles, and she actually <i>realises why</i> she has to chop it into two rectangles, and at one point she told me why to chop it into two rectangles to get the figure. And <i>then</i> when I ask her to do it, she does it totally differently. Instead of having the L-shape chopped into two rectangles, she actually makes it into a bigger rectangle. So, I think, oh hell, but we'll take her through this. And she <i>gets there</i> . She can't tell me what she is going to do — she can't tell me what she's <i>going</i> to do — but then she <i>does</i> it exactly right — and it's ever so exciting. |
| Mike | I had a similar thing, today with Sandy Brown; she was doing some work on square roots, and it's harping back to the thing we said earlier about how a kid knows they're doing something or not. She couldn't understand how you can square-root you know — 'what's the <i>opposite</i> of a square of a number? — I can <i>see</i> when I'm squaring. I <i>know</i> what to do ...' You can give them an algorithm, can't you, for squaring, but you can't give an algorithm for square-rooting. You just happen to know the answer. And, I was trying to say, well, the numbers just tumble out, and it sparked with her, and I said, well, you know, you square a number you get 25. What is the number, 5. What did you do? I don't know. It's just there, 36, 6, 81, 9. [I inferred that the pupil was responding with the 6, 9 etc.] Where's it coming from? 'I don't know. It's just there. It's just coming out'. It was nice. I don't know where the number was coming from. I suppose what she's <i>actually</i> done, is, she's run through her nine times table without even knowing about it, and found there was a pattern for the 81. That fits in there. It's nine nines. I don't know what you do. You know it straight away. Where it comes from, I don't know. |

In Data item 11, the two teachers talk about the role of the video recording, supporting remarks which I have made about reflection and resonance with experience.

What video offers	
Clare	I think if you are watching somebody else's classroom, it's a feeling of, 'Oh yes, I've <i>been</i> there. I know what that feels like from the teacher's point of view.
Mike	I was just going to say, <i>resonate</i> . It's something that really makes you — either you were in there and at the time it meant something to you, or now it means something to you; or you weren't there — like I wasn't in that lesson, but there were things that Clare was doing that I felt 'I've been there — I know exactly what's going on.' I know what the feelings are.
Clare	But, there's also seeing it from the student's point of view. That's why I'm so interested in this bit, because I want, I really want it confirmed that what was going on was what I thought was going on

Data item 11 – Extract from transcript of video stimulated recall session 4.2.87

4 *Comparison of transcripts of interviews with Clare and Mike.*

Interviews with Clare and Mike were informal and rather like conversations. However, there were differences between them. I felt that with Clare, my remarks were mostly concerned with seeking to know her perceptions and motivations. Thus a transcript of a discussion with Clare, usually had much more of her words than mine, as my remarks were usually brief. Although it was my intention to interact similarly with Mike, I found myself being drawn more readily into the issues involved. This was shown in a transcript by my contributions to the discussion being much more lengthy than with Clare.

I include two excerpts from transcripts. The first is from a discussion with Clare after lesson 13 (KMP, 6.3.87), where the focus of the conversation is her management of learning. The second is from a discussion with Mike after the third Pythagoras lesson (13.2.87), where the focus of conversation was the seeking of outcome from a classroom activity. The third participant in this, Sheila, is SH who was undertaking secondary observation in Mike's lessons. The two excerpts have been chosen because they illustrate the difference in my level of involvement, but they each manifest important issues in this research: the first contributing to a characterisation of *management of learning*, and the second to issues related to *the teacher's dilemma*.

EXCERPT FROM TRANSCRIPT OF CLARE'S KMP LESSON, 6.3.87

Barbara: Well, the main theme there for me was about classroom management and about this business of needing to talk to a lot of people individually so you don't actually get round the class and see the work that people are doing. So, more or less entirely on that theme, there is behaviour and attitude. You said at one point that there is a group over there gossiping, and the implication there was gossip is not allowed in Clare's lessons. There was the business of just being back from the ski-trip and, okay, you've been away on holiday but this is not a holiday now. So I saw you dealing with general behaviour, I saw a queue forming and right at the beginning of the lesson you had a very substantial queue and it seemed to disappear quite quickly. And so I was asking questions about how and why queues form and what your attitude to queues are, and why I never really notice you with long queues or queues particularly often. I mean having a queue at the front of your room doesn't seem to me a general feature of your lessons.

Clare: I felt it was inevitable this lesson, because firstly there were people who hadn't been in a maths lesson for a couple of weeks, I expected a much longer queue than there was because I expected people to have not brought their books, or forgotten where they were or what they were doing, that sort of thing. So actually I was quite pleased that the queue was not much longer.

Barbara: What were you mostly doing with them at that point?

Clare: Well, I was just trying to think because at the beginning that's what I was expecting, to have queues of people with administrative problems, but I didn't get them, I had instead the other things which I knew I would also get which were two or three bits of feedback from the last lesson where people had gone away and tried things, and had brought them back to show and see if they were okay. So I knew those were going to happen, I knew there were two or three people I had to see about test corrections, and I also knew that I wanted to see people about knots because I was aware that they had handed these things in hoping to get comments and have them back, and I think it's important to get them back as soon as possible if people are mature enough to develop their work. So I knew I was going to have to do all that so I knew I was going to have some queues, but actually they weren't as bad as I thought they were going to be. But I am very aware of queues. I am never quite sure what to do about them. But I don't want to rush anybody. Sometimes I will stop, it's the same with the hands up, sometimes I

will stop and clear the (inaudible) and if I have somebody who is going to be a long time, I might just ask the next two or three people what they want, or I might occasionally say to a queue, Look if you are waiting for a card to be signed go on to the next one and I will see you later. And that usually clears it a bit. But I am never really quite sure what to do. Sometimes people have to wait. It's a bit more obvious in a queue than when you're walking round the classroom.

Barbara: I believe that. I've seen situations where teachers have had much more lengthy queues, much more backlog than I see in your room so is there something about the way of working that discourages people from joining the standing in queues? Is there is some way in which they are more resourceful than other pupils might be, more independent? Is that possible?

Clare: It must be because this year we are not using (inaudible) foundation years, but in previous years, to me it has been one of the biggest problems with foundations years, what to do with the queues. So, now this lot were foundation years once, and formed enormous queues once.

Barbara: With you?

Clare: Mm, yes. So it must be something to do with getting used to the system.

Barbara: Whose system, yours? Or the schools, or the KMP system?

Clare: The KMP system, but it must be something else as well. I think it must be something to do with the fact that a lot of what I find myself saying to them is the same sort of thing anyway, and that I hope that I have said the same thing to them so many times they now to do things without having to come and ask me first.

Clare: For instance, if they get to the end of a work card they now know that I am going to ask them questions about that card, or I am going to make suggestions about things they might have thought about, or I might say Did you drop this that and the other? And so I think more of them do that without coming up to me first, now that saves a visit to the teacher, so that's training isn't it?

Barbara: Is it explicit or does it just happen because of the way you work.

Clare: Well I don't say Look, if you are doing this you only have to come and see once. I just go through the procedure of saying What did you learn on this card, and

why don't you go now and write it down in your own words. I go through that so often that some of them just begin to do it. Not all of them but most of them just begin to do it so that they actually come up prepared for the sort of things I am going to say to them. I suppose. I don't know if that is one of the answers, but I think it must be one of the answers. I try and make the students resourceful as well, I never have the right amount of the right sort of paper, but I do have my trolley and I do make sure it is properly equipped, and that cuts down the number of questions, and most of the, most of the KMP cards are there so that cuts down questions, because if people can't find a card then that's a bit of a hassle because they have to queue up and ask about that. There is a lot of the mechanical stuff, the administrative stuff which I think I've got more right than wrong, so that cuts down the queues.

Barbara: They seem to know what they are going to do, what they are supposed to do, I mean I get the impression.

Clare: I was quite surprised at the beginning of the lesson that they all knew what they were going to do, that they were that organised.

Barbara: Do they have strategies for coping with being stuck?

Clare: That is something I talk to them about explicitly, and implicitly, depending on who they are, both mostly, in that if you are stuck you check that you understood the (inaudible) that might be checking the answers or it might be just making sure that what you have written makes sense, because I find people usually get stuck about two stages further on from when they really stopped understanding, so to go back is one. And sometimes I talk to them explicitly. I have talked to all of them about using the answers, and how it is not cheating to look at the answer if you are stuck, and (inaudible) and working backwards from there to say Oh, this is what they meant, and sometimes when you do that you might end up feeling a bit of a fool, but that doesn't matter. So I talk to them about that. I talk to them about talking to each other about maths, especially somebody who is perhaps (inaudible) and I talk to them also about trying to explain things to somebody else, because that is certainly something that I have found helped me with my maths, so I try and get them to do it as well. To test out their understanding by trying to explain to somebody else. I don't get them to that that often, but it is certainly a strategy I have got up my sleeve, and if I ask them to do that I say specifically this is because if you are explaining something to somebody else you naturally think it through and make sure you understand, and sometimes you

can understand things a lot better, sort of in the middle of explaining it to somebody else. So there is those. I can't think of any more off the top of my head, but I think that is about it, so I do try and keep a collection of getting unstuck skills.

Barbara: One thing I noticed was today, when you talked to people they stood at your side, whereas last time you talked to them in much the same way you had them sit down. Do you think that makes any difference to what actually goes on?

Clare: Mm, um that was conscious, semi-conscious because I thought that as it was a first KMP lesson after a break for some I thought there would be a lot of quick questions, and when I have had people sitting down before there were very few people in the room, and I knew I was much freer to spend long chunks of time with people. The whole atmosphere of the room was different too, because all the people who mess about and who tend to waste time were ski-ing, and it isn't really that they've come back from ski-ing and were wasting time, it's just that the people who waste time tended to be the ones who went ski-ing, and so the atmosphere was different, so I would have them sitting on chairs in different circumstances. It depends on what I think is going to go on in the lesson really.

EXCERPT FROM TRANSCRIPT OF MIKE'S THIRD
PYTHAGORAS LESSON, 13.2.87

Barbara : I felt that all the kids, well most of the kids, that let's see, I've written it down, I've written down the result is important, and I felt that first of all it was very important that there was a result and secondly it was important that they knew what it was and that they could express it. And I was saying this to Sheila just now, and she was saying something about Mike thought the result was important to him, and put emphasis on it. Can we recreate that?

Sheila: I think that's why I started on the educational material because that's where I thought that Mike had indicated that he wanted a conclusion.

Barbara: That's right.

Sheila: Whereas I was not getting that to them, saying to them that we just wanted their results and I interpreted that as a definite conclusion. Now whether that is just my experience ...

Barbara: What made me write that down specifically was that when you asked them to change groups and I joined the group that I was with, what they did was to spend time trying to articulate the result which they had got, they didn't talk at all about what they had done, you know it wasn't, we tried out this and then we did that, and then we did that; none of that at all. It was, we got this result. And so they compared results and decided that they had all substantially got the same thing, and they didn't seem to be interested beyond the result. Now, would that have disappointed you, if you had been part of that would you have tried to make them talk about what they had done, or would you have been happy with that?

Mike: I was surprised by it. Given that at the outset what I was trying to do was to try to find an investigative way of approaching Pythagoras' theorem which is a result laid down by external agencies, that they have to know. Um, and that I don't like the idea of just reaching results, I wanted there to be something else on the way. And I wanted to put over what I think is important, whether it is a mathematical process and ways of thinking and ways of working in the way of doing it that had taken three lessons, three hours and it was probably going to take a bit more time, another session, probably I'm talking about four hours to get to a stage where most of them probably know the result. On Monday, they might not know it on Tuesday.

Barbara: So, coming back to these questions that I wrote down the other day, because I wondered what status the

fact that you were working towards Pythagorus' Theorem what that brought to the lesson. I'm saying this awfully badly. Um, if you had been working on some random investigation and where the important thing was going through certain mathematical processes rather than getting any particular result, would they still have been as keen to know the result, what the result is, and being able to say the result?

I am wondering if you have given them some clues in the way that you set this up and what you yourself want out of it, I mean what you want out of it is twofold, but one of the things is that you want them to learn Pythagorus' result.

Mike: Mn, well the way they come to their ideas, they may have partly, but that partly is nothing to do with me. A lot is to do with the mathematicians they have had the five years before they came and their perception of what mathematics is, if they perceive mathematics as a set of results, if that's firmly imbedded in their values system and beliefs system, then they are going to work towards that. Now suddenly destroying that and saying No, it's, there's no result, just investigate, just do it, see what happens. I would suspect judging by the response we were getting when we had a discussion with three of those students that they would be a bit unsure about that, feel What's the point if we don't know where we are going, surely you must know, you are a teacher, you must know we are going somewhere, and they are still seeing the teacher as someone who takes you to a position, a point, not a teacher who can point you down the road, and I see the latter as being quite important, a teacher being someone who can tell you which way to go and let you go on your own way and in the walking, in the getting there you develop a lot of awareness. I see the latter as being important, and therefore I would want to develop students towards this, but one of the problems of doing this is one doesn't know how long one has students for. One can take a year, half a year to get there, and quite suddenly they leave or there are problems with the timetable and you can't take them next year, then what happens? It makes one wonder a lot more about the need for a common approach, only common approaches that are the same as my approach. going, surely you know, you are a teacher.

Barbare: Let me give you a scenario. Did Clare happen to show you those little polystyrene balls with holes in? I could see the use of those to set up a fairly random investigation where with a result in mind- whatever that may be - but that, I think it could be posed in a number of ways, but the two that spring to mind are that you could set a particular investigation to do like how many ways are there of

Joining three of these together, four of these together, five of these together, and so on, or if you make a network from these what way could you represent it on paper? I mean various things like that. You could actually pinpoint specifically and ask them to investigate that, so that at the end of the day although you don't know it at this moment, I assume you don't, that there might be a result. On the other hand, just dump a pile of it on their desks and say invent some questions and explore them. Now, I think

Mike: You seem to be considering the two as distinct, different methods of teaching, different styles.

Barbara: Well, I think there are differences, what I am

Mike: I think there are differences, but I don't think differences like someone does it this way, and someone does it that way, and I was thinking then that would be a very nice piece of work to do, I mean I would have thought these, perhaps, judging from the response you get from them Oh, you just give it to us and didn't let us know, and they can now sense what it feels like doing a problem where they don't know where they are going, and they actually get somewhere. And maybe part of the teaching process is leading them towards a position of well I can maybe ... But that's part of teaching, part of teaching them not just Pythagorus Theorem, where I want to go, but also where I want to go is to see if I can develop a problem that I can develop the answer as well, you know? where I can develop the answer as problem without the question

Barbara: So you see that as part of a progression?

Mike: Sure.

Barbara: Oh, that's interesting. What I was wondering, the question in my mind was clearly there is some importance, there's some mileage in this result of Pythagorus, and there was a result, and they were saying things like 'Mike's got a result in his head', and he did, and if you had another situation where you didn't have a result in your head, and where it was genuinely exploration for its own sake rather than exploration to get Pythagorus, I mean

Mike: But it's not ...

Barbara: No, no, I mean maybe I am making an artificial split, between these two situations, but I was wondering if we could, we, I mean, feel free to ignore all of this totally. If you could set up another situation where some

of those conditions were different it would be interesting to see reactions to it and how they perceived what they were doing. I mean, whether we could then interview some of them and say 'How did your working on the Pythagorus result inform what you did here, did you still believe that Mike had a result in his head?' 'Did you believe you were working towards a particular point?' It might be at the end of the day that everybody settles down on doing the same thing, rather like billiards, because at the end of the first billiards lesson you had the same sort of, the whole thing being too disparate and people trying to explore things that they couldn't get anywhere with, and wanting to pull it together into one avenue, and in a sense there you got to a result, whether that was the result that was important in your mind to start off with I don't know.

Mike: Having a result is important in mathematics. . . .

5. *Pythagoras episode — William tells the group.*

The following episode occurred during Mike's second Pythagoras lesson and concerns one group of boys.

In the group, William, had been absent during the previous lesson, and so the others in the group explained to him what they had been doing with the two problems — Square Sums and Triangle Lengths (for more details see Chapter 6). His response was, "Oh, it's Pythagoras isn't it!" It appeared that he had met the result previously. They made him explain, since up to this point the name *Pythagoras* had not been mentioned, and they then checked out what he said and discovered that it fitted their data.

When Mike joined their group they were full of this experience. Following is an excerpt from the transcript of their conversation:

- | It's Pythagoras isn't it! | | |
|---------------------------|----|--|
| 1. | M | Right, can you explain how far you're getting? |
| | P1 | We've worked out the triangle lengths, haven't we William? (Laughter) |
| | M | How do you mean — the triangle lengths? |
| | P1 | Em, that's it (pointing to book) |
| 5. | M | What can you tell me about the triangle lengths then? |
| | P2 | Well, you take the two sides, the two right angled sides, and you square the numbers |
| | M | Can you tell me where this came from? |
| | P1 | William said .. |
| | P2 | William. We've been slogging our ideas out for days and William came in and said he knew it. |
| 10. | P1 | That's how you do it. |

Data item 12 – Extract from *Pythagoras 2* – 6.2.87

The conversation continued, Mike probing to see just how much they did in fact understand and suggesting further tasks.

My interest lay in what effect William's telling them the result had had on their thinking. I recalled that Mike, reflecting on the previous lesson, and in response to a question from me about how important it was to him that they should find the Pythagorean relationship, had responded with words

which I quoted earlier, about ‘telling’ an answer. He had gone on to say then,

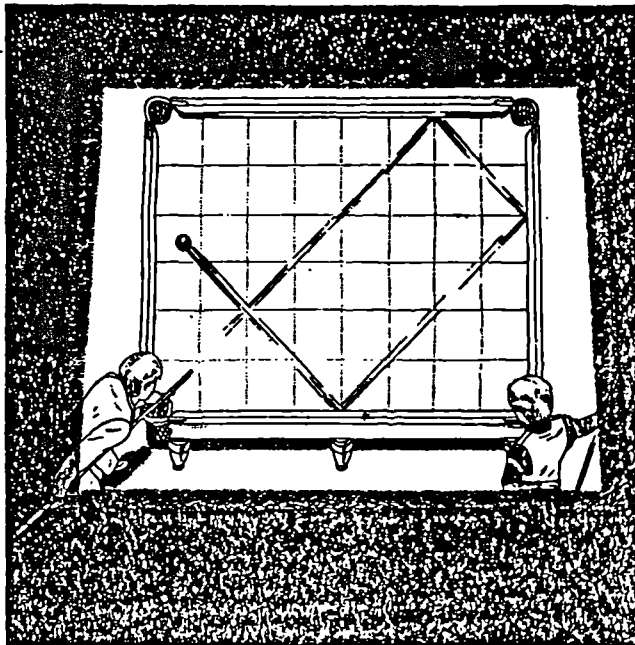
And I think that the relationship with that group is such that we can spend a session talking about what we have found, and one of the things that I will then think about needing to do is to say, well there is this which brings a lot of it together. Maybe it will be with that activity. Maybe it will be me just showing them. I think I will have to judge later on how many of them will reach that stage on their own and how many won't. But I am not going to be totally reluctant in telling them. (Mike, 30.1.87)

In the case of this group it had not been Mike who had told them, but one of their own friends. As a result, it seemed that they understood the relationship, since they were able to explain it to Mike. At the end of the lesson I asked the group whether it had mattered that William had told them the result before they could work it out for themselves, and they appeared not to be concerned about this. Ironically they seemed more concerned about the amount of work they had put into the task, as if this would not have been necessary if someone had told them the result earlier. I was intrigued by this perception. What had they gained from their extensive exploration, and how might this contrast with thinking which would have resulted if they had been presented with the result at an earlier stage? This reminds me of comments by the Amberley teachers, that pupils would prefer to be told what to do, rather than required to think for themselves.

6. *The strange billiard table lesson – introductory sheet*

The strange billiard table

The billiard table in the illustration below is a little odd. It only has four pockets and the base is divided into squares. The rules of the game are a little odd too. Only one ball is used and it is always struck from the same corner at 45° to the sides. (The ball always rebounds at 45° to the sides.)



Below are a few questions to start you off, but remember that an important part of an investigation is asking your own questions.

What would happen if the billiard table was of a different size, say 2×6 , 5×10 , 4×8 , etc?

On some tables the ball will travel over every square. Which tables?

- How many times does the ball hit the side of the table?

Which pocket does the ball fall into?

APPENDIX 5

THE PHASE 3 LESSONS AND RELATED THINKING

CONTENTS	PAGE
1. The Kathy shapes lesson.	54A
2. Offering Jenny the distributive rule	61A

1. The Kathy-shapes Lesson

The following description was written as a result of Phase 3 analysis in June 1989. The lesson itself took place on 28.9.88.

DESCRIPTION OF THE LESSON

After the 2.30 bell on Wednesday, pupils started to arrive at the room on the top floor. It seemed quite a big room until it had about twenty large fourth-year pupils in it. Then it seemed to shrink. When all thirty-two had arrived, it was very full. I had learned to choose my chair and establish my place early if there was somewhere particular that I wished to sit. The room had large windows on two opposite sides. When the sun shone it was extremely bright and the only way to work comfortably was to draw the curtains on the sunny side.

Opening

At the start of this particular lesson, as with most of their lessons at this time of day, the class were bubbly and the room noisy. They had P.E. the lesson before and this affected their mood. I sat at the side of the room, not with any particular group. Ben stood next to the white board which stretched horizontally across the full length of his front wall. When most pupils seemed to have arrived he began.

T Can we have a bit of hush. (pause) It's silence for about a minute because we're still not settled. We haven't been settled for about the last three Wednesdays, so could we just sit in silence - Lesley! - and collect our thoughts together because we're going to do some maths. Can we just sit quietly for a little time to think about what we're going to do.

There were a few coughs, but the class did indeed become silent. After a brief pause, Ben went on,

T OK. Let's just recap what we did last lesson. Last lesson we looked at finding areas of some shapes - and we looked at finding perimeters of different shapes ... And then we said there are such things as *Kathy-shapes*.

Could someone put their hand up and tell me what they remember a *Kathy-shape* was?

Various hands went up, but there was still silence. Ben said, "Rachel?"

Rachel replied, "Shape that has the same area as perimeter".

Ben replied, "Shape that has the same area as perimeter - now hang on - that's what I wrote on the board. Could someone put it into their own words?"

There were a number of suggestions. Someone asked if it had to be a square. Another asked if it had to have two lots of parallel lines. Another person said that it did not. Someone asked Ben a question which I did not hear, and he replied,

T I think any shape will do, any shape. Er, could I just remind you of shapes we looked at last time. We looked at squares; we looked at rectangles; we looked at circles; we looked at triangles; we looked at parallelograms; we looked at trapezia. There are a lot of *other* shapes, aren't there, which haven't got special names, so when we start looking for Kathy-shapes don't just get stuck on *those*. There are other ones.

One boy made a comment, which again I did not capture. (It was very windy outside, and the recorder was not picking up pupils' voices very clearly.) Ben laughed as he replied, "You've found a pattern? There's no answer to that! Write it down so that we can all understand it." The boy making this remark was Colin. He appeared to be one of the most extrovert in the group, and possibly one of the brighter pupils as he was one of the few taking GCSE a year earlier than the rest. Other's in the class laughed along with Ben's comment in a good-natured way, and Colin tipped his head in acknowledgement. No significance was associated with the word *pattern* for me at that stage.

Forming groups

There followed some talk about a forthcoming parents meeting and the need for the group to have some of their work wall-mounted for parents to see. Ben suggested that groups may like to produce examples of Kathy-squares, rectangles, circles, triangles or others.. He went on,

T Now, what I thought we'd do is maybe we'd split into groups so that three or four people could look for each area - maybe look for all the Kathy-squares, and maybe when you've found the Kathy squares you'd like to write why you can't find any more ... So, as we go through, can we just put our hands up how many would like to look at squares.

That's too many. Can we come back to that in a minute. Rectangles? I've got five, Louise, if you join that table, that's a rectangle table. OK? Circles? Would you like to lead a party looking for Kathy circles? [This question was put to a student-teacher who was also observing the lesson.] There you are, who wants to join Mr. Smith in looking for Kathy-circles?

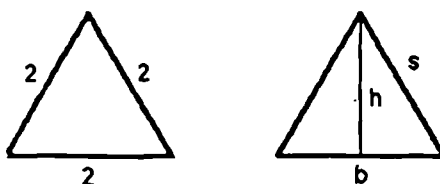
There you are. There's four on there. [The room was getting quite noisy at this stage as pupils indicated what they wanted to do and where they wanted to be. Ben's voice rose.] Who wants - we have a Kathy-triangle! Is it anything like the Bermuda triangle? Let's leave this - who wants to do squares? One, two, three, four, five. Could the squares go on there please. And the others are triangles.

Pupils began to move about and there was quite a lot of noise. Ben remonstrated, "Settle down. If you want to talk to your neighbour about

maths that's great. If I can hear your voice anywhere in the room, you must be too loud".

Vertical Height - a misconception?

Pupils gradually settled to work and the noise abated. Three girls started work on an equilateral triangle of side 2cm. One of them said that she did not know how to find the area. Another, Jenny said, "It's two times two, divided by ...". Ben, who was watching them, interrupted her, "No! Go back. Go back!" Jenny said, "H times b", and Lesley backed her up, "Height times base". Ben said, "Where's the height?"



Lesley pointed to her diagram of the triangle, "That's the height and that's the base". (See sides labelled s and b in the right-hand diagram). Ben said, "What's the height? No, that's not the height. The height is vertical."

The girls conferred, "Up there, from there to there isn't it?" They moved their fingers over the diagram. Ben asked, "So what's the height?"

"Two" was the reply.

Ben pointed to one of the sides of the triangle (the one marked s). "Is that two?" Someone said it was, so he went on, "So, that can't be two can it?" He pointed out the vertical height (side labelled ' h ' in the right-hand diagram). One of the girls said, "It'll be two and a half", and Jenny agreed with her.

I heard in Ben's voice notes of incredulity. I thought - could these girls really think that the height was either two, or more than two? This was surprising. I could have believed that many difficulties might arise in finding the area of a triangle, but it seemed so obvious that if all the sides were of length two, then the height would be less than two. What were they thinking? What concept did they have of vertical height?

They were still talking. Lesley drew another triangle, fairly accurately, and started to measure. Eventually, she said, "The sides are longer than the actual middle of it, the actual height. The height is shorter." Ben said to the others, "Do you agree with that?". They said that they did. He said, "OK. Let's go back to this. If that's two, that's two and that's two, what can you tell me about that then?" He pointed to the height? Jenny said, "It's shorter. It should be one and a half. You take half off the number?"

Ben replied, "If I said I didn't believe you, what could you do to convince me?"

Is there a pattern?

The discussion continued, Ben saying that he was not convinced, the girls starting to agree. Eventually Jenny said, "You can do lots of triangles and measure them and see if there's a pattern." Lesley said, as if reciting a mantra, "IS - THERE - A - PATTERN?" Ben chuckled. "What am I always saying?" The girls chorused, "Is there a pattern?"

Expressing ideas - mathematical challenge

One group in the class had found a Kathy-square. They claimed that there was only one. Ben asked why others did not work, and the reply came from Luke, "Because they don't!" Ben pushed Luke to explain:

Luke Because a square's only got four sides, and there's no number which equals four which times by itself which equals itself times four.

Ben Say that slowly again.

Luke There's no other number that when times by itself is the same as when its times by four.

Ben Why don't you write that down. That sounds a really good reason why there aren't any more - if its true.

Do you understand what he's saying - Danny? No? Luke, you've got a job now to convince Danny!

Ben moved away at this point leaving the boys to continue, but returned about fifteen minutes later.

Ben OK Can you explain it in your own words, what you thought he was saying?

Danny He was saying that four is the only number when times by itself and times by four that will equal the same.

Ben Can you put it in slightly different words?

Danny Umm, if you times four by four you get sixteen and then if you times four by itself you get sixteen.

Ben That's what you just said. I don't understand the difference. Do you understand the difference?

Luke I do. I know

Ben Well, could you tell me then. I don't understand. Danny's not quite sure obviously.

Luke The difference, what the difference is between, what sort of difference ..?

Ben The statement I think he said is that four times four is the same as four times four. That's what I keep hearing, which is obvious isn't it?

Luke Mr West, pick a number.
 Ben Three
 Luke Right. What is three times by itself?
 Ben Nine
 Luke And what is - because a square has got four sides, to find the area, the perimeter, you have to times that by four- three times four?
 Ben Twelve
 Luke But, if you do it with four they both equal the same number, sixteen.

These boys went onto consider other numbers and Ben asked about nought. Luke replied, "Nought times nought is nought. Nought plus nought plus nought plus nought is nought!" Ben said, "He's just found another one!" But Danny objected, "You can't draw nought as a square." They went on to discuss what a square of side nought might mean.

Danny You can't draw nought as a square
 Luke There
 Darren No, that's nought, the whole piece of paper. That's what nought means
 Danny Yeah
 Darren That's not a square
 Luke It is
 Darren It's not
 Danny That length there is not the same as that length there
 Luke Yes it is

Of course it is!

Two other boys were working on pentagons. They wondered if a pentagon of side five would be a Kathy-shape since a square of side four was. One of them said, "Well, because it's got five sides, it's five centimetres. That one's the same principle. That's got four fours are sixteen, because it's four centimetres." Ben said, "So now you're working on to see if it's true?" The other boy said, "No it's not - it's a counter-example!"

The boys went on to talk about fitting squares into pentagons and vice versa. One of them said, "Does a pentagon actually go into a square, if you do by twenty? Ben replied, "You'd have to draw that. I don't understand what you've just said. Draw it and show me."

Working as a group

Ben came back subsequently to the group of girls focusing on equilateral triangles. He suggested that they might consider what happened if they looked at different sizes of right angled triangle. "What happens if you look at a bigger one then, or a smaller one? And there might be a pattern

between those numbers, yes? If you all do the same, it's a waste of time isn't it? So can you get yourselves organised?" In response to this, girls replied, "I'll do three", and "I'll do five".

He then moved on to another group of girls and found that they had in fact already organised themselves in a way similar to that which he had just suggested. One pupil said, "We're doing a number each, instead of the same. I'm doing three by two and three by three, and she's doing four by three." In doing this they had discovered that for certain rectangles the area was smaller than the perimeter, while for others the perimeter was larger. The area and perimeter seemed to swap over near nine by three. Ben's advice was, "Now, question, you've just used whole numbers. Do you think there's one between those two where they are the same? Could you find it do you think?"

ANALYSIS OF THE LESSON

I chose Ben as a teacher to observe because I believed that I should see in operation a teacher committed to and experienced in an investigative style of teaching. My purpose in observation was to identify the practical manifestations, or characteristics, of investigative teaching of mathematics which were evident in his classroom. I chose this lesson, on *Kathy-shapes*, to present a flavour of the characteristics, because it encompasses two important aspects of mathematics teaching. They are, an approach to mathematics through question and enquiry, and an attention to syllabus content in mathematics. The first was evident in the seeking for *Kathy-shapes* and encouraging pupils to determine what shapes they would look for. The second was evident in Ben's declared aims for the lesson which included addressing concepts of area and perimeter, as well as the possibility that opportunity might arise for presentation of the Pythagorean result.

The beginning of this lesson was fairly typical of Ben's lessons. After initial settling, there was often a period of reconstruction of ideas from a previous lesson, and pupils were usually willing to offer contributions. Initial scene setting was often followed by a move into group work of some kind. I observed over many lessons that Ben rarely constructed groups himself. He commented that he wanted pupils to make decisions about how it was most appropriate for them to work. Before the lesson, Ben had said to me that he would probably split pupils into groups according to the shapes they would like to explore. I asked how he thought they would choose and he said, "I think the weaker ones will choose squares because that was my example". In the event too many pupils initially opted for squares, and so Ben dealt with the other shapes first returning to squares when other choices had been made.

Before the lesson I had asked Ben how much guidance he was going to give with regard to finding the *Kathy-shapes*. He indicated that he expected to have the opportunity to talk about processes such as specialising and simplifying. He said that pupils were into 'making conjectures'. He had said to me at some earlier point, "You should ask

them (the pupils) what it is that I'm always telling them to do", and he seemed quite confident as to the reply I should get. It emerged that the instruction was 'look for a pattern', or 'is there a pattern?' and when the girl in the triangles group asked spontaneously "IS THERE A PATTERN" he looked at me and grinned. However, the pupils were so keen on pattern spotting that they came up with patterns even when Ben was hoping to focus their attention elsewhere, as with Colin at the beginning of this lesson. There was plenty of evidence in the lesson of pupils trying out special cases, looking for patterns and making conjectures about the incidence of Kathy-shapes. At various points Ben urged pupils to check or to explain. Being convincing about what they believed, and writing it down were also important aspects of classroom rubric. When observing the work on pentagons, I was struck by one boy's use of the term *counter-example*, which is a quite sophisticated idea in mathematical proof. Mathematical vocabulary was an important aspect of Ben's mathematics lessons and there were many occasions when he took opportunities to introduce pupils to relevant vocabulary.

In our talk before the lesson, Ben had said that the activity might lead into work on Pythagoras theorem, but that he would wait and see. He could not predict where it would lead. In the event he was surprised to find that pupils had difficulties in understanding or in finding the vertical height of a triangle. He said after the lesson,

T It's quite interesting, how many are still insecure about this vertical height. ... They were drawing a triangle and the vertical height was going to be *bigger* than the side of the triangle - and they were convinced!

However, he recognised that it was important to be alerted to such fundamental misconceptions and felt that this was a particular value of allowing pupils to choose their own directions to follow. It is important to consider in what respect the girls' conception of vertical height was a *misconception*. According to the standard definition of vertical height, they were wrong when they said it should be more than two. However, if they had an alternative conception of what *height* meant, then their belief in its being more than two could have been well founded.

In working with groups Ben often asked for clarification of what pupils meant by the words which they used. In some cases it seemed to be for the sake of encouraging clear expression, while in others he seemed genuinely to need their help to understand. With Luke and Danny he said that he did actually understand what Danny meant, but felt it important enough to put emphasis on having an unambiguous statement. However, with the boys working on pentagons he did not understand what they were saying and his asking for a diagram was a genuine need for clarification. It is interesting to consider how pupils themselves perceive such requests for clarification. Subsequent talking with pupils has led me to believe that the pupils were often aware of his motivation on such occasions.

As he worked from group to group Ben sometimes advised pupils and sometimes left them to think about how to proceed. However, his advice

was as often about *how* to work as it was about what to work on. Although he often left pupils very free to work with whom they chose and on what, he often suggested how they might best organise themselves and use each other. I saw his advice to the triangle group bear fruit on a subsequent occasion when they naturally shared out the tasks which were involved in an activity. With the rectangles group he suggested that they might look for numbers in between the whole numbers which they had so far been considering. In many cases he urged pupils to write down what they were thinking. It was interesting to consider when he found it appropriate to give different levels of advice and when not.

2. Offering Jenny the distributive rule

The case study, which follows, was part of an article entitled 'Focus and Emphasis'. This was written in November, 1989, to draw attention to different levels of interpretation which occurred in the analysis of a classroom episode. It has, so far, not been published, but I have extracted the case study to provide further evidence to support analysis in Chapter 7.

A CASE STUDY

A teacher Ben, working with a fourth-year secondary class on GCSE project work, sat down at a table with a group of four girls who were working on their individual projects. They had each chosen their own topic on which to produce a project, but on this table there was considerable overlap between topics and so plenty of scope for discussion and sharing of ideas. One girl, Jenny, had chosen the topic of *picture frames*, and was at the point of developing a general formula for the length of a frame, of unit width, which would surround a given rectangular picture of integer dimensions - for examples see figures 1 and 2 below. Ben entered a conversation with Jenny. The other girls were not directly involved in the conversation, although at least one of them, Lesley, was following it very closely. I was sitting at the table with the group.

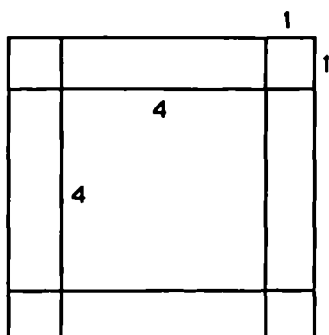


figure 1

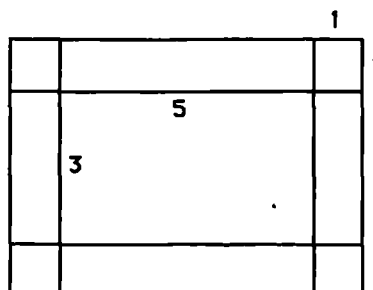


figure 2

Stages of the teacher's role

I shall first focus on the teacher's role throughout the event, and identify a number of stages separated by shifts in the behaviour of the participants. The pieces of transcript which follow came from an audio-recording of the conversation between Ben and the girls. Together they make up the seven minutes on which I report.

Stage 1. Entering into the pupils' thinking

The following piece of transcript consists of the first words between Jenny and Ben.

- 1 Ben Can I sit down? I'm getting old!
- Jen Right, to try to find a way to find a formula, (someone laughs; Ben - 'Yeah?') what I've done is, I've put the signs up here, then crossed out the ones which I ... then found the answer from numbering round the edge ...
- Ben What are you trying to find a formula for?
- Jen A quick formula, like instead of adding on four every time, em like say you wanted to find 200 by 200 you'd have to go all the way up to a 100, - no, - yeah - a 199 to find out wouldn't you? / And add 4 on every time? So, I'm trying to find a formula - (Ben - 'Yes') right? So, to find a formula when 4 by 4 equals 20, which I know because I drew that thing with the numbers round it, - yeah? Now what I've done is taken these 4 sides, then 4 add 4 add 4 that's for the whole thing ...
- 5 Ben Could you just excuse me for one minute ...
(brief interruption while he goes over to other pupils, says something to them, and returns)
- Jen Right, now er
- Ben Hang on, what this for ... - this is a four by four ..
- Jen This is an example, right?, you have to it - read that first ...
- 10 Ben OK, yes, - what's your formula for
- Jen Formula, to find the length of one of these / without drawing it.
- Ben OK, OK, yes.
- Jen So, - what I've done is / taken the example of 4×4 - and I know that equals 20 from doing the diagram.
- Ben Yes.
- 15 Ben So to try and find the formula I've taken the four basic sides.
- Ben Yes.

- 17 Jen And, - set them like that, and then we found out that was the *nearest* so I've left out those there and just doing the first one. And then I've added, looked at the diagram, and I've shown where 16 is, and I've seen that the 4 left are the corners. So, is that a good way of finding out?

At the start Jenny describes some of her own thinking in connection with finding a formula to express the length of frame for a picture of given dimensions. Ben's interjections are few. His questions *seem* to be ones aimed at finding out what she is working on - "What's your formula for?", and other interjections appear little more than acknowledgements that he is there, or of support for her to continue - "OK", "Yes", "Yes". This stage, which was common at the beginning of his interactions with pupils generally, might be characterised as '*entering into the pupil's thinking*'. . It was his custom to listen first. In commenting on this section of the transcript he remarked to me on the length of Jenny's speeches compared to his own. He said, "She's talking more than I am." His tone of voice indicated pleasure in what he observed. His comment reminded me of an earlier experience in which I had shown him a transcript where he had remarked that he was doing much more of the talking than the pupils were, and he had found this both surprising and salutary. In other conversations he had told me that he valued the ability to let the pupils talk and to resist dominating the conversation (and therefore the thinking?) himself.

Stage 2 Starting to focus the thinking

However, '*the ability to let the pupils talk and resist dominating*' implies a detachment where it is possible to observe and control your interaction while interacting. This detachment may be considered a characteristic of the next stage which begins with a shift at line 18.

- 18 Ben Yes I don't know where we're going – what happens if I have a 2 by 5?
- Jen You add 2 and 2 and 5 and 5. And then I've explained here that the formula..
- 20 Ben Hang on, slow down, have you got a bit of paper – let's write down what you've just said – there's too many things going around for me to cope. // So if I say I've got a 2 by 5, yes
- Jen Then to try and find a formula for that I've put $2 + 2 + 5 + 5$, and then I've as well ...
- Jen You see what I'm doing there is trying to find a formula for ...
- Ben How many should it be for 2 by 5?
- Jen 2 by 5? That should be 18.
- 25 Ben What does that add up to?
- Jen That's 4 and that's 10 and add another 4 for the corners – yes.
- Ben So if I've got a 3 by 7, what ...
- Jen ... 24.

- Ben Should be 24? How do you *do* it so *quickly*?
- 30 Jen ... a good brain!
- Ben Come on, tell me how you did it quickly, because obviously you've got a method you're not writing down.
- Jen Oh, well, no this is what, it's explained here – length of side plus length of side plus length of side plus length of side ...
- Ben Yes.
- 34 Jen Plus 4 for the amount of corners, and the total length.

Ben's interjections seem now more focused. In response to a question from Jenny he said, "What happens if I have a 2 by 5?", and later (line 23), "How many should it be for a 2 by 5?", and (at line 25), "What does that add up to?". He focuses her attention first onto a particular case, the 2 by 5 case; then onto 'how many', and then onto the operation of adding up lengths of the components of the frame. His questions may have a number of purposes - for example, exploring whether she can apply her thinking to *any* particular case which he suggests, or of focusing her attention onto specific examples in which he is interested or which have some relevance. Whatever their purpose, they are not unfocused and she would have to be determined to pursue some other line of thought to resist going with the questions. To this extent he is controlling what occurs.

At two points in this stage he acts determinedly to emphasise aspects of learning which he values and wants pupils to value. The first is where (at line 20) he says, "Hang on. Slow down!". My interpretation of this, as I observed the event, was that he genuinely needed to jot down what she was saying in order to understand it. Not so, as he told me later. He seemed rather pleased that I had 'been fooled'. This, he said, was a deliberate strategy to emphasise the value of stopping to write things down when they were possibly too complex all to be held mentally. He wanted his example to make this point. Again (at lines 29 and 31) where he refers to her *doing the calculation so quickly*, he said that he wanted to emphasise to her the value of slowing down sometimes to consider what you are actually doing, to express it in words in order to make your methods more overt. Both of these were deliberate strategies which he was able to employ because he was both acting and controlling his own action.

At the end of this stage Ben's focusing of Jenny's attention seems to become very much more overt.

- 35 Ben Oh, you've now confused me, cause I thought you were trying to shorten *that*!
- Jen Nooo!
- Ben 'cause I think there's an easier, shorter way.
- Jen No, that's me trying to ...

- Les That just means there *is* a shorter way, doesn't it.)¹
- 40 Ben Justify it. Yes there *is* a shorter way.)
- Les There's a shorter way to find it out.
- Jen Yes – the four square ...
- Ben No.
- Jen No.
- 45 Les (aside) Pythagoras comes into everything.
- Ben So you're just trying to explain how you came *to* it.
- Jen Yes.
- 48 Jen Is that all right?

From, "I thought you were trying to shorten that", (line 35) it is as if he tentatively explores whether he should make evident what he sees - that is the existence of a shorter way. Having already hinted at this he decides to go with it, and so states clearly what he thinks - "yes there *is* a shorter way",. This is almost a stage in itself, incorporating the teacher's decision in lines 35 to 40.

Stage 3 A decision to be made

- 49 Ben Yes, yes its OK.
- 50 Jenny Right.
- Ben hm, /// I'm just wondering / I don't know, I just don't know whether it's worthwhile, to be honest. If you write it down, you won't lose any marks, but I'm not sure how many I'd have given.
- Jen So what would you think would be best? To go into 3D cubes – find out, to put the formula to that – or do you think it would be best to investigate the formula more and try to find a shorter way?
- Les I'd try to find a shorter way – a shorter way Jenny.
)
- Ben That's your choice. /
)
- Now, er, I'm going to give you a bit of maths you don't know.
- 55 Jenny Go on then.

The teacher's tone of voice is very important here - highlighting a shift at line 49. The tone indicates uncertainty. He starts to speak more slowly and hesitantly especially in the statement beginning "hm (long pause) I'm just wondering.(pause) I don't know" (at line 51). When I asked Ben himself about them he said that he had no recollection at all of what he had been thinking or intending there - this in complete contrast to the

¹ Brackets like this indicate people speaking at the same time.

deliberate strategies which I mentioned in Stage 2 (recall lines 20, 29, 31). My story for this is that he was totally immersed here in his own thoughts, to do with the stage that Jenny had reached, to do with what help he might give her, and to do with what marks he would subsequently be able to award for her project. This total immersion excluded the possibility of his being able to account for and influence his own actions. I felt that he was thinking aloud, articulating something of a dilemma which he saw, and in consequence his remarks were not totally coherent. He was also unable to recall them later².

I need to add contextual information here. Directly before this lesson, in which pupils were given time to work on GCSE coursework which Ben would eventually mark himself and submit for moderation, he and I had talked about the marking of such work, and of how any help given to the pupil by the teacher might influence this marking. He had indicated that here was a dilemma involving many questions to which he did not as yet have answers. I use the words '*as yet*' deliberately here, and although they are my words and not his, I feel that Ben would continue to work on these questions and would subsequently come up with answers to some of them with which he was satisfied. Indeed I believe that I saw part of this process happen in the subsequent events of his interaction with Jenny and her group, which I shall come to shortly. It was a characteristic of Ben's work more generally that he was continually questioning his actions and seeking ways of becoming a more conscious practitioner.

My reason for going into this contextual information is that Ben was at a decision point here in the lesson. He agrees that this was the case. In becoming acquainted with Jenny's work he could see a possibility of enhancing the quality of what she might ultimately present if he were to point out to her some algebraic strategies that were at her disposal. In order to do this he had to give her access to a particular mathematical law which she had not yet encountered. Should he do it? In response to her questions about possible alternatives he says quietly and almost as an aside "That's your choice." (line 53), then almost immediately, his decision made, he says that he will give her some Maths she doesn't know (line 54).

Stage 4 Exposition

- 56 Ben (to me) This is what we were talking about isn't it? (laughs)
We were talking about this, you know, how much help am I
allowed to give people who do course work. Yes?
Right, now, there's a thing called the Distributive Law – yes?
We've not met the Distributive Law have we?

Jen No.

²See Mason & Davis 1988 for remarks on levels of awareness and the ability to shift between levels.

- Ben The Distributive Law says that, if I've got something like *that*, I can work it out 2 ways really – yes? (Jenny reaches for calculator.)
- Come on, you don't need a calculator – I can add these together – $3 + 4$ is
- Jen Seven.
- 60 Ben times two?
- Jen Fourteen.
- Ben So that's one way – yes? I put 2×7 . The other way says I can do this / – now do you see the pattern for that?
- Jen Ye-es.
- Les You've got 2 on the outside and then you add them together.
- 65 Ben Two times that, and two times that, and you add them together. What's the answer to that?
- Jen Fourteen.
- Ben Gives the same answer. / *Or* you could do it the other way – see – hm.
- Jen Is this the *secret* way?
- Ben Well, yes, but you can work backwards. If I had something like $4 \times 5 + 4 \times 7$ – yes?
- 70 Jen That's got nothing to do with that has it? No.
- Ben I would write that as 4 times – yes? $5 + 7$? couldn't I.
- Jen Yes.
- Ben So that might be useful to you. /// (laugh)
- Les That means you've got to do something with that in your answers (Ben laughs)
- 75 Jen But it doesn't have to be that does it? Hm, what about ...
- 76 Les He's just telling you that you see, but ... that's the answer.

I shall comment on Ben's aside to me (line 56) later as it reflects on the teacher-researcher relationship to which I referred above. However, the next stage begins with the decisively uttered word "Right!" (line 56). It is as if having decided on a course of action, he is in overt control again and ready to get on with it. He launches into exposition. His tone, as I hear it in the recording, now is now almost avuncular, certainly more patronising than it has been in earlier stages. I ask myself whether this is a feature of exposition. Even in this expository stage (lines 56 (from "Right") to 73) he does not totally dominate the conversation. There are plenty of pauses for pupils to think and comment. He certainly dominates the thinking, but Jenny enters into what he offers and appears to make sense of it. Her remark (at line 70) of, "That's got nothing to do with that has it? No.", indicates that she is following Ben's strategy in choosing examples to illustrate the law. There is a use of 'prompt-response', (a form of *clozed* questioning) to draw Jenny into participation (lines 58 and 60), and where

Ben's statements do not actually end with a question, he uses the word 'yes?' to indicate that some response is required (lines 69 and 71).

The pace of events in the different stages contributes strongly to the characterisation of the stage. This is where a transcript is unhelpful. As well as being unable to encompass voice tones or emphasis on words, it is very difficult for it to hold a real sense of pausing, of people talking at the same time, or of speed of diction. This pacing is crucial to the atmosphere prevailing, and so to the sense which an observer or participant makes of the action. In this stage there is a measured feel to interactions, possibly created by the deliberate pauses on the part of the teacher to allow pupils to contribute. Here the teacher's words and actions influence the pace, but nevertheless he allows Jenny some space to give articulation to her sense-making. On the other hand, at the shift from stage 2 to stage 3, brief words and phrases at lines 36 and 37 take up as much time as lines 38 to 47 which go past very quickly, but the transcript cannot convey this information easily.

This stage ends (at line 73) on a different note. Ben says, "So that might be useful to you.", followed by a rather deprecating laugh. It is as if he is now saying - 'There you are! Take it or leave it! He does not wish Jenny to feel constrained to use what he has offered. Without his contribution she could not have used it, hence his decision to offer, but there should be no imperative on her part to make use of it. However, did she in practice have this option?

The pupils' experience

The above paragraphs contain an attempt at characterising the action from the teaching and the teacher's perspective as I saw it. I shall give some thought now to the pupils' experience, both in terms of Jenny herself and in terms of the comments which her friend Lesley offers.

Jenny

It seems reasonable to state that Jenny took an active part in the discussion throughout. She is not a forceful member of the group but has a quiet thoughtful confidence and an ability to express her thinking. Lesley, on the other hand is much more forthcoming, ready to express opinions, to jump in with remarks which she had not had time to consider but yet which often carry a high degree of insight. The event contributed to this view which I have formed of the two girls. Jenny's responses express her mathematical thinking and her involvement in the activity. Her use of words like 'right?' and 'yeah?' (lines 4 and 8) and questions to Ben, such as "Is that a good way of finding out?", (line 16) express a confidence both in her own explanations and her relationship with the teacher. Her responses "No that's me trying to ... " ('justify it' - offered by Ben himself) (lines 38 and 40) and "But it doesn't have to be that does it?", (line 75) indicate some independence in her thinking. After Stage 4 there follows a period of about ten minutes, for which I have not provided transcript, where Ben stays with Jenny, supporting her in exploring the

rule which she has just been given in relation to her own particular examples of picture frames, and tentative attempts to offer a formula. In this time their relationship returns to being almost collegial. The avuncular tone which Ben adopted during the exposition is gone. They are now closer in status - thinking *together* rather than teacher informing. Subsequently Jenny decided to incorporate the distributive law into her project and I read a beautifully explained and expressed exposition of it in her words at a later date. Ben was delighted with this result. It reinforced his decision and provided evidence of positive mathematical outcome from his intervention.

I tried, a week later, to get some retrospective comments from Jenny about the seven minutes by showing her the transcript and asking some questions. For example I was interested in what she had understood by Ben's remarks at line 51, and in the way she felt about his response to her questions at line 52. She had very little to say. It was as if she could not re-enter the situation. Perhaps she could genuinely remember very little of it. Perhaps her absorption in what was happening at the time was too great for her to be able to recall it later. What I was asking of her seemed to leave her bemused - perhaps she could see little sense in it. Perhaps I could have recreated the situation more effectively. This paralleled the teacher's inability to recall his thinking related to his remarks at line 51. There are many questions in here about how to get access to participants' feelings about an event.

As a special case, and without other examples from which to generalise, it is impossible to illustrate how Ben's approach to Jenny in this event is particular to Jenny as an individual as much as it is representative of his overall way of working. In the course of many lessons and interactions with pupils we were able to discuss individual pupils' characteristics and how Ben moderated his approach depending on the pupil and what the pupil was trying to achieve. I believe that Ben's decision of whether to offer the distributive rule here may have gone differently given a pupil other than Jenny, or indeed might not even have exercised him as a decision with some pupils.

Lesley

The comments which at a number of points came from Lesley raised for me some very interesting and important questions. Firstly in response to Ben's indicating that he could see a shorter way, Lesley interjected, "That just means there *is* a shorter way, doesn't it?", (line 39). It is not clear whether this comment is addressed to Ben or to Jenny, or is just thrown into the arena for its own sake. Ben has not yet committed himself to their being a shorter way, he has only suggested it. It is likely that Lesley's comment was the challenge which pushed him into being more assertive - "Yes there *is* a shorter way". However, Lesley goes on, "There's a shorter way to find it out.", (line 41) It could be that this is advice for Jenny who is still trying to justify her earlier generalisation. Whatever its purpose, Lesley's comment is at a meta-level, indicating her interpretation

of what she has heard from Ben. There is another comment in similar vein later (at lines 74 and 76)

73 Ben So that might be useful to you. /// (laugh)

Les That means you've got to do something with that in your answers (Ben laughs)

Jen But it doesn't have to be that does it? Hm, what about ...

76 Les He's just telling you that you see, but ... that's the answer.

Ben's laugh (in line 73) is a recognition of what he sees happening here, and which he referred to, to me, later as "playing games". It is, he suggests, getting perilously close to 'I the teacher have particular intentions at which I shall only hint, but you the pupil must guess what I actually mean by the hints'. Lesley appears to be entering with spirit into this guessing game. However, Ben himself was worried by the thought that he might actually be caught up in such a game. There are important questions here concerning the teaching process.

The teacher-researcher relationship

The issue of the teacher giving help to pupils who are working on GCSE projects which he subsequently has to mark was very much in our minds as Ben and I went into this lesson involving project work. Thus when Ben said to me (line 56), "this is what we were talking about isn't it", I had no doubt as to what he was referring. His further words confirmed this - declaring to the pupils what we had been talking about earlier.

In the ten minutes following the episode of explaining the distributive law, Ben and Jenny continued to discuss how this new law could be related to her current work. Their discussion followed Jenny's focus, but often it was Ben's emphasis which influenced the detail of their thinking. However, as their discussion drew to an end, Ben with a long, drawn out, emphatic "No-ow!", continued, "I've done a hell of a lot here, haven't I? Do you understand it?" Jenny replied, "Yeah! Am I still allowed to include it?", and Ben laughed. He said again, this time to us all, almost jubilantly, "That's what we were talking about".

It seemed that it needed only the slightest trigger to recall for him our conversation from before the lesson. The discussion between Ben and the pupils which followed could have been conditioned largely by this earlier conversation. They discussed whether the help which he had given Jenny was *fair* in terms of GCSE assessment and also in terms of what was given to other pupils. One comment from a pupil suggested that although Ben had made many suggestions it was in fact Jenny who had done all the work. Whether this was what they believed, was what they wanted to believe, or was what they thought the teacher would want to hear, it was nevertheless an opportunity for pupils to think about and comment on an issue which caused the teacher some hard thinking. The situation contained a degree of explicitness which I was coming to expect in this classroom, in terms of the teacher's sharing of his objectives and concerns with his pupils. Of course I had never had access to conversation in the

classroom when I was not present, so I could not say whether this explicitness was particularly influenced by my presence. The pupils probably got used to my asking them questions about their learning and it was possible that they were more alert to this meta-level of thinking when I was around. Ben said on a number of occasions that my presence caused *him* to think more deeply about aspects of his practice than he might otherwise do. There are many questions here to do with the observer's influence on what occurred.